Analytical approach to dynamics of transformed rotating-wave approximation with dephasing

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We study the dynamics of the Jaynes–Cummings model within transformed rotating-wave approximation (TRWA). We analyze this model coupled to a dephasing reservoir, through the Lindblad formalism in the master equation. Then, we examine the expectation value of the number operator. Finally, we investigate the validity of this model under dephasing using the Mandel parameter and the total number of quanta.

Keywords: atom–photon interaction, Jaynes–Cummings model, Mandel parameter

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1. Introduction

The Jaynes–Cummings model (JCM) is a very attractive model proposed in the physics of photon–atom interaction and has long been of interest to physicists. This model with the rotating-wave approximation (RWA), where the antirotating term $g(a^{\dagger}\sigma_{+}+a\sigma_{-})$ is neglected, is described by a Hermitian Hamiltonian which is exactly solvable.[1–3] This approximation is used in the weak coupling regime ($g/\omega \ll 1$) and weak field amplitude.[4,5]

The antirotating term ignored under RWA is wrongly interpreted as a non-conservative term which violates the energy conservation.[1,2,5,6] However, according to

$$\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{1}{\mathrm{i}\hbar}[H,H]+\frac{\partial H}{\partial t}=0, \tag{1}$$

which means that H is time independent; the antirotating term does not change the total energy.

However, it was found later that this term does not conserve the total number of quanta in the system,[7] defined by the operator

$$N=a^{\dagger}a+\sigma_{x}+I. \tag{2}$$

In recent years, much effort has been devoted to numerical solutions of JCM without RWA in the quantum optical literature. Furthermore, a number of researchers have shown that it is possible to obtain an approximate solution to the JCM without RWA.[8–10] On the other hand, the authors in Ref. [11] proposed a transformed rotation wave approximation (TRWA) using a unitary transform. They also claimed that this new approximation has the effect of counter-rotating terms but with a simple mathematical structure of the rotating wave approximation. The main goal of the present paper is to check the validity of the results obtained in the recent work by Gan and Zheng.[11] As we show, the absence or presence of the dephasing, has no effect on the Mandel parameter. While we expect that our results would be affected by noise.

The rest of this paper is organized as follows. In Section 2, we first review the interaction between atom and field in the system and the TRWA model. In Section 3, we study the expectation value of $a^{\dagger}a$ and the Mandel parameter and prove that the number of photons in the presence of dephasing is a decreasing function of time. In Section 4, we present the expectation value of σ_x and study the behavior of the expectation value of σ_x versus time. Finally, we demonstrate that our results are inconsistent with what we expected. Our conclusions are summarized in Section 5.

2. Transformed RWA

In this section, we introduce the transformed rotating-wave approximation (TRWA). It is well known that the Hamiltonian of the JCM consisting of both rotating and counter-rotating terms is given by[11]

$$H=\frac{1}{2}\Omega\sigma_{x}+\omega a^{\dagger}a+\frac{1}{2}g\sigma_{z}(a+a^{\dagger}), \tag{3}$$

where ω and Ω are the field and atomic transition frequencies, respectively and g is the coupling constant, $a(a^{\dagger})$ is the annihilation (creation) operator of the EM field, while σ_i $(i=x,y,z$ is the atomic operator defined as $\sigma_x|e\rangle=|e\rangle, \sigma_x|g\rangle=-|g\rangle$, where $|e\rangle$ and $|g\rangle$ represent the excited and ground states, respectively.

In order to investigate the effect of the counter-rotating terms, we apply

$$s=\frac{g\varepsilon}{2\omega}\sigma_{z}(a^{\dagger}-a) \tag{4}$$

to Eq. (3) and obtain $H'=\exp(s)H\exp(-s)$.[11] Using Eqs. (3) and (4), we obtain

$$H' = H'_0+H'_1+H'_2, \tag{5}$$

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$$H_0' = \frac{1}{2}\eta\Omega\sigma_x + \omega a^\dagger a - \frac{g^2}{4\omega}\varepsilon(2-\varepsilon), \quad (6)$$

$$H_1' = \frac{1}{2}g(1-\varepsilon)(a^\dagger + a)\sigma_z + \frac{1}{2}\eta\Omega\,\mathrm{i}\sigma_y\frac{g}{\omega}\varepsilon(a^\dagger - a), \quad (7)$$

$$H_2' = \frac{1}{2}\Omega\sigma_x\left\{\cosh\left[\frac{g}{\omega}\varepsilon(a^\dagger - a)\right] - \eta\right\} + \frac{1}{2}\Omega\,\mathrm{i}\sigma_y\left\{\sinh\left[\frac{g}{\omega}\varepsilon(a^\dagger - a)\right] - \eta\frac{g}{\omega}\varepsilon(a^\dagger - a)\right\}. \quad (8)$$

