Comparison of entanglement trapping among different photonic band gap models*

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(Received 6 December 2012; revised manuscript received 19 March 2013)

We investigate the roles of different qubit–environment decoherence models on the entanglement trapping of two qubits. By considering three environmental models (the single photonic band gap model, the common photonic band gap model, and the two independent photonic band gaps model), we note that the final values of entanglement trapping are determined by these different models. We also give the conditions of obtaining the larger entanglement trapping by comparing two-qubit entanglement dynamics in different decoherence models. Moreover, the comparison of entanglement trapping between two Bell-like states in the same decoherence model are also carried out.

Keywords: entanglement trapping, quantum decoherence, photonic band gap, pseudomode method

PACS: 03.65.Ta, 03.65.Yz, 03.67.Mn, 05.70.Fh **DOI:** 10.1088/1674-1056/22/9/090307

1. Introduction

Realistic quantum systems are affected by decoherence and entanglement losses because of the unavoidable interaction with their environments.[1] For example, in Markovian environments, in spite of an exponential decay of the single-qubit coherence, the entanglement between two qubits may completely disappear at a finite time.[2–5] This phenomenon, known as entanglement sudden death and experimentally proven to occur,[6,7] limits the time when entanglement can be exploited for practical purposes. Thus, the issue of how to preserve quantum system entanglement in a realistic physical system is especially important. Up to now, several ways have been proposed for keeping the atomic entanglement for a long time by suppressing spontaneous emission. One widely applied method is to place the qubits in a structured environment, e.g., a microcavity.[8,9] Other methods that use dynamic manipulation, such as mode modulation[10] or the quantum Zeno effect,[11] can be regarded as extensions of the so-called bang–bang method.[11]

Besides the above methods, Refs. [12]–[15] examined the possible conditions that lead to preservation of entanglement by placing the atoms in the photonic band gap. Among realistic physical situations, this scheme is known to occur for atoms embedded in the band gap of photonic crystal materials as an environment,[16] and high values of entanglement trapping can be achieved. Such a system is easily and realistically manipulated to perform quantum computation and quantum information processing. From Refs. [14] and [15], it is interesting to find that entanglement can be trapped in the band-gap environmental models.

In this article, we will concentrate on the question: what kind of local qubit–environment decoherence model can result in larger entanglement trapping in the dynamical process? We consider a noninteracting two-qubit system under the influence of the photonic band gap environments. Three qubit–environment models (the single photonic band-gap model, the common photonic band-gap model, two independent photonic band-gaps model) are studied. By analytical and numerical analysis, we find that the initial entanglement between two qubits via these three different decoherence models all would display entanglement trapping. By comparing the roles of different decoherence models on the atomic entanglement trapping for the same initial entangled state, we examine the conditions in which larger entanglement trapping can be achieved. Moreover, we also compare the entanglement trapping of two Bell-like states $|\Phi\rangle = \cos\theta|\mathrm{eg}\rangle + \sin\theta|\mathrm{ge}\rangle$ and $|\Psi\rangle = \cos\theta|\mathrm{ee}\rangle + \sin\theta|\mathrm{gg}\rangle$ in the same photonic band-gap model.

The pseudomode approach[17–20] is applied in this paper. It is exact and does not rely on either Born or Markov approximation, and it is still possible to solve the dynamics without any approximation. The analytic solution presented by using this approach in this paper is valid for a general initial state of the two-atom system.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61178012 and 11247240), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20123705120002), the Open Project of State Key Laboratory of Crystal Material in Shandong University, China (Grant No. KF1103), the Natural Science Foundation of Shandong Province, China (Grant No. ZR2012FQ024), the Youth Funds from Qufu Normal University, China (Grant No. XJ201219), and the Scientific Research Foundation for Doctors of Qufu Normal University, China (Grant No. BSQD20110132).

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2. Theoretical model

Consider a system composed of two initially entangled atoms A and B, in the ground and excited states $|\mathrm{g}\rangle$ and $|\mathrm{e}\rangle$, with a transition frequency of ω_0. Due to unavoidable interaction with the environment, the open system is affected by decoherence and entanglement losses. In this paper, we start by considering three different atom–environment models, which we refer to as model 1, model 2, and model 3. In the model 1, only one of two atoms is coupled with a photonic band gap, while the other atom is isolated from all environments. In the model 2, two atoms are interacting with a common photonic band gap. An exactly solvable model for two atoms interacting with two independent photonic band gaps is called model 3.

