Effects of three-body interaction on dynamic and static structure factors of an optically-trapped Bose gas*

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We investigate how three-body interactions affect the elementary excitations and dynamic structure factor of a Bose–Einstein condensate trapped in a one-dimensional optical lattice. To this end, we numerically solve the Gross-Pitaevskii equation and then the corresponding Bogoliubov equations. Our results show that three-body interactions can change both the Bogoliubov band structure and the dynamical structure factor dramatically, especially in the case of the two-body interaction being relatively small. Furthermore, when the optical lattice is strong enough, the analytical results, combined with the sum-rule approach, help us to understand that: the effects of three-body interactions on the static structure factor can be significantly amplified by an optical lattice. Our predictions should be observable within the current Bragg spectroscopy experiment.

Keywords: dynamic structure factor, three-body interaction, optical lattice

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1. Introduction

A Bose–Einstein condensate (BEC) trapped in an optical lattice provides a routine platform for investigating many coherence phenomena,[1,2] such as Bloch oscillations,[3–5] elementary excitations,[6] Landau-Zener tunnelling,[7,8] swallow-tail features of the band structure,[9–13] gap solitons,[14,15] and the quantum phase transitions to the Mott insulator phase.[16] Moreover, it is well accepted that such cold atomic gases are a test bed for understanding novel quantum phases and non-equilibrium many-body dynamics.[2]

At the mean-field level, a BEC can be well described by the Gross-Pitaevskii equation (GPE) which is a cubic non-linear Schrödinger equation. In the GPE, the cubic non-linearity models the two-body interaction, which can be tuned conveniently by Feshbach resonance.[17] At the low-density limit, two-body s-wave scattering should be dominant and the three-body interactions can accordingly be neglected. Hence, the GPE with only cubic non-linearity is adequate to describe almost all properties of a BEC at zero temperature.

However, for the case of higher densities, the three-body elastic collisions are expected to play a more important role, as indicated by experiments.[18] The studies on effects of the three-body interaction on the properties of a BEC have followed two lines of directions based on the origin of the interaction. The first line of research focuses on the intrinsic three-body interaction. In general, the three-body interaction arising from triple collisions can be obtained by studying the three-body scattering problem and is characterized by the three-body-coupling constant within the context of pseudo potentials. The theoretical determination of the three-body-coupling constant in a dilute BEC has a long history of research, dating back to 1959 when Wu[19] predicted a general form, $g_2 = 16\pi\hbar^2 a_s^4(4\pi - 3\sqrt{3})\ln(C|\Psi|^2 a_s^3)/m$, for a Bose gas of hard spheres. The constant C has been determined only recently by Braaten and Nieto[20,21] using effective-field theory. This general expression for g_2 has been confirmed by Köhler in a paper[22] in which the explicit three-body contact potential for a dilute Bose gas is derived from microscopic theory.

On the other hand, most of the current research follows the second line, which explores how to identify experimentally accessible systems that have extrinsic three-body or higher interactions. In particular, Büchler *et al.*[23] have reported that polar molecules interacting via dipolar interactions driven by microwave fields can manifest themselves as a Bose–Hubbard model with two- and three-body interactions between nearest neighbors. However, for on-site interactions, Johnson[24] has shown that the two-body collisions of atoms confined to the lowest vibrational states of a three-dimensional optical lattice can generate effective three-body and higher interactions. Among works along this research line, a highlight is the investigations of the Bose–Hubbard model with a three-body constraint, which arises naturally due a dynamic suppression of three-body loss of atoms occupying a single lattice site,[25–29] and can be well realized experimentally.[30]

Regardless whether the three-body interaction is intrinsic or extrinsic, it is clear that the description based on two-body interactions is no longer sufficient[31] in above-mentioned cases and the effects due to three-body interactions must be considered properly.[32–34] In this work, the motivation is to investigate within a mean-field approach, how the three-body

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interactions affect the elementary excitation and the dynamic structure factor of a BEC trapped in a one-dimensional (1D) optical lattice at the mean-field level.

It's well known that a key property of a quantum gas is its excitation spectrum, which is directly accessible in experiments through two-photon Bragg spectroscopy.[35,36] Theoretically, Bragg spectroscopy can be characterized by the dynamic structure factor. Much work[37–41] has been done along this research line, which mainly focuses on how the dynamical structure factor is affected by the two-body interactions and the external tapped potential. To our best knowledge, the investigation of how the three-body interaction modifies the dynamical structure factor is still lacking.

