Pseudoscalar Cornell potential for a spin-$1/2$ particle under spin and pseudospin symmetries in $1+1$ dimension

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The Cornell potential that consists of Coulomb and linear potentials has received a great deal of attention in particle physics. In this paper, we present the exact solutions of the Dirac equation with the pseudoscalar Cornell potential under spin and pseudospin symmetry limits. The energy eigenvalues and corresponding eigenfunctions are given in closed form.

Keywords: Dirac equation, pseudoscalar Cornell potential, spin and pseudospin symmetry

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1. Introduction

Relativistic symmetries of the Dirac Hamiltonian were discovered many years ago, but only recently these symmetries have been recognized empirically in nuclear and hadronic spectroscopy.[1] The cases in which the mean field is composed of a vector ($V_{\rm V}$) potential and a scalar ($V_{\rm S}$) potential are usually pointed out as a necessary condition for the occurrence of spin and pseudospin symmetries in nuclei. Within the framework of the Dirac equation, pseudospin symmetry used to feature deformed nuclei and superdeformation, and to establish an effective shell model[2–4] and spin symmetry, is relevant to mesons.[5] Spin symmetry occurs when the scalar potential $V_{\rm S}$ is nearly equal to the vector potential $V_{\rm V}$, and pseudospin symmetry occurs when $V_{\rm S} \approx -V_{\rm V}$.[6–9]

The Cornell potential that consists of Coulomb potential and linear potential, i.e.,

$$V(x) = -\frac{a}{x} + bx, \tag{1}$$

has received a great deal of attention in particle physics. The Cornell potential was used with considerable success in models describing systems of bound heavy quarks.[10–13] The potential includes the short-distance Coulombic interaction of quarks, known from perturbative quantum chromodynamics (QCD), and the long-distance quark confinement, known from lattice QCD.[14,15] It should be stressed that a specific situation can emerge when the parameter b is simultaneously small, and leads to a particular atomic description of a perturbed Coulomb problem.[16] Ghalenovi *et al.*[17] studied the strange, charmed and beauty baryons mass in the Cornell potential by using the variational approach.

The purpose of this work is to solve the Dirac equation for the pseudoscalar Cornell potential. To this end, we shall first briefly introduce the Dirac equation with scalar-vector-pseudoscalar potential in view of spin and pseudospin symmetries in Section 2. In Section 3, we solve the Dirac equation with pseudoscalar potential under spin and pseudospin symmetries in the $1+1$ dimension. Finally, Section 4 contains the summary and concluding remarks.

2. Dirac equation in the 1+1 dimension

The $(1+1)$-dimensional time-independent Dirac equation for a spin-$1/2$ particle of rest mass m under the action of vector ($V_{\rm V}$), scalar ($V_{\rm S}$), and pseudoscalar ($V_{\rm P}$) potentials can be written, in terms of the combinations $\Sigma = V_{\rm V} + V_{\rm S}$ and $\Delta = V_{\rm V} - V_{\rm S}$, as (in units of $\hbar = c = 1$)[18–22]

$$\begin{aligned} &H\psi = E\psi, \\ &H = \sigma_1 p + \sigma_3 m + \frac{1+\sigma_3}{2}\Sigma + \frac{1-\sigma_3}{2}\Delta + \sigma_2 V_{\rm P}, \end{aligned} \tag{2}$$

where σ_1, σ_2, and σ_3 denote the Pauli matrices. If we assume the wave function to be

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix},$$

we have two first-order equations for the upper ψ_+ and the lower ψ_- components of the spinor

$$-{\rm i}\frac{{\rm d}\psi_-}{{\rm d}x} + m\psi_+ + \Sigma\psi_+ - {\rm i}V_{\rm P}\psi_- = E\psi_+, \tag{3}$$

$$-{\rm i}\frac{{\rm d}\psi_+}{{\rm d}x} - m\psi_- + \Delta\psi_- + {\rm i}V_{\rm P}\psi_+ = E\psi_-. \tag{4}$$

In the spin symmetry limit, $\Delta = 0$ and $E \neq -m$.[1,18] The Dirac equation becomes

$$\psi_- = -{\rm i}\frac{\left(\dfrac{{\rm d}\psi_+}{{\rm d}x} - V_{\rm P}\psi_+\right)}{E+m}, \tag{5a}$$

$$\begin{aligned} &-\frac{{\rm d}^2\psi_+}{{\rm d}x^2} + \left[(E+m)\Sigma + V_{\rm P}^2 + \frac{{\rm d}V_{\rm P}}{{\rm d}x}\right]\psi_+ \\ &\quad = (E^2 - m^2)\psi_+. \end{aligned} \tag{5b}$$

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For the pseudospin symmetry case, $\Sigma = 0$ and $E \neq m$.[1,18] The Dirac equation can be obtained from the previous one by taking $\psi_+ \leftrightarrow \psi_-$, $m \to -m$, $\Sigma \to \Delta$, and $V_{\rm P} \to -V_{\rm P}$. In spin and pseudospin symmetry, the solution of the relativistic problem is mapped into a Sturm–Liouville problem in such a way that solutions can be found by solving a Schrödinger-like problem.[18,19]

3. Dirac equation with pseudoscalar Cornell potential

In this section we will solve the Dirac equation under spin and pseudospin symmetry with pseudoscalar Cornell potential.