We first consider the model 1 (the single photonic band-gap model) in which atom A is coupled to a photonic band gap, while the other atom B is isolated from all environments. The Hamiltonian of the system can be written by

$$H^1 = \frac{\omega_0}{2}\sigma_z^{\mathrm{A}} + \frac{\omega_0}{2}\sigma_z^{\mathrm{B}} + \sum_{k=1}^{N}\omega_k a_k^\dagger a_k + \Big(\sigma_+^{\mathrm{A}}\sum_{k=1}^{N} g_k a_k + \mathrm{h.c.}\Big), \tag{1}$$

where $a_k^\dagger$ and a_k are the creation and annihilation operators of quanta of the reservoir, $\sigma_+^j = |e_j\rangle\langle g_j|$ and $\sigma_-^j = |g_j\rangle\langle e_j|$ are the inversion operators of the j-th atom ($j = \mathrm{A}, \mathrm{B}$), ω_j is the transition frequency of the j-th atom, ω_k and g_k are the frequency of the mode k of the reservoir and its coupling strength with two atoms. Then, in the model 2 (the common photonic band-gap model), we consider two atoms interacting with a common photonic band gap. Thus, the Hamiltonian of such a system is given by H^2 expressed as

$$H^2 = \frac{\omega_0}{2}\sigma_z^{\mathrm{A}} + \frac{\omega_0}{2}\sigma_z^{\mathrm{B}} + \sum_{k=1}^{N}\omega_k a_k^\dagger a_k + \Big[(\sigma_+^{\mathrm{A}} + \sigma_+^{\mathrm{B}})\sum_{k=1}^{N} g_k a_k + \mathrm{h.c.}\Big]. \tag{2}$$

However, in the model 3 (two independent photonic band gaps model), two atoms interact with their independent photonic band gap. We describe the Hamiltonian of this system as follows:

$$H^3 = \frac{\omega_0}{2}\sigma_z^{\mathrm{A}} + \frac{\omega_0}{2}\sigma_z^{\mathrm{B}} + \sum_{k=1}^{N}\omega_k a_k^\dagger a_k + \sum_{k=1}^{N}\omega_k b_k^\dagger b_k + \Big[\sigma_+^{\mathrm{A}}\sum_{k=1}^{N} g_k a_k + \sigma_+^{\mathrm{B}}\sum_{k=1}^{N} g_k' b_k + \mathrm{h.c.}\Big]. \tag{3}$$

In order to study the dynamics of two atoms, we solve the master equation by using the pseudomode approach. This exact master equation describes the coherent interaction between the atoms and the pseudomodes in the presence of decay of the pseudomodes due to the influence of a Markovian reservoir. The number of pseudomodes relies on the shape of the reservoir spectral distribution. We focus on an idealized photonic band gap model, in which both Lorentzians are centered at the same frequency, and the second is given by a negative weighting, so that[18]

$$D(\omega) = \frac{W_1\Gamma_1}{(\omega-\omega_c)^2 + \big(\Gamma_1/2\big)^2} - \frac{W_2\Gamma_2}{(\omega-\omega_c)^2 + \big(\Gamma_2/2\big)^2},$$

where the weights of the two Lorentzians satisfying $W_1 - W_2 = 1$ and $\Gamma_2 < \Gamma_1$ ensure positivity of D. The ω_c is the center of the spectrum, and Γ_1 and Γ_2 are the full width at half maximum of two Lorentzians, respectively. There are two poles in D, which are located at $\omega_\mathrm{c} - \mathrm{i}\Gamma_1/2$ and $\omega_\mathrm{c} - \mathrm{i}\Gamma_2/2$, and there is a change in sign of the residues of D between these poles. Thus, the photonic band gap model includes two pseudomodes a_1 and a_2 decaying with decay rates: $\Gamma_1' = W_1\Gamma_2 - W_2\Gamma_1$ and $\Gamma_2' = W_1\Gamma_1 - W_2\Gamma_2$, respectively.

For three different atom–environment models, let us consider that the photonic band gap is initially in the vacuum state $|\bar{0}\rangle = \prod_{k=1}^{N}|0_k\rangle$, corresponding to the pseudomode theory, $|\bar{0}\rangle$ equals $|0_1\rangle|0_2\rangle$. In the model 1, the exact pseudomode master equation is given by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\big[H_{\mathrm{eff}}^1, \rho\big] - \frac{\Gamma_1'}{2}\big[a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1\big] - \frac{\Gamma_2'}{2}\big[a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2\big], \tag{4}$$

where

$$H_{\mathrm{eff}}^1 = \omega_0\sigma_+^{\mathrm{A}}\sigma_-^{\mathrm{A}} + \omega_0\sigma_+^{\mathrm{B}}\sigma_-^{\mathrm{B}} + \omega_\mathrm{c}a_1^\dagger a_1 + \omega_\mathrm{c}a_2^\dagger a_2 + \Omega\big[a_2^\dagger\sigma_-^{\mathrm{A}} + a_2\sigma_+^{\mathrm{A}}\big] + V\big(a_1^\dagger a_2 + a_1 a_2^\dagger\big), \tag{5}$$

where ρ is the density operator for two atoms and the pseudomodes, and the two pseudomodes are coupled with the strength of the coupling $V = \sqrt{W_1W_2}(\Gamma_1 - \Gamma_2)/2$. The atom interacts coherently with the second pseudomode (the strength of the coupling Ω), which is in turn coupled to the first one.