In this paper, we calculate the Bogoliubov band structure and the corresponding dynamic structure factor of a BEC with two-body and three-body interactions loaded in an optical lattice. Our results show that the three-body interactions dramatically modify the Bogoliubov band structure, i.e., they do not only change the lowest band structure but also the higher band. Meanwhile, we also find that the three-body interactions dramatically affect the lowest excitation strength $Z_1(p)$, and play a suppressing role on the lowest-excitation strength and the static structure factor. Finally, arguments based on the sum-rule approach are employed to analytically explain the behavior of the static structure factor at small momenta. The conditions for possible experimental realization of our scenarios are also discussed.

The paper is organized as follows. In Section 2, we present the physical models. In Section 3, the Bogoliubov band structure and dynamic structure factor are considered. In Section 4, we consider the lowest-excitation strength to the lowest Bogoliubov band and the main numerical and analytical results are presented. In Section 5, the static structure factor is studied for different three-body interactions. Possible experimental scenarios and our main conclusions are presented in Section 6.

2. Basic model

We focus on a BEC with both two- and three-body interactions loaded into a 1D optical lattice, where the motion along the radial direction is tightly confined. At sufficiently low temperature, such a BEC system can be regarded as a Hamiltonian system governed by the grand canonical Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{r}\left[\frac{\hbar^2}{2m}|\nabla\Psi(\boldsymbol{r})|^2 + V_{\mathrm{ext}}(\boldsymbol{r})|\Psi(\boldsymbol{r})|^2 + \frac{g_1}{2}|\Psi(\boldsymbol{r})|^4 + \frac{g_2}{3}|\Psi(\boldsymbol{r})|^6\right], \quad (1)$$

with Ψ being the wave function of the condensate. In Eq. (1), the term $g_1 = 4\pi\hbar a_{\mathrm{s}}/m$ represents the two-body interaction at the mean-field level, with a_{s} being the s-wave scattering length and m the atomic mass. The last term $g_2|\Psi|^6/3$ represents the higher-order effects due to the three-body scattering interaction. We like to mention that g_2 is complex in general, i.e., when the three-body interaction is considered, the three-body dissipation should be taken into account properly. In typical experiments for ^{87}Rb, the real part $\mathrm{Re}[g_2]$ and the imaginary part $\mathrm{Im}[g_2]$ are given by $\hbar\times10^{-26}$ cm^6/s–$\hbar\times10^{-27}$ cm^6/s and $\hbar\times10^{-30}$cm^6/s[42–45] respectively, which means $\mathrm{Re}[g_2]/\mathrm{Im}[g_2]\approx10^3$–$10^4$.[46] Therefore, we can safely ignore the imaginary part of g_2 on time scales comparable to the duration of a three-body elastic.

We assume that the BEC is trapped by an external potential given by the sum of a harmonic trap of magnetic origin V_{ho} and a stationary optical potential V_{opt} modulated along the x axis. The resulting potential in Eq. (1) is given by

$$V_{\mathrm{ext}}(\boldsymbol{r}) = \frac{1}{2}m\left(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2\right)+V_0E_{\mathrm{R}}\sin^2(kx), \quad (2)$$

where ω_x, ω_y, and ω_z are the frequencies of the harmonic trap, $k=2\pi/\lambda_{\mathrm{L}}$ is fixed by the wave length of the laser light creating the stationary 1D lattice along the x axis, λ_{L} is the laser wave length, $E_{\mathrm{R}}=\hbar^2k^2/2m$ represents the recoil energy. The optical lattice has periodicity $d=\pi/k=\lambda_L/2$. In the following, we focus the case of $\omega_y=\omega_z\gg\omega_x$. For example, in Ref. [47] the axial (ω_x) and the radial ($\omega_{y,z}$) frequencies are 8.7 Hz and 85.7 Hz respectively; the strength of an optical lattice V_0 can be varied up to $15E_{\mathrm{R}}$ with $E_{\mathrm{R}}=3.9$ kHz. In such a case, the sample is deeply in the 1D regime with less than 1% population of the radial excited state and Ψ in Eq. (1) can be written as $\Psi=\phi_0\psi$, where ϕ_0 is the ground state of the radial motion $\phi_0=\sqrt{1/\pi a_\perp}\exp[-(y^2+z^2)/2a_\perp^2]$ with $a_\perp=\sqrt{\hbar/m\omega_{x,y}}$. Following the standard procedure, we obtain the following quasi-1D GPE