If we introduce ε and η as

$$\eta = \exp\left[-\frac{g^2}{2\omega^2}\varepsilon^2\right], \quad \varepsilon = \frac{\omega}{\omega + \eta\Omega}, \quad (9)$$

we have

$$H_1' = \eta\Omega\frac{g}{\omega}\varepsilon[a^\dagger\sigma_- + a\sigma_+]. \quad (10)$$

So far, no approximation is adopted. Here, we neglect H_2', because this term describes double and multi boson transitions which are related to $O(g^4)$. As pointed out in Ref. [11], the exact ground state of Eq. (5) is $|g,0\rangle$ with the energy of the ground state explained by[11]

$$E_g = -\frac{1}{2}\eta\Omega - \frac{g^2}{4\omega^2}\varepsilon(2-\varepsilon). \quad (11)$$

The energy of the excited states are

$$E_{2n+1} = \left(n+\frac{1}{2}\right)\omega - \frac{1}{2}\sqrt{(\omega-\eta\Omega)^2 + g'^2(n+1)} - \frac{g^2}{4\omega^2}\varepsilon(2-\varepsilon), \quad (12)$$

$$E_{2n+2} = \left(n+\frac{1}{2}\right)\omega + \frac{1}{2}\sqrt{(\omega-\eta\Omega)^2 + g'^2(n+1)} - \frac{g^2}{4\omega^2}\varepsilon(2-\varepsilon), \quad (13)$$

where $g' = 2g\eta\Omega/(\omega+\eta\Omega)$. We should notice that all expecting values of parameters in our paper depend on the difference between the atomic transition frequency and the field frequency in a different way. By showing η to be less than 1, we could have introduced $(|\omega - \eta\Omega|)$. This quantity is larger than the difference between the atomic transition frequency and the field frequency that is calculated in the JCM with RWA. Therefore, we prefer to use conditions allowing us to be far from RWA regime, instead of the RWA and its limitations. We use the TRWA, which does not rely on the assumption of near resonance and weak coupling, essentially. Next, in addition to checking the validity of this approximation, we calculate the Mandel parameter.

3. Calculation of the expectation value of $a^\dagger a$ and the Mandel parameter

An interesting subject in physics is to determine whether the behavior of a system is governed by classical or quantum mechanics in various conditions. Experimental observations confirmed the quantum nature of light.[12] In quantum optics, the observation of sub-Poissonian statistics is a clear signature of the quantum nature of light. Authors in Ref. [13] stated that the field in the JCM exhibits many nonclassical effects in time evolution, such as sub-Poissonian behavior. In this direction, the Mandel parameter measures the deviation from the Poissonian distribution and the negative value of this parameter shows sub-Poissonian statistics. On the other hand, Poissonian statistics yields zero value for the Mandel parameter whereas the super Poissonian statistics corresponds to the positive value for it. We define the Mandel parameter,[9,12] which determines the sub- or super-Poissonian behavior of a system as follows:

$$Q(t) = \frac{\langle(\Delta n)^2\rangle - \langle n\rangle}{\langle n\rangle}, \quad (14)$$

where $\langle(\Delta n)^2\rangle = \langle n^2\rangle - \langle n\rangle^2$, and n is the number operator. With this construction explained above, we calculate . First of all, we focus on the time derivation of $\langle a^\dagger a\rangle$ from $|g,0\rangle$. As we know, we can drive the time evolution of the number operator according to

$$\langle a^\dagger a\rangle = \langle g,0|\exp(-s)B\exp(s)|g,0\rangle, \quad (15)$$

with

$$B = A^+\exp(s)a^\dagger a\exp(s)A \quad (16)$$

and $A = \exp(s)\exp(-\mathrm{i}Ht)\exp(-s)$. After some calculations, we obtain

$$B = \exp(\mathrm{i}H't)\left[a^\dagger a - \frac{\varepsilon g}{2\omega}(a+a^\dagger) + \frac{\varepsilon^2 g^2}{4\omega^2}\right]\exp(-\mathrm{i}H't). \quad (17)$$