3.1. Spin symmetry limit

The exact spin symmetry occurs in the Dirac equation when $\mathrm{d}\Delta/\mathrm{d}r = 0$ or $\Delta = C_s$ =constant.[23–26] In this part we take $\Sigma = 0$, $C_{\rm s} = 0$, and pseudoscalar potential as Cornell potential in Eq. (1). Therefore, equation (5b) becomes

$$\frac{\mathrm{d}^2\psi_+}{\mathrm{d}x^2} - \left[b^2x^2 + \frac{a(a+1)}{x^2} + \xi^2\right]\psi_+ = 0, \tag{6}$$

where $\xi^2 = -b(2a-1) - \varepsilon^2$, and $\varepsilon^2 = E^2 - m^2$. To obtain the solution of Eq. (6), by using transformation $s = x^2$, we rewrite it as follows:

$$\frac{\mathrm{d}^2\psi_+}{\mathrm{d}s^2} + \frac{1}{2s}\frac{\mathrm{d}\psi_+}{\mathrm{d}s} - \frac{1}{4s^2}\left[b^2s^2 + \xi^2 s + a(a+1)\right]\psi_+ = 0. \tag{7}$$

This second differential equation can be solved by the Nikiforov–Uvarov method.[27–30] Following Eqs. (28)–(35) in Ref. [28], one can obtain the energy equation as

$$b(4n+2+\sqrt{4a(a+1)+1}) + \xi^2 = 0, \tag{8}$$

or equivalently

$$\begin{aligned} E &= \pm\left[m^2 + 2b(2n+a+3/2) - b(2a-1)\right]^{1/2} \\ &= \pm\left(m^2 + 4b(n+1)\right)^{1/2}. \end{aligned} \tag{9}$$

In Table 1, we list the bound state energy eigenvalues with $m = 5$ fm^{-1} for some nuclear states. To find the eigenfunction of the spin symmetry case, we follow Eqs. (36)–(39) in Ref. [28] and obtain the upper spinor as

$$\psi_+(x) = N\,\mathrm{e}^{-bx^2/2}x^{a+1}L_n^{a+1/2}(bx^2), \tag{10}$$

where N is the normalization constant determined by[31]

$$N = \sqrt{\frac{2b^{a/2+3/2}n!}{(n+a/2+3/2)!}}. \tag{11}$$

Table 1. Bound state energy eigenvalues (in units of fm^{-1}) with $m = 5$ fm^{-1}.

n	$b = 0.1$		$b = 0.5$		$b = 1$	
	spin symmetry	pseudospin symmetry	spin symmetry	pseudospin symmetry	spin symmetry	pseudospin symmetry
0	±5.039841	±5.000000	±5.196152	±5.000000	±5.385165	±5.000000
1	±5.079370	±5.039841	±5.385165	±5.196152	±5.744563	±5.385165
2	±5.118593	±5.079370	±5.567764	±5.385165	±6.082762	±5.744562
3	±5.157519	±5.118593	±5.744563	±5.567764	±6.403124	±6.082762
4	±5.196152	±5.157519	±5.916080	±5.744563	±6.708204	±6.403124
5	±5.234501	±5.196152	±6.082762	±5.916080	±7.000000	±6.708204

3.2. Pseudospin symmetry limit

The exact pseudospin symmetry occurs in the Dirac equation when $\mathrm{d}\Sigma/\mathrm{d}r = 0$ or $\Sigma = C_{\rm ps}$ =constant.[32–35] In this part we take $\Delta = 0$, $C_{\rm ps} = 0$, and pseudoscalar potential as the Cornell potential in Eq. (1). As we mentioned, in the pseudospin symmetry case, $\Sigma = 0$ and $E \neq m$,[1] and the Dirac equation can be obtained from the previous one by taking $\psi_+ \leftrightarrow \psi_-$, $m \to -m$, $\Sigma \to \Delta$, and $V_{\rm P} \to -V_{\rm P}$. Therefore, the energy equation can be obtained as

$$b(4n+2+\sqrt{4a(a-1)+1}) + \tilde{\xi}^2 = 0, \tag{12}$$

where $\tilde{\xi}^2 = -b(2a+1) - \varepsilon^2$. We can also write the energy eigenvalues as

$$\begin{aligned} E &= \pm\left[m^2 + 2b(2n+a+1/2) - b(2a+1)\right]^{1/2} \\ &= \pm\left(m^2 + 4bn\right)^{1/2}. \end{aligned} \tag{13}$$

