Feedback control scheme of traffic jams based on the coupled map car-following model*

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Based on the pioneering work of Konishi *et al.* [*Phys. Rev. E* (1999) **60** 4000], a new feedback control scheme is presented to suppress traffic jams based on the coupled map car-following model under the open boundary condition. The effect of the safe headway on the traffic system is considered. According to the control theory, the condition under which traffic jams can be suppressed is analyzed. The results are compared with the previous results concerning congestion control. The simulations show that the suppression performance of our scheme on traffic jams is better than those of the previous schemes, although all the schemes can suppress traffic jams. The simulation results are consistent with theoretical analyses.

Keywords: traffic jam, coupled map car-following model, control scheme, safe headway

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1. Introduction

Traffic jams are one serious traffic problem in modern cities, which has attracted much attention recently. The optimal velocity (OV) model proposed by Bando *et al.*,[1] one of the favorable car-following models for studying traffic flow, has successfully revealed the dynamical evolvement process of traffic congestion in a simple way. Hereafter, much work has been done using this model.[2–11] The coupled map (CM) car-following model is a corresponding discrete version of the OV model, which plays an important role in suppressing traffic jams.[12,13] In 1999, according to the CM car-following model, Konishi *et al.*[14] proposed a decentralized delayed-feedback control scheme for suppressing traffic jams. In 2006, Zhao and Gao[15] presented a simple strategy for congested traffic induced by bottlenecks in the traffic system. After that, many important achievements related to the control scheme[16–22] have been obtained.

Although these control schemes can suppress traffic jams effectively, they just use velocity difference between successive vehicles of an arbitrary number of preceding vehicles as a feedback control signal to control the speed of the studied car for suppressing traffic jams. However, they cannot be used to study the suppression performance of the safe headway effect on traffic jams. In fact, when the headway between a following vehicle and its front one is less than the safe headway, the following driver pays more attention to the headway of his car, and continuously controls speed of his car for keeping a certain distance with the leading car to avoid collision. Therefore, a control scheme considering the safe headway effect can also suppress traffic jams. In view of the above reasons, we consider the influence of the safe headway on traffic flow. Moreover, the present work introduces a new feedback control method including the safe headway effect.

The rest of the present paper is organized as follows. The CM car-following model and its stability analysis are reviewed in Section 2. A new feedback control scheme for suppressing the congested traffic is introduced in Section 3, along with the theoretical analysis. In Section 4, our simulation results are presented and compared with the previous work concerning congestion control. Finally, some conclusions are drawn from the present study in Section 5.

2. CM car-following model and its stability analysis

2.1. CM car-following model

Now, we consider the CM car-following model under the open boundary condition. The leading vehicle is described as follows:[14]

$$x_0(n+1) = v_0 T + x_0(n), \tag{1}$$

where $x_0(n) > 0$ is the position of the leading vehicle at time $t = nT$, $v_0 > 0$ is its velocity which is a constant, T is the sampling interval, and $x_0(0) > 0$ is the initial leading vehicle position.

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The following vehicle i is described as[14]

$$x_i(n+1) = v_i(n)T + x_i(n), \quad (i = 1,2,\ldots,N), \tag{2}$$

where $x_i(n) > 0$ and $v_i(n) > 0$ are the position and speed of the $i-$th vehicle at time $t = nT$, respectively, and N is the total number of vehicles. The speeds of the following vehicles are governed by[14]

$$v_i(n+1) = \alpha_i \left[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)\right]T + v_i(n), \tag{3}$$

where $\alpha_i > 0$ is the sensitivity of the i-th vehicle driver, $V_i^{\mathrm{OP}}(y_i(n))$ is the OV function, which depends on the headway distance in the form of

$$y_i(n) = x_{i-1}(n) - x_i(n). \tag{4}$$

The OV function is given by[14]

$$V_i^{\mathrm{OP}}(y_i(n)) = \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\mathrm{sat}}\left(2\frac{y_i(n)-\eta}{\varsigma_i}\right)\right], \tag{5}$$

where $v_i^{\max} > 0$ is the maximum speed, $\eta > 0$ is the safe headway, and $\zeta_i > 0$ is a parameter. The saturation function $\bar{H}_{\mathrm{sat}}(\cdot)$ is described as

$$\bar{H}_{\mathrm{sat}} = \begin{cases} +1, & \text{if } \rho > +1, \\ \rho, & \text{if } -1 \le \rho \le +1, \\ -1, & \text{if } \rho < -1. \end{cases} \tag{6}$$