On the other hand, using the above relations and Eq. (4), the effect of the $\exp(s)$ on the ground state is expressed as follows:

$$\exp(s)|g,0\rangle = \exp\left(-\frac{\varepsilon^2 g^2}{4\omega^2}\right)\left\{\sum_{n=1}^{\infty}\left(\frac{\varepsilon g}{2\omega}\right)^{2n}\frac{1}{\sqrt{2n!}}|g,2n\rangle + \left(\frac{\varepsilon g}{2\omega}\right)^{2n-1}\frac{1}{\sqrt{(2n-1)!}}|e,2n-1\rangle + |g,0\rangle\right\}. \quad (18)$$

Using Eqs. (18), (11)–(13), we obtain

$$\left[a^\dagger a - \frac{\varepsilon g}{2\omega}(a+a^\dagger)\sigma_z + \frac{\varepsilon^2 g^2}{4\omega^2}\right]\exp(-\mathrm{i}H't)\,\mathrm{e}^s|g,0\rangle = \mathrm{e}^{-\varepsilon^2 g^2/8\omega^2}\left\{\sum_{n=1}^{\infty}\left(\frac{\varepsilon g}{2\omega}\right)^{2n}\frac{1}{\sqrt{2n!}}\mathrm{e}^{-\mathrm{i}E_{2n}t} \times\left[\left(2n+\frac{\varepsilon^2 g^2}{4\omega^2}\right)|g,2n\rangle - \frac{\varepsilon g}{2\omega}(\sqrt{2n}|e,2n-1\rangle + \sqrt{2n+1}|e,2n-1\rangle)\right] + \left(\frac{\varepsilon g}{2\omega}\right)^{2n-1}\frac{1}{\sqrt{(2n-1)!}}\mathrm{e}^{-\mathrm{i}E_{2n-1}t}$$

$$\times\left[\left(2n-1+\frac{\varepsilon^2 g^2}{4\omega^2}\right)|e,2n-1\rangle - \frac{\varepsilon g}{2\omega}(\sqrt{2n-1}\,|g,2n-2\rangle+\sqrt{2n}\,|g,2n\rangle)\right] + \mathrm{e}^{-\mathrm{i}E_g t}\left(\frac{\varepsilon^2 g^2}{4\omega^2}|g,0\rangle-\frac{\varepsilon g}{2\omega}|e,1\rangle\right)\Bigg\}. \quad (19)$$

Therefore, inserting Eqs. (16) and (19) into Eq. (15), we obtain

$$\begin{aligned}
&\langle a^\dagger a\rangle \\
&= \Bigg\{\sum_{m=1} -\left(\frac{\varepsilon g}{2\omega}\right)^{4m+2}\frac{1}{2m!} \\
&\times \exp\left[\mathrm{i}t\left(-\omega+\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 m}\right.\right. \\
&\left.\left.+\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2(m+1)}\right)\right] \\
&+\sum_{m=1} -\left(\frac{\varepsilon g}{2\omega}\right)^{4m}\frac{1}{(2m-1)!}2\cos[\sqrt{(\omega-\eta\Omega)^2+g'^2 m}]t \\
&+\sum_{m=2} -\left(\frac{\varepsilon g}{2\omega}\right)^{4m}\frac{1}{(2m-2)!} \\
&\times \exp\left[\mathrm{i}t\left(\omega-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 m}\right.\right. \\
&\left.\left.-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2(m-1)}\right)\right] \\
&-\left(\frac{\varepsilon g}{2\omega}\right)^2 2\cos\left[\frac{\omega+\eta\Omega}{2}-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2(m-1)}t\right] \\
&-\left(\frac{\varepsilon g}{2\omega}\right)^4\exp\left[\mathrm{i}t\left(\frac{\omega+\eta\Omega}{2}-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2}\right)\right]\Bigg\} \\
&\times\exp\left(-\frac{\varepsilon^2 g^2}{4\omega^2}\right)+\left(\frac{\varepsilon g}{2\omega}\right)^2\left[2+2\exp\left(-\frac{\varepsilon^2 g^2}{4\omega^2}\right)\right]. \quad (20)
\end{aligned}$$

Consequently, imposing the initial conditions on Eq. (20), we find the time evolution of $\langle a^\dagger a\rangle$. For instance, we present two different conditions: resonance condition where $\omega=\Omega=1$, $\varepsilon=0.5$, $g=0.1$ (Fig. 1(a)), and condition where $\omega=0.5$, $\Omega=1$, $\varepsilon=0.5$, $g=0.1$ (Fig. 1(b)). Through the ideal system without any damping as shown in Fig. 1, both Figs. 1(a) and 1(b) encompass alternate features.