Some numerical results can be found in Table 1. The corresponding lower spinor eigenfunctions can be obtained as

$$\psi_-(x) = \tilde{N}\,\mathrm{e}^{-bx^2/2}x^{a}L_n^{a-1/2}(bx^2), \tag{14}$$

where $\tilde{N}$ is the normalization constant determined by[31]

$$N = \sqrt{\frac{2b^{a/2+1/2}n!}{(n+a/2+1/2)!}}. \tag{15}$$

4. Conclusions

In this paper, we investigated the energy levels and wave function of the Dirac equation with the pseudoscalar Cornell potential under spin and pseudospin symmetry limits. We found that the Cornell potential with pseudoscalar potential has exact solutions, even if it has no exact solution as scalar or vector potential role.

References

[1] Ginocchio J N 2005 *Phys. Rep.* **414** 165
[2] Bohr A, Hamamoto I and Mottelson B R 1982 *Phys. Scr.* **26** 267
[3] Dudek J, Nazarewicz W, Szymanski Z and Leander G A 1987 *Phys. Rev. Lett.* **59** 1405
[4] Troltenier D, Bahri C and Draayer J P 1995 *Nucl. Phys. A* **586** 53
[5] Page P R, Goldman T and Ginocchio J N 2001 *Phys. Rev. Lett.* **86** 204
[6] Hecht K T and Adler A 1969 *Nucl. Phys. A* **137** 129
[7] Arima A, Harvey M and Shimizu K 1969 *5* **30** 517
[8] Ginocchio J N, Leviatan A, Meng J and Zhou S G 2004 *Phys. Rev. C* **69** 034303
[9] Ginocchio J N 1997 *Phys. Rev. Lett.* **78** 436
[10] Quigg C and Rosner J L 1979 *Phys. Rep.* **56** 167
[11] Chaichian M and Kokerler R 1980 *Ann. Phys.* **124** 61
[12] Bykov A A, Dremin I M and Leonidov A V 1984 *Sov. Phys. Usp.* **27** 321
[13] Plante G and Antippa A F 2005 *J. Math. Phys.* **46** 062108
[14] Stack J D 1984 *Phys. Rev. D* **29** 1213
[15] Bali G S, Schilling K and Wachter A 1997 *Phys. Rev. D* **56** 2566
[16] Bessis D, Vrscay E R and Handy C R 1987 *J. Phys. A: Math. Gen.* **20** 419
[17] Ghalenovi Z, Rajabi A A and Hamzavi M 2011 *Acta Phys. Pol. B* **42** 1849
[18] Castro L B, de Castro A S and Hott M 2007 *Int. J. Mod. Phys. E* **16** 3002
[19] Castro L B, de Castro A S and Hott M 2007 *Europhys. Lett.* **77** 20009
[20] Alhaidari A D 2011 *Phys. Lett. B* **699** 309
[21] Alhaidari A D 2010 *Int. J. Mod. Phys. A* **25** 3703
[22] Alhaidari A D 2010 *Found. Phys.* **40** 1088
[23] Zhou S G, Meng J and Ring P 2003 *Phys. Rev. Lett.* **91** 262501
[24] He X T, Zhou S G, Meng J, Zhao E G and Scheid W 2006 *Eur. Phys. J. A* **28** 265
[25] Song C Y, Yao J M and Meng J 2009 *Chin. Phys. Lett.* **26** 122102
[26] Song C Y and Yao J M 2010 *Chin. Phys. C* **34** 1425
[27] Nikiforov A F and Uvarov V B 1988 *Special Functions of Mathematical Physics* (Berlin: Birkhausr)
[28] Hamzavi M, Rajabi A A and Hassanabadi H 2011 *Int. J. Mod. Phys. A* **26** 1363
[29] Hamzavi M, Hassanabadi H and Rajabi A A 2010 *Mod. Phys. Lett. A* **25** 2447
[30] Hamzavi M, Hassanabadi H and Rajabi A A 2010 *Int. J. Mod. Phys. E* **19** 2189
[31] Aydoğdu O and Sever R 2009 *Phys. Scr.* **80** 015001
[32] Ginocchio J N 1999 *Phys. Rep.* **315** 231
[33] Ginocchio J N 1999 *Nucl. Phys. A* **654** 663
[34] Meng J, Sugawara-Tanabe K, Yamaji S and Arima A 1999 *Phys. Rev. C* **59** 154
[35] Meng J, Sugawara-Tanabe K, Yamaji S, Ring P and Arima A 1998 *Phys. Rev. C* **58** R628