Then, we treat model 2 in which two atoms are coupled with a common photonic band gap; the pseudomode master equation can be written as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\big[H_{\mathrm{eff}}^2, \rho\big] - \frac{\Gamma_1'}{2}\big[a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1\big] - \frac{\Gamma_2'}{2}\big[a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2\big], \tag{6}$$

where

$$H_{\mathrm{eff}}^2 = \omega_0\sigma_+^{\mathrm{A}}\sigma_-^{\mathrm{A}} + \omega_0\sigma_+^{\mathrm{B}}\sigma_-^{\mathrm{B}} + \omega_\mathrm{c}a_1^\dagger a_1 + \omega_\mathrm{c}a_2^\dagger a_2 + \Omega[a_2^\dagger(\sigma_-^{\mathrm{A}} + \sigma_-^{\mathrm{B}}) + a_2(\sigma_+^{\mathrm{A}} + \sigma_+^{\mathrm{B}})] + V\big(a_1^\dagger a_2 + a_1 a_2^\dagger\big). \tag{7}$$

According to model 3, in the case of two atoms interacting with their independent photonic band gap, the dynamics of two atoms and pseudomodes obey the following master equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\big[H_{\mathrm{eff}}^{3\mathrm{A}} + H_{\mathrm{eff}}^{3\mathrm{B}}, \rho\big] - \frac{\Gamma_1'}{2}\big[a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1\big] - \frac{\Gamma_2'}{2}\big[a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2\big]$$

$$-\frac{\Gamma_1'}{2}\left[b_1^\dagger b_1\rho - 2b_1\rho b_1^\dagger + \rho b_1^\dagger b_1\right]$$
$$-\frac{\Gamma_2'}{2}\left[b_2^\dagger b_2\rho - 2b_2\rho b_2^\dagger + \rho b_2^\dagger b_2\right], \quad (8)$$

where

$$H_{\rm eff}^{\rm 3A} = \omega_0\sigma_+^{\rm A}\sigma_-^{\rm A} + \omega_{\rm c}a_1^\dagger a_1 + \omega_{\rm c}a_2^\dagger a_2 + \Omega_{\rm A}[a_2^\dagger\sigma_-^{\rm A} + a_2\sigma_+^{\rm A}] + V(a_1^\dagger a_2 + a_1 a_2^\dagger), \quad (9)$$

$$H_{\rm eff}^{\rm 3B} = \omega_0\sigma_+^{\rm B}\sigma_-^{\rm B} + \omega_{\rm c}b_1^\dagger b_1 + \omega_{\rm c}b_2^\dagger b_2 + \Omega_{\rm B}[b_2^\dagger\sigma_-^{\rm B} + b_2\sigma_+^{\rm B}] + V(b_1^\dagger b_2 + b_1 b_2^\dagger). \quad (10)$$

To illustrate the role of different atom–environment models on the entanglement dynamics of two initially entangled atoms, we consider the case that two atoms are initially in the Bell-like states $|\Phi\rangle = \cos\theta|{\rm eg}\rangle + \sin\theta|{\rm ge}\rangle$ and $|\Psi\rangle = \cos\theta|{\rm ee}\rangle + \sin\theta|{\rm gg}\rangle$. These initial conditions are of fundamental importance in quantum information theory since they include the four Bell states that can be used on their own for universal quantum computation and are associated with an unit of entanglement.

For the case in which the total system initially contains only one excitation $|\Phi\rangle\otimes|\bar{0}\rangle$, the evolutional density matrix of atoms and pseudomodes ρ can be obtained by Eqs. (4), (6), and (8). While for the case of $|\Psi\rangle\otimes|\bar{0}\rangle$, there exist at most two excitations in the total system, the evolutional density matrix of atoms and pseudomodes ρ still can be acquired by Eqs. (4) and (6) in the models 1 and 3. However, for the common photonic band-gap model 2, we must modify the calculation method. Due to Ref. [20], in the basis $\{|0\rangle = |{\rm gg}\rangle, |+\rangle = (|{\rm eg}\rangle + |{\rm ge}\rangle)/\sqrt{2}, |-\rangle = (|{\rm eg}\rangle - |{\rm ge}\rangle)/\sqrt{2}, |2\rangle = |{\rm ee}\rangle\}$, and using the pseudomode approach, the dynamics of two atoms interacting with a common photonic band gap can be treated in the following pseudomode master equation:

$$\frac{{\rm d}\rho}{{\rm d}t} = -{\rm i}[H_{\rm eff}^{2'}, \rho] - \frac{\Gamma_1'}{2}[a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1] - \frac{\Gamma_2'}{2}[a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2], \quad (11)$$

where

$$H_{\rm eff}^{2'} = 2\omega_0|2\rangle\langle 2| + \omega_0|+\rangle\langle +| + \omega_{\rm c}a_1^\dagger a_1 + \omega_{\rm c}a_2^\dagger a_2 + \sqrt{2}\Omega(a_2^\dagger|0\rangle\langle +| + a_2^\dagger|+\rangle\langle 2| + {\rm h.c.}) + V(a_1^\dagger a_2 + a_1 a_2^\dagger). \quad (12)$$

For the case where there are at most two excitations in the total system, we can solve the differential equations obtained from Eqs. (11) and (12). Now we can completely solve the evolutional density matrix of atoms and pseudomodes ρ in the initial states $|\Phi\rangle\otimes|\bar{0}\rangle$ and $|\Psi\rangle\otimes|\bar{0}\rangle$. By tracing the matrix of the pseudomode degrees of freedom, we obtain the density matrix of the reduced atomic system for these two different initial states.

To quantify the entanglement, we use the Wootters concurrence[21] defined as $C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where $\{\lambda_i\}$ are the eigenvalues of the matrix $M = \rho(\sigma_y^{\rm A}\otimes\sigma_y^{\rm B})\rho^*(\sigma_y^{\rm A}\otimes\sigma_y^{\rm B})$, with ρ^* denoting the complex conjugate of ρ, $\sigma_y^{\rm A}$ and $\sigma_y^{\rm B}$ are the Pauli matrices for qubits A and B. This quantity attains its maximum value 1 for maximally entangled states and vanishes for separable states. In this paper, the initial reduced density matrix of the atomic system has an X form,[22] with nonzero elements only along the main diagonal and antidiagonal. Due to the structure of the master equations (4), (6), (8), and (11), the X form is preserved during the evolution. Thus, the concurrence has a simple analytic expression $C(t) = 2\max\{0, |\rho_{00,11}| - \sqrt{\rho_{01,01}\rho_{10,10}}, |\rho_{10,01}| - \sqrt{\rho_{00,00}\rho_{11,11}}\}$, where $\rho_{ij,kl} = \langle ij|\rho_{\rm AB}(t)|kl\rangle$ is the element of the atomic density matrix $\rho_{\rm AB}(t)$.

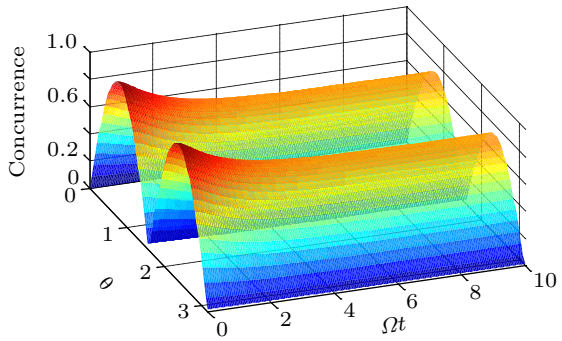


Fig. 1. (color online) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt and θ in the model 1 for the initial state $|\Phi\rangle$ or $|\Psi\rangle$. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$.

3. Numerical results and discussion

For two kinds of Bell-like states $|\Phi\rangle = \cos\theta|{\rm eg}\rangle + \sin\theta|{\rm ge}\rangle$ and $|\Psi\rangle = \cos\theta|{\rm ee}\rangle + \sin\theta|{\rm gg}\rangle$, we plot the time evolution of concurrence as a function of both the parameter θ and the dimensionless quantity Ωt, and mainly look at the dynamics in the weak-coupling regime. As we know, the role of the Lorentzian with negative weight is to introduce a dip into the density of states function $D(\omega)$. When we have $W_1/\Gamma_1 = W_2/\Gamma_2$, there exists a perfect gap $D(\omega_{\rm c}) = 0$, for the case where two atoms are resonant with the gap in the weak coupling regime, there will appear the single-atom excited state population trapping during the dynamics process.[9] From Figs. 1–3, in particular we choose the parameters $\Gamma_1 = 11\Omega$, $\Gamma_2 = \Omega$ and $W_1 = 1.1$, $W_2 = 0.1$, in which the photonic band gap satisfies a perfect gap, and two atoms are resonant with the gap $\omega_0 = \omega_{\rm c}$; the initial entanglement between two atoms via these three different atom–environment models all would display entanglement trapping. Figure 1 shows that the entanglement dynamics in model 1 is independent of the initial Bell-like states. That is to say, in this case the initial entanglement states $|\Phi\rangle$ and $|\Psi\rangle$ have the same dynamic process.