To guarantee the existence of traffic flow, it is assumed that the velocity of the leading vehicle is less than the maximum velocity among all the following vehicles.

To avoid the collisions and backward motions, all the vehicles adopt the full braking action if

$$y_i(n) < y_i^{\min}, \tag{7}$$

then

$$x_i(n+1) = x_i(n), \quad v_i(n+1) = 0. \tag{8}$$

This action implies that the i-th vehicle stops suddenly when the headway $y_i(n)$ is less than a minimum headway $y_i^{\min}$.

2.2. Stability analysis

For simplicity, we regard the CM car-following model as a simple system. If the leading vehicle runs with the speed v_0. The dynamic behavior of the i-th vehicle can be given by

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)]T + v_i(n), \\ y_i(n+1) &= v_0 T - v_i(n)T + y_i(n). \end{aligned} \tag{9}$$

The steady state of the vehicle system (9) is

$$[v_i^*, y_i^*]^{\mathrm{T}} = \left[v_0, \frac{v_0}{r_i} - \frac{\zeta_i}{2} + \eta_i\right]^{\mathrm{T}}, \tag{10}$$

where $r_i = v_i^{\max}/\zeta_i$. Obviously, we can ascertain that the essential condition for the existence of the steady state is $v_0 < v_i^{\max}$. According to the control theory, Konishi *et al.*[14] directly obtained the following essential condition for the system (9) to be stable. All the roots of

$$P_i(z) = z^2 + a_i z + b_i = 0 \tag{11}$$

are within a unit circle, where $a_i = \alpha_i T - 2$ and $b_i = 1 - \alpha_i T + \alpha_i r_i T^2$. From this condition, the ranges of the parameters are given by

$$0 < r_i < \frac{1}{T}, \quad 0 < a_i < \frac{4}{T(2 - r_i T)}. \tag{12}$$

This condition is a necessary and sufficient one to keep the system (9) stable. If the traffic system is unstable, the amplitude of a disturbance blows up with time evolution, while the disturbance will gradually disappear in a stable system. When the preceding vehicle is not running with the constant speed v_0, the dynamics of the i-th vehicle can be given by

$$\begin{aligned} v_i(n+1) &= \alpha_i \left[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)\right]T + v_i(n), \\ y_i(n+1) &= v_{i-1}T - v_i(n)T + y_i(n). \end{aligned} \tag{13}$$

Also according to the control theory, Konishi *et al.*[14] directly derived the following condition of no traffic jam on the assumption that all the vehicles satisfy the following conditions:

$$\max_{|z|=1} |G_i(z)| \le 1, \quad (i = 1,2,\ldots,N). \tag{14}$$

From this condition, the ranges of the parameters are given by

$$\frac{8 + \alpha_i T(\alpha_i T - 8)}{\alpha_i T^2(\alpha_i T - 6)} < r_i < \frac{\alpha_i}{2 + \alpha_i T}. \tag{15}$$

This condition means that the traffic system is robust. The robustness is to enhance the stability of the system, under which the amplitude of the transient disturbance decreases with time even if the structure and parameters of the system are variational. We confirm that traffic congestion cannot occur when all the vehicles satisfy the conditions (12) and (15). In the next section, we will propose a new control scheme to suppress traffic jams.