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\left[\frac{m}{2}\omega_x^2x^2+\frac{V_0}{2}\cos(2kx)\right]\psi + \frac{g_1}{2\pi a_\perp^2}|\psi|^2\psi+\frac{g_2}{3\pi^2a_\perp^4}|\psi|^4\psi. \quad (3)$$

All variables in Eq. (3) can be re-scaled to be dimensionless as follows: $\psi\sim\psi/kN$, $x\sim kx$, $\varepsilon=V_0/2E_{\mathrm{R}}$, $c=g_1kN/2\pi a_\perp^2E_{\mathrm{R}}$, $\lambda=g_2k^2N^2/3\pi^2a_\perp^4E_{\mathrm{R}}$. Then, omitting the x-direction harmonic trapping, we can obtain the following normalized and time-independent quasi-1D GPE in an optical lattice with cubic and quintic non-linearities

$$-\frac{\partial^2}{\partial x^2}+\varepsilon\cos(2x)\psi+c|\psi|^2\psi+\lambda|\psi|^4\psi=\mu\psi. \quad (4)$$

Here, μ is the chemical potential. For convenience of the following numerical calculation, we choose $g_2\approx\hbar\times10^{-26}$ cm^6/s, $a_\perp\approx2$ μm, $\lambda_{\mathrm{L}}=800$ nm, and $N\sim4000$–5000. With these parameters, the three-body interaction is calculated to be with a value of $0<\lambda<1$.

Based on Eq. (1), the linearized time-dependent GPE can straightforwardly derived following the usual procedure. In particular, the elementary excitations of the model system can be captured by using the following two Bogoliubov equations:

$$\left[-\frac{\partial^2}{\partial x^2}+\varepsilon\cos(2x)+2c|\psi|^2+3\lambda|\psi|^4-\mu\right]u_{jq}$$
$$\left(c\psi^2+2\lambda\psi^2|\psi|^2\right)v_{jq}=\omega_j(q)u_{jq}, \tag{5}$$

and

$$\left[-\frac{\partial^2}{\partial x^2}+\varepsilon\cos(2x)+2c|\psi|^2+3\lambda|\psi|^4-\mu\right]v_{jq}$$
$$+\left(c\psi^{*2}+2\lambda\psi^{*2}|\psi|^2\right)u_{jq}=-\omega_j(q)v_{jq}, \tag{6}$$

where ψ is the ground-state solution of Eq. (4) and the Bogoliubov amplitudes u_{jq} and v_{jq} satisfy the normalization and orthogonality condition $\int_{-\pi/2}^{\pi/2}(u^*_{\sigma'}u_\sigma-v^*_{\sigma'}v_\sigma)\mathrm{d}x=\delta_{\sigma',\sigma}$. Equations (5) and (6) are two linear equations and exhibit the usual translation symmetry for periodic problems. By applying Bloch's theorem for the solutions u_{jq} and v_{jq}, i.e., $u_{jq}(x)=\exp(\mathrm{i}qx)\tilde{u}_{jq}(x)$ and $v_{jq}(x)=\exp(\mathrm{i}qx)\tilde{v}_{jq}(x)$ with q being the quasi-momentum of the excitation, $\tilde{u}_{jq}(x)$ and $\tilde{v}_{jq}$ have the same periodicity as the optical lattice in space. For each value of q in the first Brillouin zone, an infinite set of eigenfunctions (ω_j) are found, forming a Bogoliubov band labeled with j. Variation of q provides the energy bands ω_j for the elementary excitations of the system. We will discuss the elementary excitation and the dynamic structure factor of the system by adjusting the system's parameters, which include the two-body interactions c and three-body interactions λ for a fixed optical-lattice depth ε.