In Fig. 2(a), we show the entanglement dynamics for two atoms in a common photonic band gap (the model 2) for the initial state $|\Phi\rangle$. Depending on θ, different dynamical behaviors are clearly visible. We firstly find that Fig. 2(a) seems to indicate the presence of entanglement sudden birth

for $\theta = 0, \pi/2$, and π. A physical interpretation of the result is that the common photonic band gap plays a role as a bridge to create this atomic entanglement. When the initial state is $|eg\rangle \otimes |0_1\rangle|0_2\rangle$ or $|ge\rangle \otimes |0_1\rangle|0_2\rangle$, the evolutional state can be composed of $|eg\rangle \otimes |0_1\rangle|0_2\rangle$, $|ge\rangle \otimes |0_1\rangle|0_2\rangle$, $|gg\rangle \otimes |1_1\rangle|0_2\rangle$, and $|gg\rangle \otimes |0_1\rangle|1_2\rangle$. The decoherence-free subspace (DFS) state $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$ is included in this evolutional states, so the generation of stationary entanglement can occur when $\theta = 0$ and π such as entanglement sudden birth for a Markovian common reservoir.[20]

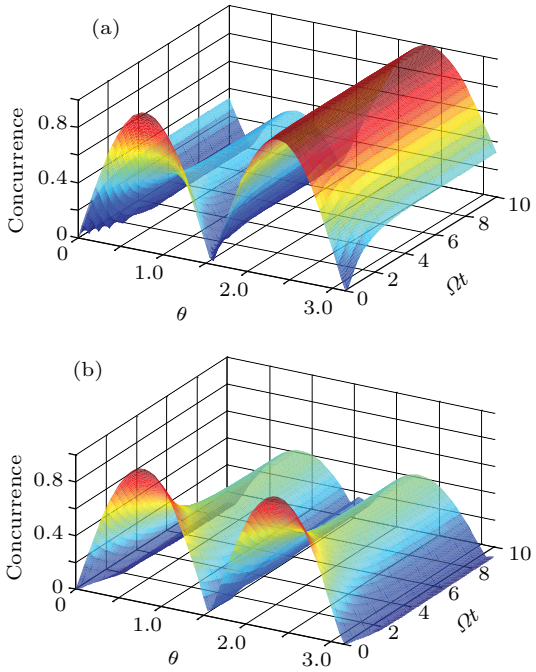


Fig. 2. (color online) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt and θ in model 2 for (a) the initial state $|\Phi\rangle$ and (b) the initial state $|\Psi\rangle$. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$.

From Fig. 2(a), we also note that the values of entanglement trapping in the case of $[0, \pi/2]$ are much smaller than that for the region $[\pi/2, \pi]$. This result is basically due to the fact that the initial state $|\Phi\rangle$ can be written as

$$|\Phi\rangle = \frac{\cos\theta + \sin\theta}{\sqrt{2}}(|eg\rangle + |ge\rangle) + \frac{\cos\theta - \sin\theta}{\sqrt{2}}(|eg\rangle - |ge\rangle).$$

It is easy to see that the weight of $|\phi^-\rangle$ in the initial Bell-like state $|\Phi\rangle$ is bigger in the case $[\pi/2, \pi]$ than in the region $[0, \pi/2]$. As an example, for $\theta = 3\pi/4$, the weight of $|\phi^-\rangle$ in the initial Bell-like state $|\Phi\rangle$ is 1, so the initial entanglement is unchanged during the dynamical process. The concurrence corresponding to the initial Bell-like state $|\Psi\rangle$ in model 2 is shown in Fig. 2(b) as a function of the dimensionless time Ωt for various values of θ. This figure displays that the values of entanglement trapping are the same in the regions $[0, \pi/2]$ and $[\pi/2, \pi]$ for the same initial entanglement $C_{|\Psi\rangle}(0) = |\cos 2\theta|$. From Fig. 2, we can know that the phenomenon of atomic entanglement sudden death can never occur for any pure Bell-like states in the common photonic band gap model. The reason for this result is that, when the photonic band gap satisfies a perfect gap ($W_1/\Gamma_1 = W_2/\Gamma_2$) and two atoms are resonant with the gap, the single-atom excited state population trapping will appear during the dynamics process.