Essentially, due to the dissipative system, to represent an exact solution of a real physical problem, we pay attention to dephasing. In other words, we have the loss of nonclassicality as a result of decoherence. Decoherence is a phenomenon of the quantum theory where the loss of coherence occurs and the interference effects are lost. The decoherence has irreversibly converted quantum behavior into classical behavior.

By using the quantum trajectory approach,[14] the Hamiltonain of the system under dephasing is described by

$$H''=H-\frac{\mathrm{i}\gamma_{\mathrm{ph}}c^+c}{2}, \quad (21)$$

where $c(c^\dagger)$ is a lowering (raising) operator of a system. In our procedure it is determined by σ_z , and γ_{ph} is a dissipative rate under dephasing. Then, we have

$$H''=H-\frac{\mathrm{i}\gamma_{\mathrm{ph}}I}{2}. \quad (22)$$

Eventually, with the same process, which is used in the previous section, we find

$$\langle a^\dagger a\rangle=\exp(-\gamma_{\mathrm{ph}}t)\langle a^\dagger a\rangle_I, \quad (23)$$

where $\langle a^\dagger a\rangle_I$ is exactly the same as Eq. (19).

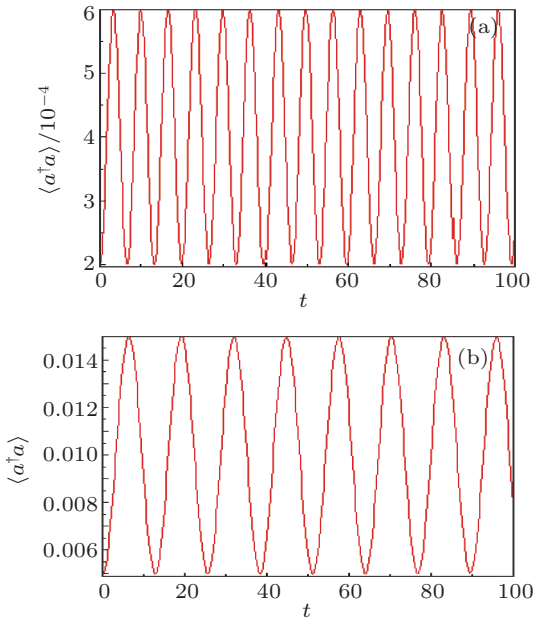


Fig. 1. (color online) Time variations of $\langle n\rangle$ with initial conditions of (a) $\omega=\Omega=1$, $\varepsilon=0.5$, $g=0.1$ and (b) $\Omega=1$, $\omega=0.5$, $\varepsilon=0.5$, $g=0.1$.

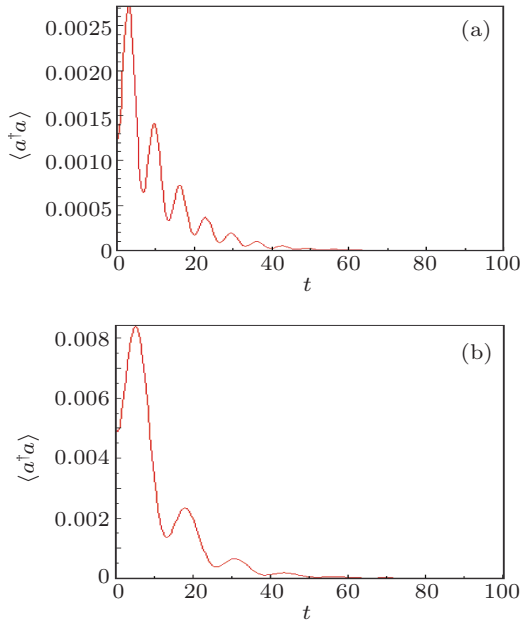


Fig. 2. (color online) Time variations of $\langle n\rangle$ in the presence of dephasing in two different initial conditions: (a) $\omega=\Omega=1$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\mathrm{ph}}=0.1$, and (b) $\Omega=1$, $\omega=0.5$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\mathrm{ph}}=0.1$.

To illustrate the time evolution of $\langle a^\dagger a\rangle$ in the presence of dephasing, we present the schematic of this procedure with

$\gamma_{\rm ph} = 0.1$ in Fig. 2. Note that other parameters are the same as those in Fig. 1. As shown in Fig. 2, the number of photons in the presence of dephasing is a decreasing function of time. This result is not in agreement with the prediction in Ref. [7] in which it was indicated that the JCM without the RWA increases with time.

