Tunability of graded negative index-based photonic crystal lenses for fine focusing*

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Graded negative refractive index-based photonic crystal (PC) lenses are designed by gradually modifying the sizes of air holes along the transverse direction for focusing the incident plane wave. To study the tunability of the graded negative index-based PC, we introduce filling factor *A*, gradually tune the filling factor, and use the finite-difference and time-domain (FDTD) algorithm for numerical calculation. Our calculation results indicate that the focal length and the spot size increase with *A* increasing. For the same *A* value, the focal length of a PC with elliptical air holes is the longest, and those of PC with square and rectangular air holes are the shortest. Moreover, when the focal length is greater than 1 μm, the focal parameters of the PC are highly insensitive to the variation of *A*. When the focal length is less than 1 μm, the PC lenses have higher transmittances and all well focus with a beam spot size breaking the diffraction limit. This feature possibly makes the graded negative index-based PC lenses have some new applications in optoelectronic systems.

Keywords: photonic crystals, graded negative index, focusing, flat lenses

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1. Introduction

Photonic crystals (PCs) have periodic dielectric artificial structures. PCs are excellent candidate structures for controlling an electromagnetic wave because of their photonic and tunable dispersions within allowed bands. In order to control the wave in a small size, graded structure graded index based PCs have been studied. In the past few years, the graded PCs were explored for focusing,[1–4] mirage,[5] superbending,[5] imaging,[6] beam splitting,[7,8] and waveguides.[9] By appropriate gradual modifications of the lattice constant of PC, the filling factor, material parameters, and the oriented angle of air holes, the graded PC can be achieved.[1–13]

One of the predominant properties of a PC is the negative refraction, due to which a sub-wavelength focused beam spot size can be achieved through appropriate modulations within some frequency regions.[14–20] Luo *et al.*[17] proved that a PC with circular air holes has the all-angle negative refraction behavior. By varying the air-hole radii along the light propagation direction, the PC with an effective negative index of $n_{\text{eff}} = -1$ can be obtained.[14] Sun *et al.*[18] proved that the geometric parameters of the PC element shape can change the effective phase index of the PC structure by greatly changing the shape of the equal-frequency contour. Then they obtained the PC with an effective negative refractive index $n_{\text{eff}} = -1$ by adjusting the widths of the rectangular air holes. However, the PC slab they designed does not possess a focal length and cannot be used to focus the incident plane waves. This drawback greatly influences the PC's further development for practical applications in industry. To overcome this shortage, Wu *et al.*[19] and Zhu *et al.*[20] respectively designed flat lenses, which were constructed by the graded negative index-based PCs with circular air holes. The graded negative index was achieved by varying the air-hole radii along the transverse direction accordingly.

However, modulation via different shapes of air holes was not fully studied in the reports mentioned above. For instance, there are no graded negative index-based flat lenses made of PCs consisting of rectangle air holes, which will be reported in the present paper. Such a structure may provide more options and freedoms for both the design and the fabrication of a PC with significant performances. Considering this, we design hexagonal lattice PCs with different air-hole shapes, i.e., circular, elliptical, square, and rectangular, to investigate and compare their tuning abilities. The PC can act as a lens for finely focusing in free space. Our calculation results demonstrate that they all present negative refraction behaviors. By varying the geometrical parameters of the PC-based lenses in the transverse direction, the PC-based lenses with the graded negative index are derived through our specific design for focusing the incident plane wave in the visible regime. By gradually varying the change rule of the PC's parameters, we investigate the relationship between the change rule and the focal property. It may be useful to design and process flat lenses made of PCs.

2. Design of PC lenses

We construct four types of two-dimensional (2D) hexagonal lattice PCs constituted by the same dielectric medium but with different air-hole shapes. The dielectric medium is sili-

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con. The lattice constant is empirically set to be 150 nm.[20] The air-hole shapes are tuned to be circular, elliptical, square, and rectangular, respectively. The schematic illustrations for the PCs and air holes are shown in Figs. 1(a) and 1(b). The normal direction of the lenses (x direction) is chosen to be along the $\Gamma \rightarrow M$ direction, which is the direction of wave propagation. The transverse direction of the lenses (y direction) is chosen to be along the $\Gamma \rightarrow K$ direction. Here r_1, r_2, r_3, and r_4 are the variable parameters of the air holes.