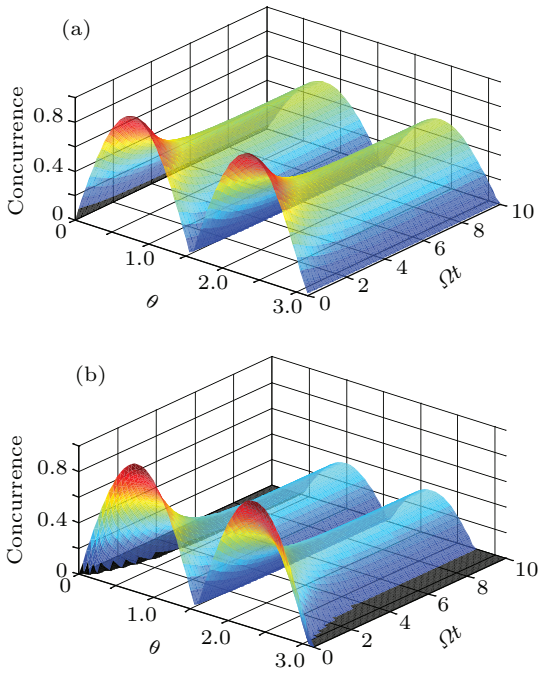


Fig. 3. (color online) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt and θ in the model 3 for (a) the initial state $|\Phi\rangle$ and (b) the initial state $|\Psi\rangle$. The black region means that the two-atom entanglement occurs suddenly. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$.

Figure 3 shows the entanglement trapping of concurrence in the model 3 where two atoms interact with their independent photonic band gap by the same coupling coefficients between the atoms and their band gaps $\Omega_A = \Omega_B = \Omega$. We note that either in the initial state $|\Phi\rangle$ or $|\Psi\rangle$, the values of entanglement trapping are the same in two regions of θ for the equal initial entanglement, as shown in Figs. 1 and 2(b). We also find that the smaller initial entanglement leads to lower asymptotic values of entanglement trapping, and the entanglement trapping phenomenon can occur for the whole range of θ in the case of $|\Phi\rangle$ (see Fig. 3(a)). However, in Fig. 3(b), the black regions $\theta \in [0, \arctan 0.3626]$ and $[\pi - \arctan 0.3626, \pi]$ show that the initial Bell-like state $|\Psi\rangle$ would undergo entanglement sudden death; that is to say, entanglement trapping could not occur in these areas. Then, we will consider different entanglement trappings due to different coupling coefficients. By choosing $\Omega_A = \Omega$, Fig. 4(a) shows the time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt and Ω_B/Ω_A for the initial state $|\Phi\rangle$ in model 3. We can obviously find that the lager values of Ω_B/Ω_A can lead to the lower asymptotic values of entanglement trapping. It can be also shown in Fig. 4(b) that, for $\Omega_B = 0.5\Omega_A$, $\Omega_B = \Omega_A$, and $\Omega_B = 2\Omega_A$, the values of entanglement trapping are getting

smaller and smaller. If $\Omega_B/\Omega_A = 0$ (model 3 can be simplified to model 1), the largest entanglement trapping can be acquired. For another type of atomic initial state $|\Psi\rangle$, the relationship between entanglement trapping and Ω_B/Ω_A is the same as the initial state $|\Phi\rangle$.

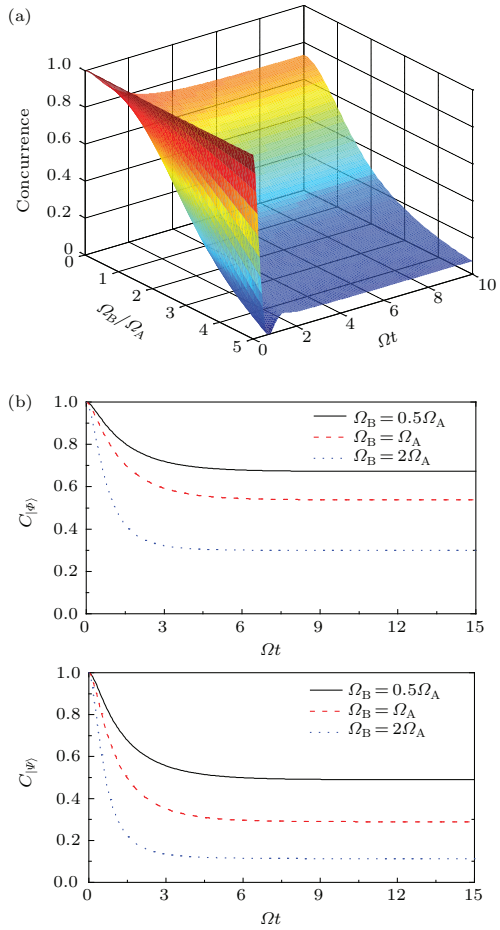


Fig. 4. (color online) (a) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt and Ω_B/Ω_A in the model 3 for the initial state $|\Phi\rangle$. (b) Entanglement trapping for two atoms in different coupling coefficients Ω_B/Ω_A in model 3. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, $\Gamma_2 = \Omega$, $\Omega_A = \Omega$, and $\theta = \pi/4$.