3. Control scheme

The aim of this paper is to put forward a new control scheme for suppressing the traffic jam in the CM car-following model. In the actual traffic system, the safe headway can affect the driving behavior of the driver. Indeed, when the headway of the considered car is less than the safe headway, the driver of the considered car has to continuously control the velocity of his car based on the information about the safe headway and the velocity difference between the considered vehicle and its front one to avoid collision, and the safe headway is the first factor to be considered by the driver. However, when the headway of the considered car is larger than the safe headway, its driver adjusts the velocity according to the information about velocity difference to keep up with the leading one, and does not need to consider the safe headway effect. Therefore, a new feedback control signal $u_i(n)$ is designated as follows:

$$u_i(n) = kv_i(n) - (1-k)H(y_i(n) - \eta)(\eta - y_i(n)), \tag{16}$$

where $0 < k < 1$ is the feedback gain, η is the safe headway, the function $H(\cdot)$ is described by

$$H(y_i(n)-\eta) = \begin{cases} 0, & (y_i(n)-\eta > 0), \\ 1, & (y_i(n)-\eta \le 0). \end{cases} \tag{17}$$

When $y_i(n)-\eta > 0$, our feedback control signal $u_i(n)$ can be reduced to Zhao's control signal as follows:[15]

$$u_i(n) = kv_i(n). \tag{18}$$

When $y_i(n)-\eta \le 0$, our control signal is designed as follows:

$$\widetilde{u}_i(n) = kv_i(n) - (1-k)(\eta - y_i(n)). \tag{19}$$

The control signal term is added to traffic system (9) as follows:

$$\begin{aligned} v_i(n+1) &= \alpha_i\left[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)\right]T + v_i(n) + \widetilde{u}_i(n), \\ y_i(n+1) &= v_{i-1}T - v_i(n)T + y_i(n). \end{aligned} \tag{20}$$

In the control signal $\widetilde{u}_i(n)$, we take the velocity difference between the $v_{i-1}(n)$ in front and $v_i(n)$ into account, besides the difference between the and η at time $t = nT$. We should note that the control signal acts in the system only if the traffic state is unstable, and the signal vanishes in the stable system. Around the steady state, the controlled system (20) can be described by

$$\begin{aligned} \begin{pmatrix} \delta v_i(n+1) \\ \delta y_i(n+1) \end{pmatrix} &= \begin{pmatrix} 1-\alpha_i T - k & \alpha_i r_i T + 1 - k \\ -T & 1 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix} \\ &\quad + \begin{pmatrix} k \\ T \end{pmatrix} \delta v_{i-1}(n), \\ \delta v_i(n) &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix}. \end{aligned} \tag{21}$$

The transfer function $\overline{G}_i(z)$ from $\delta v_{i-1}(n)$ to $\delta v_i(n)$ is described by

$$\begin{aligned} \overline{G}_i(z) &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z-1+\alpha_i T + k & k-1-\alpha_i r_i T \\ T & z-1 \end{pmatrix}^{-1} \begin{pmatrix} k \\ T \end{pmatrix} \\ &= \frac{k(z-1) + \alpha_i r_i T^2 + T - kT}{\overline{P}_i(z)}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} &\overline{P}_i(z) = z^2 + \overline{a}_i z + \overline{b}_i, \ \ \overline{a}_i = \alpha_i T + k - 2, \\ &\overline{b}_i = 1 - \alpha_i T - k + T - kT + \alpha_i r_i T^2. \end{aligned}$$

In order to suppress the traffic jam in the CM car-following model, the feedback gain k is designed such that

(i) $P_i(z)$ is stable;

(ii) $\max\limits_{|z|=1} |G_i(z)| \le 1, \ \ (i = 1, 2, \ldots, N)$.

Theorem 1 There is no traffic jam in the controlled system (20) in the following conditions.