3. Bogoliubov band structure and dynamic structure factor

As the first step, we plan to calculate numerically the lowest three Bogoliubov bands taking three-body interactions into account. To this end, we first need to obtain the ground-state wave function of ψ in Eqs. (5) and (6), which is a Bloch state at the center of the Brillouin zone. In fact, there are several numerical methods to find the Bloch wave ψ. The method adopted in this work is to expand ψ in a Fourier series and to minimize the Hamiltonian (1) in space to obtain the Fourier coefficients. Once the function ψ is obtained, we proceed to numerically solve Eqs. (5) and (6) and plot the lowest Bogoliubov bands in Fig. 1.

As shown in Fig. 1, we choose to fix the values of both the lattice depth ε and the two-body interaction c and then investigate how the three-body interactions λ affect the lowest three Bogoliubov Bands as follows: (i) Since sound speed can be regarded as a long-wavelength response of a system to a perturbation, the sound speed can be extracted from the slope of the lowest Bogoliubov band at $p=0$. It is clearly seen from Fig. 1 that the sound speed of the system is largely increased with λ increasing. (ii) The higher the Bogoliubov bands are, the smaller is the influence of λ. This suggests that, for an optically trapped BEC, the three-body interaction plays a more important role in the lowest energy excitation than in the higher ones.

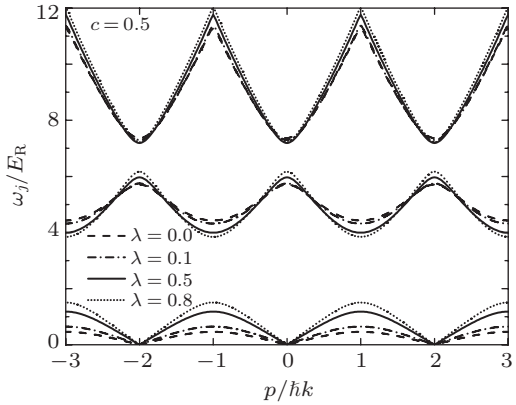


Fig. 1. The lowest three Bogoliubov bands with lattice depth $\varepsilon=5$ for different values of λ.

Next, we focus on how the three-body interaction affects the dynamic structure factor of our model system. In general, the dynamical structure factor characterizes the density-density response of a system to an external probe, delivering a momentum p and an energy ω. We suppose that there is an external density probe which transfers momentum $\boldsymbol{p}=p\boldsymbol{e}_x$ ($\boldsymbol{e}_x$ is the unit vector along the lattice direction) and energy ω. In the linear response theory, the corresponding dynamic structure factor of an optical trapped BEC can be written as

$$S(p,\omega)=\sum_j Z_j(p)\delta[\omega-\omega_j(p)]. \tag{7}$$

The weight factors $Z_j(p)$ is the excitation strength to the j-band and $\omega_j(p)$ are the corresponding excitation energies. Note that while the excitation frequency $\omega_j(p)$ is a periodic function of p, this is not true for the excitation strength $Z_j(p)$.

Before any further concrete calculations, we use the sum rule approach to analyze the basic properties of the dynamic structure factor. The simplest sum rule by the integral of the dynamic structure factor $S(p,\omega)$ gives the static structure factor $S(p)$[40]

$$S(p)=\int S(p,\omega)\mathrm{d}\omega. \tag{8}$$

As we will see later, $S(p)$ is strongly affected by the combined presence of two-body interactions, three-body interactions and the optical lattice. The dynamic structure factor in Eq. (7) also obeys the f-sum rule

$$\int\hbar\omega S(p,\omega)\mathrm{d}\omega=\frac{p^2}{2m}. \tag{9}$$

Here, the f-sum rule (9) is a direct consequence of particle conservation of the model system and represents a statement

of the conservation law. Another important sum rule is the compressibility sum rule

$$\lim_{p\to 0}\int \frac{S(p,\omega)}{\hbar\omega}\,\mathrm{d}\omega = \frac{\kappa}{2}, \tag{10}$$

with $\kappa = [n(\partial\mu/\partial n)]^{-1} = 1/(m^* v^2)$ being the thermodynamic compressibility. Here, v represents the sound speed of the system and m^* is the effective mass. We emphasize that our following numerical calculations have been double-checked by checking whether they satisfy the f-sum rule (9).

Now, we are well equipped to study the combined effects of the two-body interaction c, the three-body interaction λ, and the optical lattice ε on $S(p,\omega)$. For this purpose, we concentrate on two scenarios: first, we set the optical lattice strength to zero and find the dynamic structure factor in the sole presence of interactions. Then, we turn on all three parameters and study their combined effect on the dynamic structure factor.