Then, by investigating the subsystem of two atoms, we shall compare the effects of different atom–environment models on the atomic entanglement trapping. In Fig. 5, by choosing the model 1, model 2, and model 3 as the decoherent channel, we plot the time evolution of the concurrence for two atoms as a function of the dimensionless quantity Ωt. Figures 5(a) and 5(c) are for $\theta = \pi/4$ and $\theta = 3\pi/4$ choosing the initial Bell-like state $|\Phi\rangle$. When $\theta \in [0, \pi/2]$, we acquire that in model 1, the value of the atomic entanglement trapping is much larger than the other two decoherent models, and the value of entanglement trapping in model 2 is the smallest (as shown in Fig. 5(a)). While for $\theta = 3\pi/4$, Fig. 5(c) shows that the value of entanglement trapping in the model 2 is the largest. When two atoms go through their own photonic band-gap decoherent channels (model 3), the value of entanglement trapping is smaller than that in tmodels 1 and 2. However, in Figs. 5(b) and 5(d), the atomic entanglement dynamics is given for $\theta = \pi/4$ and $\theta = 3\pi/4$ for the initial Bell-like state $|\Psi\rangle$. We find that in the case of the initial state $|\Psi\rangle$, the value of entanglement trapping is the largest in model 1 and the smallest value would be acquired in model 3. This result is independent of the θ region. From the above analysis, according to the initial state $|\Phi\rangle$, we can acquire the larger entanglement trapping in decoherent model 1 for $\theta \in [0, \pi/2]$, and the largest entanglement occurs in decoherent model 2 for $\theta \in [\pi/2, \pi]$. However, for the initial state $|\Psi\rangle$, whether $\theta \in [0, \pi/2]$ or $\theta \in [\pi/2, \pi]$, the initial entanglement can be trapped in the bigger entanglement when only one of two atoms is coupled with a photonic band gap (model 1).

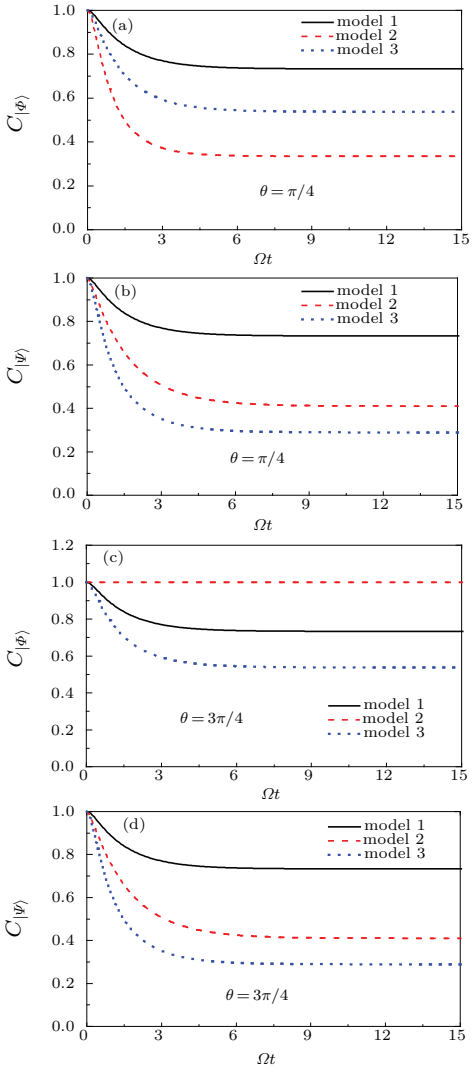


Fig. 5. (color online) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt for (a) the initial state $|\Phi\rangle$, with $\theta = \pi/4$, (b) the initial state $|\Psi\rangle$, with $\theta = \pi/4$, (c) the initial state $|\Phi\rangle$, with $\theta = 3\pi/4$, and (d) the initial state $|\Psi\rangle$, with $\theta = 3\pi/4$. The parameters used are $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, and $\Gamma_2 = \Omega$.