(i) If

$$\alpha_i T - 2 < 0 \quad \text{and} \quad \alpha_i r_i T - \alpha_i + 1 < 0, \tag{23}$$

then the feedback gain k is set as

$$\frac{1-\alpha_i T + T + \alpha_i r_i T^2}{1+T} < k < \frac{4 - 2\alpha_i T + T + \alpha_i r_i T^2}{2+T}, \tag{24}$$

else if

$$0 < \alpha_i r_i T^2 < 2 \quad \text{and} \quad (\alpha_i T - 2)(\alpha_i r_i T - \alpha_i + 1) > 0, \tag{25}$$

then the feedback gain k is set as

$$\max\left\{\frac{-\alpha_i T + T + \alpha_i r_i T^2}{1+T}, R_1\right\} < k < \frac{1 - \alpha_i T + T + \alpha_i r_i T^2}{1+T}, \tag{26}$$

where

$$R_1 = \frac{2T - \alpha_i^2 T^2 + 2\alpha_i r_i T^2 + \alpha_i^2 r_i T^3 + \alpha_i T^2}{2T + 2\alpha_i T + \alpha_i T^2}. \tag{27}$$

(ii) If

$$2 < \alpha_i r_i T^2 < 4 - 3T + \alpha_i T^2 \ \text{ and } \ (3 - \alpha_i T)(\alpha_i T - 2) > 0, \tag{28}$$

then the feedback gain k is set as

$$\max\left\{\frac{-\alpha_i T + T + \alpha_i r_i T^2}{1+T}, R_1\right\} < k < \frac{4 - 2\alpha_i T + T + \alpha_i r_i T^2}{2+T}. \tag{29}$$

(iii) If

$$\alpha_i r_i T^2 = 2 \ \text{ and } \ (\alpha_i T - 2)(\alpha_i r_i T - \alpha_i + 1) > 0, \tag{30}$$

then the feedback gain k is set as

$$\max\left\{\frac{-\alpha_i T + T + 2}{1+T}, R_1\right\} < k < \frac{3 - \alpha_i T + T}{1+T}. \tag{31}$$

(iv) If

$$\alpha_i T = 2 \quad \text{and} \quad 0 < \alpha_i r_i T^2 < 2, \tag{32}$$

then the feedback gain k is set as

$$\frac{1 - \alpha_i T + T + \alpha_i r_i T^2}{1+T} < k < \frac{4 - 2\alpha_i T + T + \alpha_i r_i T^2}{2+T}. \tag{33}$$

This theorem provides us with the ability to design the feedback gain k to suppress the traffic jam in the CM car-following model.

Remark 1 Compared with the scheme of Konishi *et al.*,[14] our scheme has a simple form: just one parameter of the feedback gain k needs to be designed. Besides, the feedback gain in our scheme is not fixed at an exact value, but set in a range, which means that it makes the process of formulating control strategies more flexible. Finally, our scheme is more reasonable than that proposed by Konishi *et al.* We represent the interaction between a vehicle and its front vehicle (the control scheme of Zhao *et al.*). Meanwhile, the safe headway effect is considered. Therefore, the suppression performance in traffic congestion of our scheme is more effective and reliable than those of the two previous schemes. The suppression performance of our scheme on traffic jams is examined in the following section.

4. Simulation results

The aim of this section is to investigate whether the results from the theoretical analysis in Section 3 holds. Our simulations are based on the CM car-following model under open boundary conditions. The CM car-following model parameters are set as follows:[14]

$$\eta = 25.0 \text{ m}, \ \ \zeta_i = 23.3 \text{ m}, \ \ v^{\max} = 33.6 \text{ m/s},$$

$$\alpha_i = 2.0\ \mathrm{s}^{-1},\ \ v_0 = 20\ \mathrm{m/s},\ \ T = 0.1\ \mathrm{s},$$
$$y^{\min} = 7.02\ \mathrm{m},\ \ r_i = v^{\max}/\zeta \approx 1.44\ \mathrm{s}^{-1}.$$

It is assumed that all vehicles have the same parameters. The initial conditions are the steady state of the model. The initial positions and speeds are set to be

$$x_i(0) = \sum_{j=i+1}^{N} y_j^*,\ \ y_j(0) = y_j^*,\ \ v_i(0) = v_i^*,\ \ i = 1,2...,N. \quad (34)$$