In the first scenario, where the lattice is absent, after some straight forward calculation, the excitation spectrum is $\omega_{\mathrm{B}}(p) = \sqrt{p^2[p^2 + 2(c+2\lambda)]}$. In this a case, the static structure factor obeys the Feynman relation as follows

$$S_{\mathrm{B}}(p) = \frac{p^2}{\omega_{\mathrm{B}}(p)}. \tag{11}$$

The value of $\omega_{\mathrm{B}}(p)$ increases with in increase of λ. Hence, we can see from Eq. (11) that the three-body interactions play a suppressing role on the static structure factor. In Fig. 2, we show the relationship between $S(p)$ and p. It is clearly seen that the three-body interactions modify the static structure factor dramatically. Our numerical results (the discrete dots on each line) in Fig. 2 exactly coincide with the analytical results.

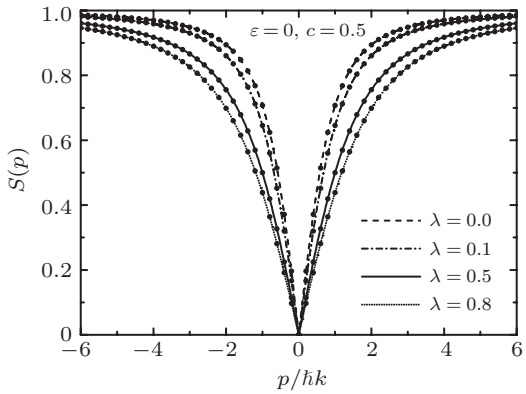


Fig. 2. Analytical results of the static structure factor $S(p)$ in a uniform Bose gas according to Eq. (11) for different values of λ. The dark dots on each line correspond to the numerical results.

In the second scenario, the optical lattice is turned on. In this case, the behavior of the dynamic structure factor will change in a drastic way. For a fixed value of the momentum transfer p, it is possible to excite several states, corresponding to different bands. An important consequence is that, on one hand, with a small momentum transfer p, it is possible to excite high Bogoliubov bands and, on the other hand, that the lowest Bogoliubov band can also be excited by the high momentum transfer p that outside the first Brillouin zone.

In general, the dynamic structure factor has to be calculated numerically. Starting from the solutions of Eqs. (5) and (6), the excitation strength Z_j is to the j-th band for a given momentum transfer p:

$$Z_j(p) = \left| \int_{-\pi/2}^{\pi/2} [u_{jq}^*(x) + v_{jq}^*(x)]\, \mathrm{e}^{\mathrm{i}px}\psi(x)\,\mathrm{d}x \right|^2, \tag{12}$$

where q belongs to the first Brillouin zone and is fixed by the relation $p = q + 2l$ with l an integer. This equation shows that, by solving Eqs. (5) and (6) within the first Brillouin zone, one can also calculate the strength $Z_j(p)$ for values of p outside the first Brillouin zone.

4. Excitation strength to the lowest Bogoliubov bands

We first discuss the behavior of the first excitation strength $Z_1(p)$. From Fig. 3, it is clear that the three-body interactions modify the amplitude of $Z_1(p)$ dramatically, especially if the two-body interactions are small (Fig. 3(b)). In this case, $Z_1(p)$ responses to the three-body interactions in a sensitivity way. However, when the two-body interactions are relatively large, the effects arising from the three-body interactions are relatively small (Fig. 3(a)). Meanwhile, Fig. 3 shows some common properties: two-body and three-body interactions all play an overall suppressing role on $Z_1(p)$, and $Z_1(p)$ exhibits characteristic oscillations. The zero of $Z_1(p)$ at $p = 2l$ (l integer) directly reflects the phonon behavior of the Bogoliubov band, which vanishes at the same value as shown in Fig. 1.