Finally, a comparison between the dynamical process of Bell-like states $|\Phi\rangle$ and $|\Psi\rangle$ in these three different decoherent environmental models is shown in Fig. 6. Since the dynamics of the atomic entanglement is symmetrical between $\theta \in [0, \pi/4]$ and $\theta \in [\pi/4, \pi/2]$, we choose $\theta = \pi/10$ and $\theta =$

$\pi/5$ both belonging to $[0,\pi/4]$. In model 1, the atomic entanglement dynamics are the same for these two different initial states $|\Phi\rangle$ and $|\Psi\rangle$ (as shown in Figs. 6(a) and 6(d)). According to model 2, Figs. 6(b) and 6(e) reveal that for $\theta \in [0,\pi/4]$, the larger entanglement trapping appears in the case of the initial state $|\Psi\rangle$. However, for the model 3, from Figs. 6(c) and 6(f), we find that larger entanglement trapping can be acquired for the initial state $|\Phi\rangle$ than $|\Psi\rangle$ for $\theta \in [0,\pi/4]$. In Fig. 6, we only analyze the comparison of dynamical process between different Bell-like states for $\theta \in [0,\pi/2]$. However, for the case of $\theta \in [\pi/2,\pi]$, there is only an evident different result in model 2 that the larger entanglement trapping can be acquired in the initial state $|\Phi\rangle$ (see Figs. 2(a) and 2(b)).

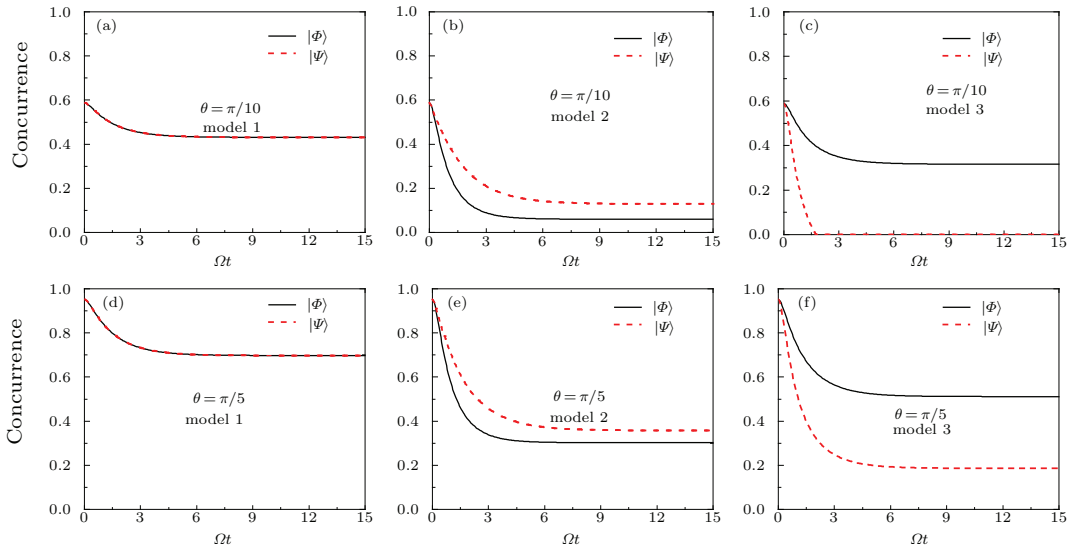


Fig. 6. (color online) Time evolution of the concurrence of two atoms as a function of the dimensionless quantity Ωt for the initial Bell-like states $|\Phi\rangle$ and $|\Psi\rangle$ for (a) model 1, with $\theta = \pi/10$, (b) model 2, with $\theta = \pi/10$, (c) model 3, with $\theta = \pi/10$, (d) model 1, with $\theta = \pi/5$, (e) model 2, with $\theta = \pi/5$, and (f) model 3, with $\theta = \pi/5$. The parameters used are: $W_1 = 1.1$, $W_2 = 0.1$, $\Gamma_1 = 11\Omega$, $\Gamma_2 = \Omega$.

4. Conclusion

In conclusion, we have studied entanglement trapping in different decoherence environments, and considered a two-atom system interacting with the photonic band-gap models: the single photonic band-gap model, the common photonic band-gap model, and the two independent photonic band-gaps model. Firstly, by choosing the parameters such that the photonic band gap satisfies a perfect gap, we find that the initial entanglement between two atoms via these three different decoherence models all would display entanglement trapping. However, from Ref. [23], we can know that the initial entanglement between two atoms would gradually disappear by considering the common environmental model. Thus, the phenomenon of entanglement trapping can only occur in the photonic band-gap models. Then, we compare the effects of different decoherence models on the atomic entanglement trapping for the same initial entangled state, and acquire the conditions for getting the larger entanglement trapping. In addition, we also compare the entanglement trapping of two Bell-like states in the same photonic band-gap models. These findings demonstrate that the values of entanglement trapping can be controlled by establishing the qubit–environment models, and are helpful for preserving or trapping quantum entanglement experimentally.

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