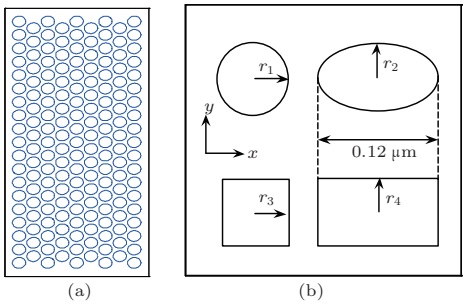


Fig. 1. (color online) Layout of the PC lenses. (a) A schematic diagram of the PC lenses with air holes. (b) Schematic illustrations of four different shapes of air holes: circular, elliptical, square, and rectangular.

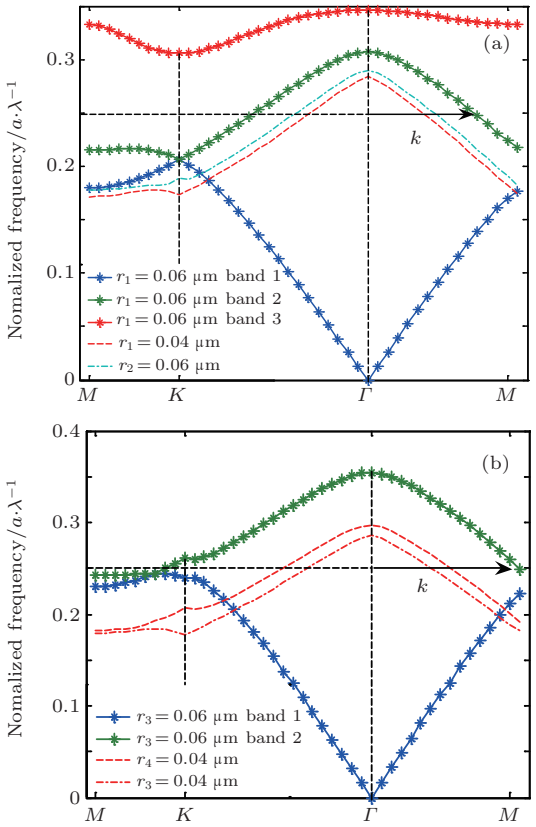


Fig. 2. (color online) Band structures of the designed hexagonal lattice PCs (a) with circular and elliptical air holes and (b) with square and rectangular air holes.

We employ commercial software (FDTD Solution 7.5 from Lumerical Inc.) to calculate the photonic band structures of the designed hexagonal lattice PCs working in the transverse magnetic (TM) mode, and the first three photonic bands are shown in Figs. 2(a) and 2(b). To work in the second photonic band structures for negative refraction, we choose a frequency of $0.25a/\lambda$ as the working frequency, and the corresponding wavelength is 600 nm.

Each PC lens with different air holes can be characterized by an effective refractive index $n = -|\boldsymbol{k}|c/\omega$. To focus the incident plane wave, the sizes of the air holes in the transverse direction vary in such a way that their size is smallest in the center of the PC lens and gradually increases to the edges, as shown in Fig. 3. To make sure that the PCs work in the second photonic band, we change the variable parameter of the air holes ranging from 40 nm to 60 nm following the function

$$r(y) = \frac{A}{2}y^2 + \left(\frac{2}{135} - \frac{675}{1000}A\right)|y| + 0.04. \quad (1)$$

The filling factor A is the squared gradient constant of the air holes. To find the relationship between the value of A and the focal parameter, we change the value of filling factor A from -0.02 to 0.02 in steps of 0.001. The whole functions are shown in Fig. 4. It can be seen that the graded parameter $\partial r/\partial y$ increases with filling factor A increasing. And the variation rule of the graded parameter presents a symmetrical function of y. For the two limit cases of $A = 0.02$ and -0.02, the variation rule of parameter r (changing from r_1, r_2, r_3, to r_4) as a function of y has a reversible symmetry.