In the presence of dephasing, for two different initial conditions, Mandel parameter is shown in Fig. 3. As we can see, both systems have a sub-Poissonian behavior ($Q < 0$). In this case, we see that in the presence of dephasing, the quantum correlation appears. Although we expect that, under dephasing, the system leads to the loss of quantum effects. So we believe that in the presence of the dephasing, quantum behavior must be converted into classical behavior, and in this condition, the Mandel parameter yields non-negative value in some times.

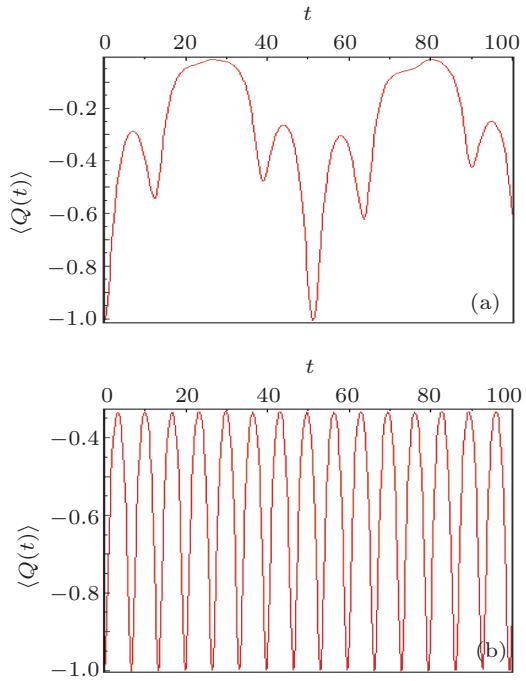


Fig. 3. (color online) Mandel parameters in two different initial conditions: (a) $\omega = \Omega = 1$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$, and (b) $\Omega = 1$, $\omega = 0.5$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$.

4. Expectation value of σ_x

For studying the time evolution of the number of quantas (N), not only does $\langle a^\dagger a\rangle$ play an important role, but also $\langle\sigma_x\rangle$ does. Thus in this section we focus on calculating the time evolution of $\langle\sigma_x\rangle$, essentially.

By using a calculation method similar to that in the previous section, we compute the following expression:

$$\langle\sigma_x\rangle = \langle g,0|\, {\rm e}^{-s} A^+ {\rm e}^{s}(\sigma_x) {\rm e}^{-s} A {\rm e}^{s}\, |g,0\rangle\,, \tag{24}$$

where

$${\rm e}^{s}(\sigma_x){\rm e}^{-s} = D\left(\frac{\varepsilon g}{\omega}\right)\sigma_+ + D\left(-\frac{\varepsilon g}{\omega}\right)\sigma_-, \tag{25}$$

with $D(\beta) = \exp[\beta(a^\dagger - a)]$. Using Eqs. (24) and (25), we obtain

$$\begin{aligned}\langle\sigma_x\rangle \;=\; & {\rm e}^{-\varepsilon^2 g^2/(4\omega^2)}\Bigg\{ {\rm e}^{{\rm i}E_g t}\,\langle g,0| \\ & + \sum_{m=1}\Bigg[{\rm e}^{{\rm i}E_{2m}t}\left(\frac{\varepsilon g}{2\omega}\right)^{2m}\frac{1}{\sqrt{(2m)!}}\,\langle g,2m| \\ & + {\rm e}^{{\rm i}E_{2m-1}t}\left(\frac{\varepsilon g}{2\omega}\right)^{2m-1}\frac{1}{\sqrt{(2m-1)!}}\,\langle e,2m-1|\Bigg] \\ & \times\left| D\left(\frac{\varepsilon g}{\omega}\right)\sigma_+ + D\left(-\frac{\varepsilon g}{\omega}\right)\sigma_-\right| {\rm e}^{-{\rm i}E_g t}\,|g,0\rangle \\ & + \sum_{n=1}\Bigg[{\rm e}^{{\rm i}E_{2n}t}\left(\frac{\varepsilon g}{2\omega}\right)^{2n}\frac{1}{\sqrt{(2n)!}}\,|g,2n\rangle \\ & + {\rm e}^{{\rm i}E_{2n-1}t}\left(\frac{\varepsilon g}{2\omega}\right)^{2n-1}\frac{1}{\sqrt{(2n-1)!}}\,|e,2n-1\rangle\Bigg]\Bigg\}. \end{aligned}\tag{26}$$