Consider a case where the leading vehicle stops suddenly four times:[14] $v_0(n) = 0$, nT = 100–103, 120–123, 140–143, and 160–163. The external disturbance is used to examine the control effect of our scheme on traffic jams. Now, we set the feedback gain k in our scheme. In order to suppress the congested traffic, the feedback signal is designated as follows: the model parameters α_i, r_i, and T satisfy Eq. (25). Then the feedback gain k is chosen according to Eq. (26). Substituting the parameter values into Eq. (26), we have $0.801 < k < 0.961$. The feedback gain is fixed at $k = 0.85$. In the control scheme of Konishi *et al.*, the procedure to design four feedback gains can be found in Ref. [14].

Figures 1(a), 2(a), 3(a), and 4(a) show the space–time plot of the traffic flow in an uncontrolled system (the CM car-following model), controlled systems of Konishi *et al.* and Zhao *et al.*, and our scheme, respectively. It can be seen that the stop disturbance propagates backwards in the uncontrolled system. From Figs. 2(a), 3(a), and 4(a), we observe that there is neither oscillating behavior nor a traffic jam when control signals are introduced in the system. All the vehicles are running smoothly without full braking in the three controlled systems. However, it should be noted that the headway fluctuation is gentle in Fig. 4(a) compared with those in Figs. 2(a) and 3(a). Figures 1(b), 2(b), 3(b), and 4(b) show the speeds of the first and the 100-th vehicles corresponding to Figs. 1(a), 2(a), 3(a), and 4(a). In Fig. 1(b), it is found that when traffic flow is disturbed, the 100-th vehicle's velocity perturbation is amplified with time and finally evolves into stop-and-go state. From Figs. 2(b), 3(b), and 4(b), we can see that the 100-th vehicle is moving smoothly as the amplitude of speed fluctuation decreases with increasing vehicle number i. However, it should be noted that the amplitude of speed fluctuation is smaller in the system controlled by our control signal than in the traffic system controlled by the control signals of Konishi *et al.* and Zhao *et al.* The simulation results indicate that three control schemes can be used to suppress traffic jam, and the traffic system using our scheme exhibits better temporal behavior.

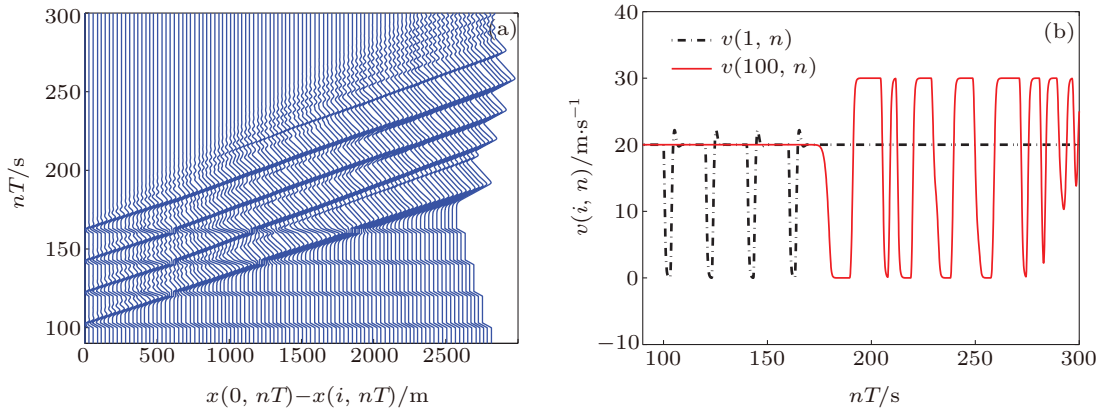


Fig. 1. (color online) Numerical simulations for an uncontrolled system. The leading vehicle stops four times suddenly. (a) Space–time plot of an uncontrolled system. (b) Temporal velocity behavior of the first and 100-th vehicles.