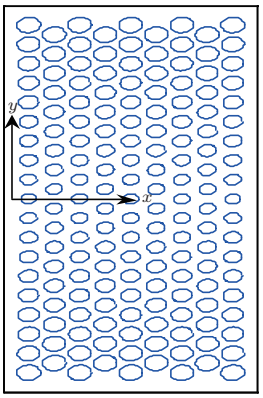


Fig. 3. (color online) Schematic diagram for the graded negative index-based PC lenses with the graded air-hole sizes described by Eq. (1).

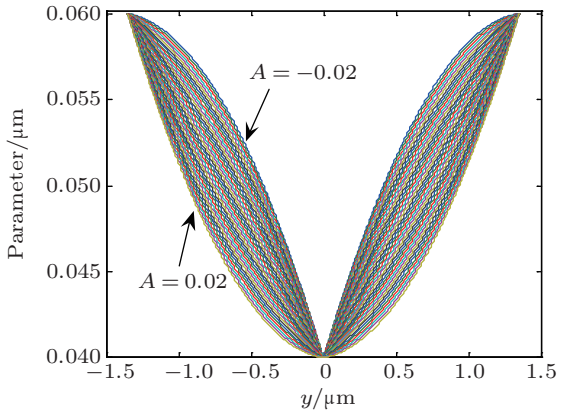


Fig. 4. (color online) Air-hole parameters (r_1, r_2, r_3, and r_4 shown in Fig. 1(b)) as functions of filling factor A, varied in steps of 0.001.

3. Results and analyses

Figure 5 shows the numerical calculation results regarding the focusing properties of the lenses. The focal lengths are measured in terms of the locations where the field intensity along the optical axis is maximum. The spot sizes are derived at the sites of full width and half maximum (FWHM). The amplitudes are the values of maximum field intensity along the optical axis.

With the increase of value A, the focal lengths of the flat lenses with elliptical air holes remain nearly unchanged, but those of others increase subsequently, as shown in Fig. 5(a). It demonstrates that the flat lens with elliptical air holes is insensitive to the hole size variation. Moreover, it can be seen that tuning ranges of the lenses with circular, square, and rectangular air holes are larger than that with the elliptical air holes. For the lenses with square and rectangle air holes, generally, the focal lengths increase as the line-like rule with the increase of filling factor A. For the lenses with circular air holes, the increase of the focal length is so small that it can be ignored, and the value of the focal length has a big change at about A=0.003. The focal lengths of the lenses with square and rectangular air holes are shorter than those of the others. The wave field after the PC lens can be considered as the superposition of propagating waves and evanescent waves. The evanescent wave contains a lot of near field fine structural information, but its propagation length is very short. For flat lenses with elliptical air holes, the focal length is longer than that of the others, and the main factor affecting the wave field is the propagating wave. The propagating wave contains little information about the near field fine structure. It may be the reason why it is highly insensitive to the variation of A. For the lenses with circular air holes, when $A < 0.003$, the amplification effect of the evanescent wave is significant, and the maximum value of the wave field is located in the region where the evanescent wave exists. When $A > 0.003$, the maximum value of the wave field is that of the propagating wave. It may be the reason why the value of the focal length has a big change at about $A = 0.003$. With the increase of A, the spot sizes increase, as shown in Fig. 5(b). The variation trend of the spot sizes is similar to that of the focal lengths. The lens with rectangular air holes presents a strong tuning ability in comparison with the others due to its band structure (see Fig. 2(b)). The lenses with square and rectangular air holes and part of the circular air holes have the ability to break through the diffraction limit, that is, the spot sizes can be smaller than half of the incident wavelength, as shown in Fig. 5(b). Especially for the rectangular air-hole-based lenses, the minimum finely focused spot size can reach as small as 60 nm. The amplitudes of the lenses are shown in Fig. 5(c). For the lenses with circular and elliptical air holes, the variation of amplitude is less than that of the other two cases. The lens with a higher amplitude has a larger transmittance. As described before,[19,20] the negative refraction of the PC lens is mainly determined by band 2 in the band structure, i.e., the main contribution for the negative refraction comes from band 2. As can be shown in Figs. 2(a) and 2(b), the width of band 2 for the cases of square and rectangular air holes is wider than that for the other two cases, and the corresponding negative refraction behavior is stronger accordingly. It may be the reason why the former corresponds to a higher amplitude.