Finally, we obtain

$$\begin{aligned}\langle\sigma_x\rangle \;=\; & \Bigg\{\sum_{n=1}\exp[{\rm i}(E_g - E_{2n-1})t]\left(\frac{\varepsilon g}{2\omega}\right)^{2n-1} \\ & \times\frac{\langle g,0|\,D(-\beta)\,|g,2n-1\rangle}{\sqrt{(2n-1)!}} \\ & + \sum_{\substack{m=1\\n=1}}\exp[{\rm i}(E_{2m} - E_{2n-1})t]\left(\frac{\varepsilon g}{2\omega}\right)^{2m+2n-1} \\ & \times\frac{\langle g,2m|\,D(-\beta)\,|g,2n-1\rangle}{\sqrt{(2m)!(2n-1)!}} \\ & + \sum_{m=1}\exp[{\rm i}(E_{2m-1} - E_g)t]\left(\frac{\varepsilon g}{2\omega}\right)^{2m-1} \\ & \times\frac{\langle e,2m-1|\,D(\beta)\,|e,0\rangle}{\sqrt{(2m-1)!}} \\ & + \sum_{\substack{m=1\\n=1}}\exp[{\rm i}(E_{2m-1} - E_{2n})t]\left(\frac{\varepsilon g}{2\omega}\right)^{2m-1+2n} \\ & \times\frac{\langle e,2m-1|\,D(\beta)\,|e,2n\rangle}{\sqrt{(2n)!(2m-1)!}}\Bigg\}\,{\rm e}^{-(\varepsilon g/(2\omega))^2}. \end{aligned}\tag{27}$$

In this study, we note that we need the numbers of initial and final states of photons, in addition to the use of the particularity of Laguerre polynomials. It means that, in the 2nd and the 4th terms of Eq. (27), we should consider the decrease and the increase of photons in each transition. Thus, three different conditions may appear as follows:

$$\begin{aligned} &\text{I)}\quad 2m-1 < 2n \;\text{ and }\; 2m-1 < 2n-2, \\ &\text{II)}\quad 2n > 2m-1 > 2n-2, \\ &\text{III)}\quad 2m-1 > 2n \;\text{ and }\; 2m-1 > 2n-2. \end{aligned}\tag{28}$$

Therefore, assumptions I and III describe the conditions in which the initial and final numbers of quantas are much different. On the other hand, assumption II satisfies the situation that initial and final numbers of quantas are nearly small.

Therefore, in this approach, equation (27) has three different constructions

$$\begin{aligned}\langle\sigma_x\rangle = & \exp\left(-\frac{\varepsilon^2 g^2}{4\omega^2}\right)\left\{\sum_{n=1}\left(\frac{\varepsilon g}{2\omega}\right)^{2n-1}\frac{1}{(2n-1)!}(-\bar{\beta})^{2n-1}\right.\\ & \times\exp\left[\mathrm{i}t\left(-\frac{\eta\Omega}{2}-\left(n-\frac{1}{2}\right)\omega\right.\right.\\ & \left.\left.+\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 n}\right)\right]\\ & +\sum_{m=1}\left(\frac{\varepsilon g}{2\omega}\right)^{2m-1}\frac{1}{(2m-1)!}(\beta)^{2m-1}\\ & \times\exp\left[\mathrm{i}t\left(\left(m-\frac{1}{2}\right)\omega\right.\right.\\ & \left.\left.\left.-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 m}+\frac{\eta\Omega}{2}\right)\right]+K\right\},\end{aligned}\tag{29}$$

where K can be described by assumptions so that if assumption I is satisfied, we have

$$\begin{aligned}K = & \sum_{\substack{m=1\\ n=1}}\left(\frac{\varepsilon g}{2\omega}\right)^{2m+2n-1}\left\{\left[\exp(\mathrm{i}t(M_+-N_-))\right]\right.\\ & \times\frac{(-\bar{\beta})^{2n-2m-1}}{(2n-1)!}L_{2m}^{2n-2m-1}(|\beta|^2)\\ & +[\exp(\mathrm{i}t(M_--N_+))]\\ & \left.\times\frac{(-\bar{\beta})^{2n-2m+1}}{(2n)!}L_{2m-1}^{2n-2m+1}(|\beta|^2)\right\}.\end{aligned}\tag{30}$$