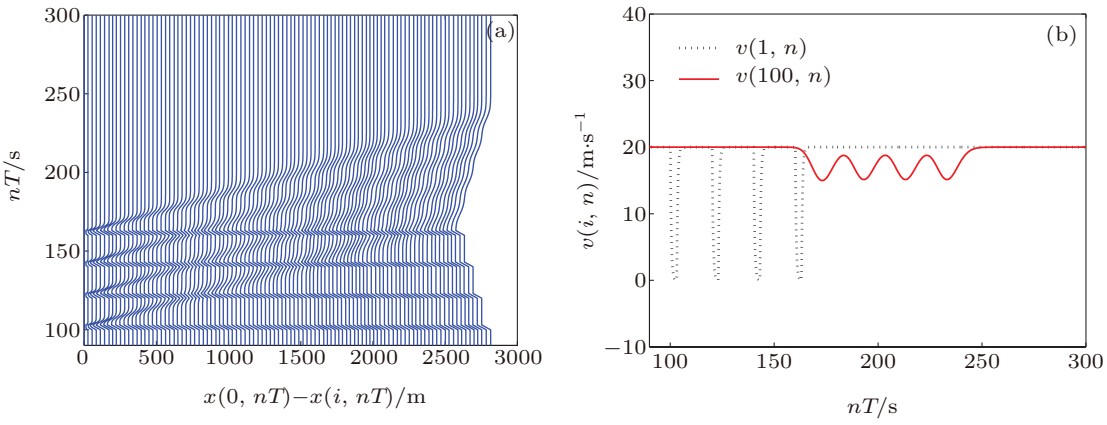


Fig. 2. (color online) Numerical simulations for the system controlled by the schem of Konishi *et al.* The leading vehicle stops four times suddenly. (a) Space–time plot of the system controlled by the control signal of Konishi *et al.* (b) Temporal velocity behaviors of the first and 100-th vehicles.

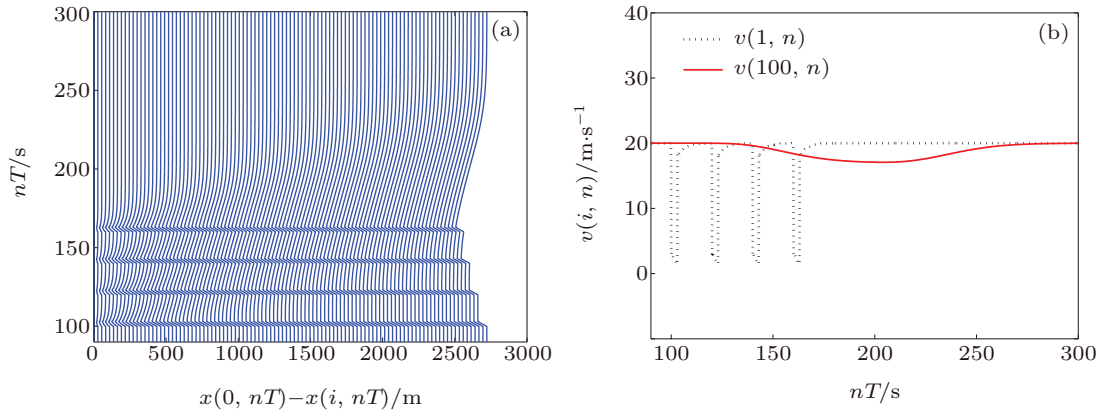


Fig. 3. (color online) Numerical simulations for the system controlled by the scheme of Zhao *et al*. The leading vehicle stops four times suddenly. (a) Space–time plot of the system controlled by the control signal of Zhao *et al*. (b) Temporal velocity behaviors of the first and 100-th vehicles.