When the optical lattice is deep enough, it is possible to analytically study the behavior of $Z_1(p)$ in the tight-binding approximation. In this limit, one can approximate the solutions of Eqs. (5) and (6) as follows:

$$u_q(x) = U_q \sum_n \mathrm{e}^{\mathrm{i}qn\pi}\phi(x - n\pi), \tag{13}$$

and analogously for $v_q(x)$. Here, $\phi(x)$ is a function localized near the bottom of the optical potential, and n labels the potential wells. Moreover, in the tight-binding limit, the dispersion law of the lowest band takes the Bogoliubov form[48]

$$\omega(p) = \sqrt{\varepsilon(p)[\varepsilon(p) + 2\kappa^{-1}]}, \tag{14}$$

with

$$\varepsilon(p) = 2\sigma \sin^2\left(\frac{p}{2}\right). \tag{15}$$

In the above equations, $\varepsilon(p)$ is the lowest Bloch band, describing the energy per particle of a condensate with quasi-momentum p and σ is the tunnelling rate between adjacent sites. The parameter κ labels the compressibility which can be obtained as $\kappa^{-1} = (c+2\lambda')\int_{-\pi/2}^{\pi/2}\phi^4(x)\mathrm{d}x$ and $\lambda' \sim 0.1\lambda$, as discussed in Ref. [48].

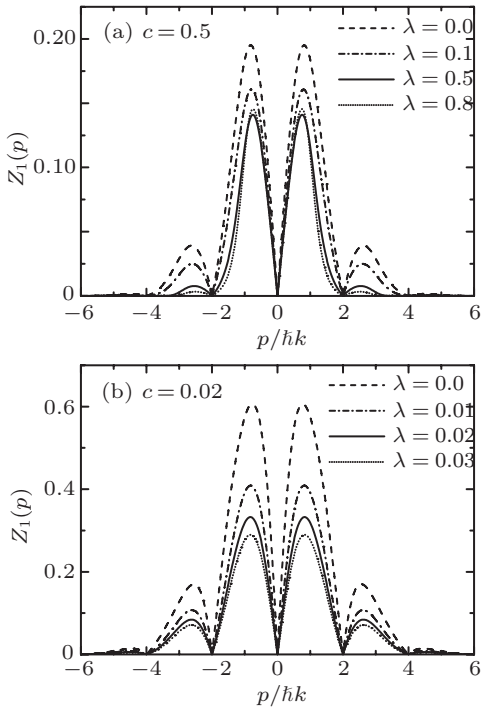


Fig. 3. $Z_1(p)$ for different three-body interactions with $\varepsilon = 5$ for (a) $c = 0.5$ and (b) $c = 0.02$.

We choose $\phi(x)$ with the Gaussian $\phi(x) = \exp(-x^2/2\zeta^2)/(\pi^{1/4}\sqrt{\zeta})$ to obtain an analytical result of $Z_1(p)$. After some tedious calculations, we obtain the following result:

$$Z_1(p) = \frac{\varepsilon(p)}{\omega(p)}\exp\left(-\frac{\zeta^2 p^2}{2}\right), \tag{16}$$

which corresponds to the excitation strength to the first Bogoliubov band. Here, the width ζ can be obtained numerically by minimization of the ground-state energy and behaves like $\zeta \sim (2\varepsilon)^{-1/4}$ for $2\varepsilon \gg 1$, which is also shown in Ref. [40]. Equation (16) produces numerical results with good accuracy obtained by solving the Bogoliubov equations for relatively deep optical lattices. From Eq. (16), the oscillating behavior of $Z_1(p)$ (see Fig. 3) arises from the term $\varepsilon(p)/\omega(p)$, and the suppression of the strength at large p through the Gaussian term. For fixed two-body interactions c, it is clearly shown that the term $\varepsilon(p)/\omega(p)$ decreases with increase of the three-body interactions (λ), due to the fact that the term κ^{-1} in $\omega(p)$ monotonously increases with λ, as shown in Figs. 3(a) and 3(b). Especially, if we tune the two-body interactions very small, the three-body interactions will play a dominate role, as shown in Fig. 3(b). The strength Z_1 has a maximum close to the edge of the first Brillouin zone, where it approximates the value $Z_1 \approx \sqrt{\kappa\sigma/(\kappa\sigma+1)}$. We see that Z_1 is quenched both by increasing two-body and three-body interactions ($\kappa \to 0$). This tendency is clear when we compare Fig. 3(a) with Fig. 3(b). The influence of three-body interactions on $Z_1(p)$ could be measured in Bragg spectroscopy experiments. In order to observe three-body-interaction effects, we need to tune the two-body interactions to be relatively small, if the two-body interactions are large, the influence of three-body interactions will be masked.