If assumption II is satisfied, then

$$\begin{aligned}K = & \sum_{\substack{m=1\\ n=1}}\left(\frac{\varepsilon g}{2\omega}\right)^{2m+2n-1}\left\{[\exp(\mathrm{i}t(M_+-N_-))]\right.\\ & \times\frac{(\beta)^{2m-2n-1}}{(2m)!}L_{2n-1}^{2m-2n-1}(|\beta|^2)\\ & +[\exp(\mathrm{i}t(M_--N_+))]\\ & \left.\times\frac{(-\bar{\beta})^{2n-2m+1}}{(2n)!}L_{2m-1}^{2n-2m+1}(|\beta|^2)\right\}.\end{aligned}\tag{31}$$

Finally, if the previous conditions are not satisfied, the third condition is tested

$$\begin{aligned}K = & \sum_{\substack{m=1\\ n=1}}\left(\frac{\varepsilon g}{2\omega}\right)^{2m+2n-1}\left\{\left[\exp(\mathrm{i}t(M_+-N_-))\right]\right.\\ & \times\frac{(\beta)^{2m-2n-1}}{(2m)!}L_{2n-1}^{2m-2n-1}(|\beta|^2)\\ & +[\exp(\mathrm{i}t(M_--N_+))]\\ & \left.\times\frac{(\beta)^{2m-2n+1}}{(2m-1)!}L_{2n}^{2m-2n+1}(|\beta|^2)\right\}.\end{aligned}\tag{32}$$

In the above equations, we have

$$M_+ = \left(m-\frac{1}{2}\right)\omega+\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 m},$$

$$M_- = \left(m-\frac{1}{2}\right)\omega-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 m},$$

$$N_+ = \left(n-\frac{1}{2}\right)\omega+\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 n},$$

$$N_- = \left(n-\frac{1}{2}\right)\omega-\frac{1}{2}\sqrt{(\omega-\eta\Omega)^2+g'^2 n}.$$

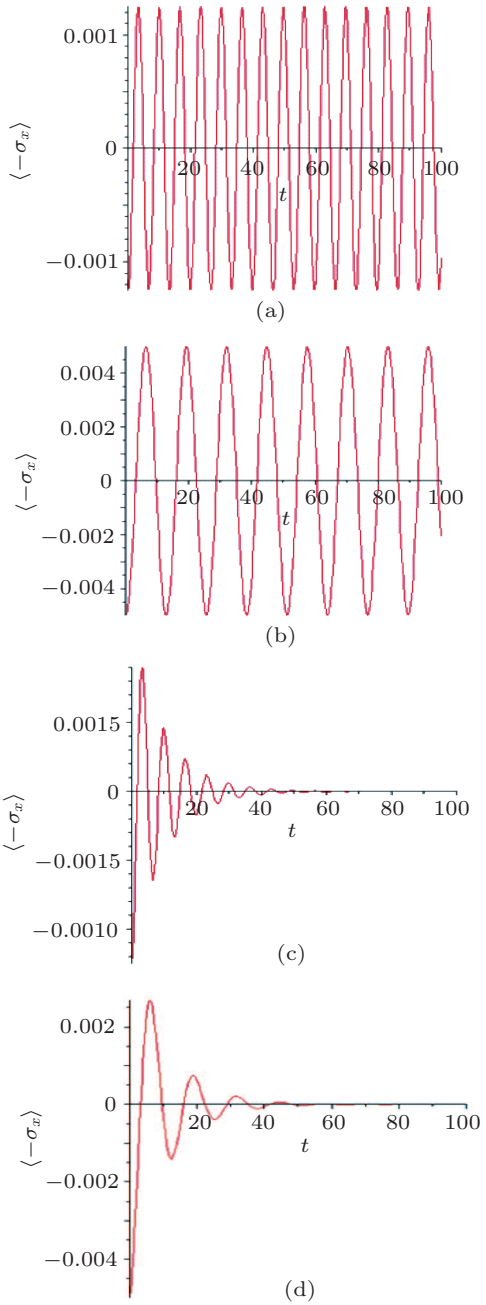


Fig. 4. (color online) Time variations of $\langle-\sigma_x\rangle$ with initial conditions (a) $\omega=\Omega=1$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\rm ph}=0$, (b) $\Omega=1$, $\omega=0.5$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\rm ph}=0$, (c) $\omega=\Omega=1$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\rm ph}=0.1$, and (d) $\Omega=1$, $\omega=0.5$, $\varepsilon=0.5$, $g=0.1$, $\gamma_{\rm ph}=0.1$.

So far, we have presented an analytical solution of $\langle\sigma_x\rangle$. In order to illustrate more clearly, in the presence and the absence of dephasing, $\langle-\sigma_x\rangle$ versus t with different initial conditions are shown in Fig. 4. As an example, we assume $n=1$ and $m=5$.