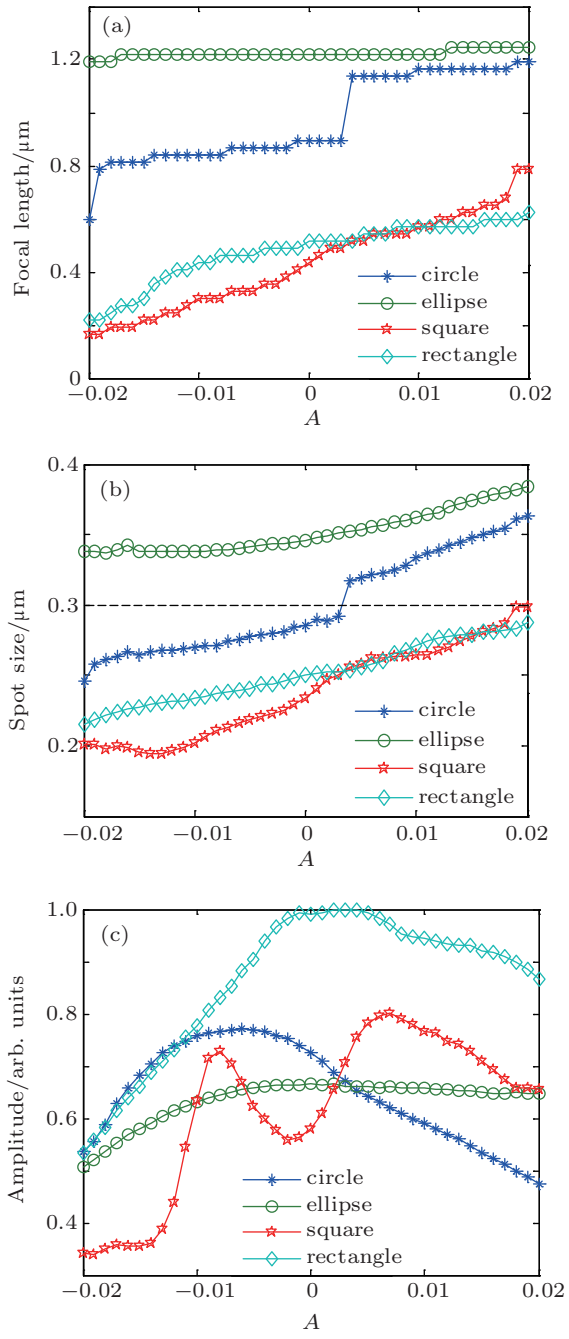


Fig. 5. (color online) Analyses of focusing properties ((a) focal length, (b) spot size, and (c) amplitude) versus filling factor A.

4. Conclusion

In this paper, we design hexagonal lattice PC lenses tuned with different filling factors and air-hole shapes. They can

present negative refraction behavior and focus the incident plane wave behind the lenses. By gradually changing the parameters of the PC lenses, we find the relationship and the variation rule between the value of A and the focal properties. The focal lengths and spot sizes of the lenses with circular and elliptical air holes are larger than those of the lenses with square and rectangular air holes. The focal parameters of the lenses with elliptical air holes are designed to be highly insensitive to the value of A. In the case of the lenses with circular air holes, the variation of focal parameters is small with A increasing. In the case of the lenses with square and rectangular air holes, they can break through the diffraction limit.

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