5. Static structure factor in an optical lattice

In the presence of an optical lattice, the static structure factor in Eq. (8) is obtained by summing up the excitation strengths $Z_j(p)$ over all bands as follows:

$$S(p) = \sum_j Z_j(p). \tag{17}$$

The corresponding results are presented in Figs. 4(a) and 4(b) for different values of the two-body interactions. When the two-body interactions are very weak (Fig. 4(b)), even presence of a small three-body interaction will lead to a great modification of the static dynamic structure, which mainly comes from the contribution of $Z_1(p)$. In contrast, the effect of three-body interactions is less pronounced for larger value of the two-body interactions.

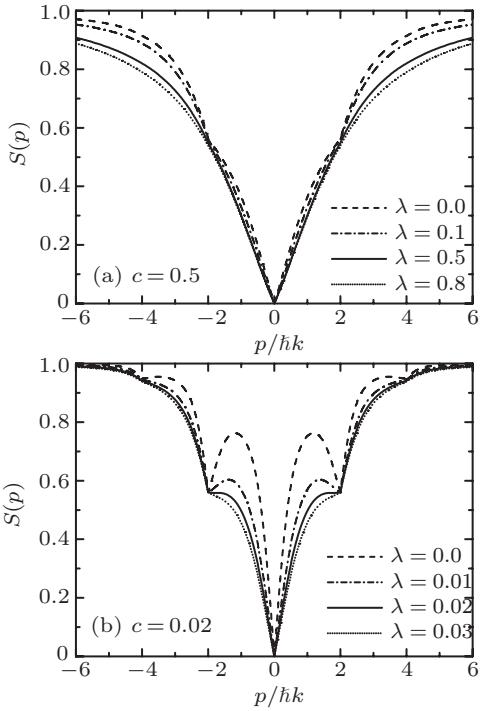


Fig. 4. Static structure factor $S(p)$ for different three-body interactions with $\varepsilon = 5$ for (a) $c = 0.5$ and (b) $c = 0.02$.

As discussed in Ref. [40], the behavior of $S(p)$ for small momenta can be described exactly using sum-rule arguments from Eqs. (8)–(10). The result is that in the presence of two-body and three-body interactions, the low p behavior of the

static structure factor behaves like

$$S(p) \sim \frac{|p|}{2m^*v}, \tag{18}$$

where $m^*v = \sqrt{m^*\kappa^{-1}}$. For a relatively deep optical lattice, as discussed in Ref. [49], the effective mass m^* approximately does not depend on the number of atoms, nor on the mean-field interaction. However, κ^{-1} increases as the three-body interactions become stronger, so that $S(p)$ decreases with the increase of three-body interactions. This tendency is clearly seen in Fig. 4.

6. Possible experimental scenarios and conclusions

This work is based on an extended GPE that highlights the competition between the two-body and three-body interactions. Particularly, the physics of our model is characterized by the interplay of three parameters: the two-body s-wave interaction c, the three-body interaction λ, and the lattice strength ε. With the state-of-the-art technologies, all these parameters can experimentally be controlled almost at will. First, the interatomic interactions can be turned in a very versatile manner via the technology of Feshbach resonances, whereas the depth of an optical lattice can be changed from shallow to sufficiently strong. Second, there has been progress in identifying experimental accessible system exhibiting three-body interactions. In particular, in Ref. [23], it has been pointed out that, by loading polar molecules into the optical lattice and adding an external microwave field, one can achieve a situation where only observable three-body interactions among neighboring atoms exit. In addition, for on-site interactions, Johnson[24] has recently shown that there are effective three-body collisions of atoms confined to the lowest vibrational states of a three-dimensional optical lattice. Finally, Bragg spectroscopy has offered the possibility of studying the full momentum structure of the excitations of a BEC in an optical lattice.[50] Consequently, the phenomena discussed in this paper should be observable with the current experimental possibilities.

In summary, we have found that three-body interactions can dramatically change the elementary excitation spectrum and the corresponding dynamic structure factor of a BEC loaded in an optical lattice. In particular, our results show that the three-body interactions dramatically affect the lowest excitation strength $Z_1(p)$, and play a suppressing role on the lowest excitation strength and the static structure factor. Moreover, based on the sum rule approach, the behavior of the static structure factor at small momenta is analytically explained. We hope that our study will help Bragg scattering experiments to probe these important phenomena induced by the three-body interactions.

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