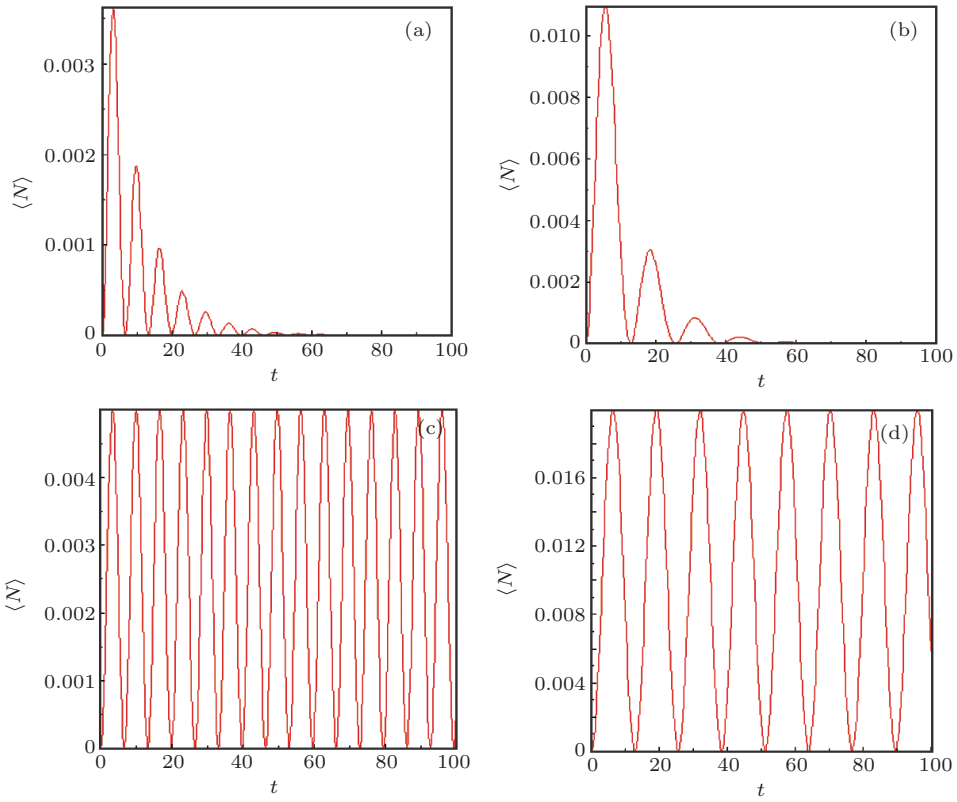


Fig. 5. (color online) Time evolutions of $\langle N\rangle$ in two different Initial conditions: (a) $\omega = \Omega = 1$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$, (b) $\Omega = 1$, $\omega = 0.5$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$, (c) $\omega = \Omega = 1$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0$, and (d) $\Omega = 1$, $\omega = 0.5$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0$.

Finally, in regard to Eq. (2), we present time evolutions of $\langle N\rangle$ (the number of quanta) for two different initial conditions, ($\omega = \Omega = 1$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$; $\Omega = 1$, $\omega = 0.5$, $\varepsilon = 0.5$, $g = 0.1$, $\gamma_{\rm ph} = 0.1$), with and without dephasing in Fig. 5. As we see in Figs. 5(a) and 5(b), the total number of quanta is a decreasing function of time and is not conserved. In other words, the annihilation of the total number of quanta can be viewed as a result of dephasing. On the other hand in the absence of dephasing the number of quanta is conserved.

5. Conclusion

In this paper, using the TRWA used in Ref. [11], we investigate the dynamics of the Jaynes–Cummings model with transformed RWA under dephasing. we also calculate the time evolutions of the number state and σ_x. It is shown that $\langle a^\dagger a\rangle$ in the absence of dephasing is an oscillating function of time, while in the presence of dephasing it has a damping behavior. Finally, considering the definition of the Mandel parameter, we find the plot of this parameter versus time. It is shown that the system has a quantum behavior with and without dephasing. However, as expected, the quantum systems under the noise, due to the decoherence effect, should be close to the classical regime. In addition, we conclude that the total number of quanta in the presence of dephasing is a decreasing function of time and is not conserved. However, in the absence of dephasing, the number of quanta is conserved. Thus, the results obtained here indicate that by using the TRWA, the nature of the system has changed and the results are inconsistent with those of a similar system which is not under the RWA.[7,13]

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