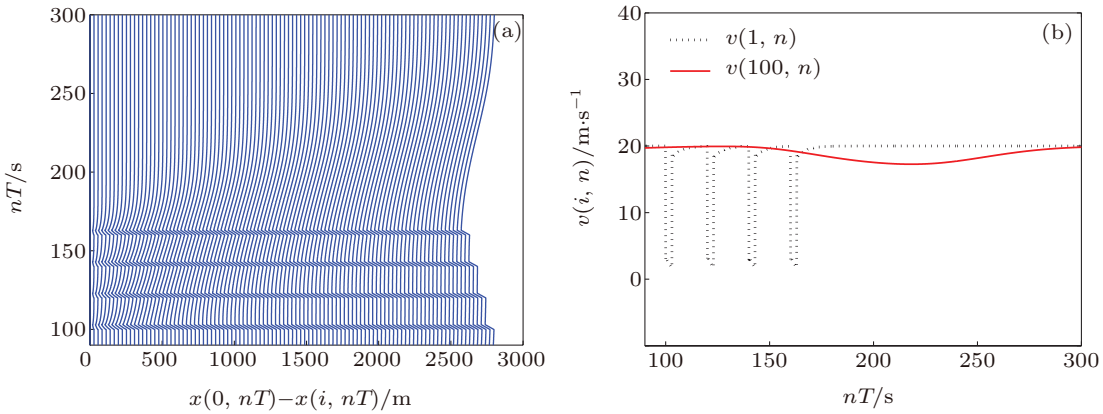


Fig. 4. (color online) Numerical simulations for the system controlled by our scheme proposed. The leading vehicle stops four times suddenly. (a) Space–time plot of the system controlled by our control signal proposed. (b) Temporal velocity behaviors of the first and 100-th vehicles.

For more details, we let the leading-vehicle stop suddenly for a short time: $v_0(n) = 0$, $nT = 100$–102 s, and observe three vehicles: the first, the 25-th, and the 50-th. Figures 5(a), 6(a), and 7(a) show the space–time plots of the system controlled by the control signals of Konishi *et al*. and Zhao *et al*., and our control signal, respectively. Figures 5(b), 6(b), and 7(b) show the temporal velocity behaviors of the three vehicles corresponding to Figs. 5(a), 6(a), and 7(a) respectively. From Figs. 5, 6, and 7, we can see that the suppression performance of our scheme on traffic jams is better than those of the other two schemes, although all three control schemes can suppress the traffic jam.

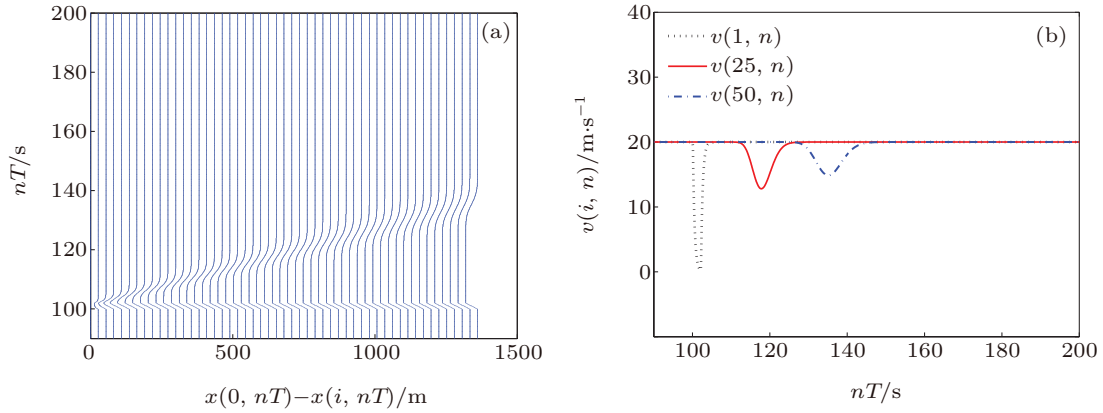


Fig. 5. (color online) Numerical simulations for the system controlled by the scheme of Konishi *et al*. The leading vehicle stops at $nT = 100$–102 s. (a) Space–time plot of the system controlled by the control signal of Konishi *et al*. (b) Temporal velocity behaviors of the first, 25-th, and 50-th vehicles.

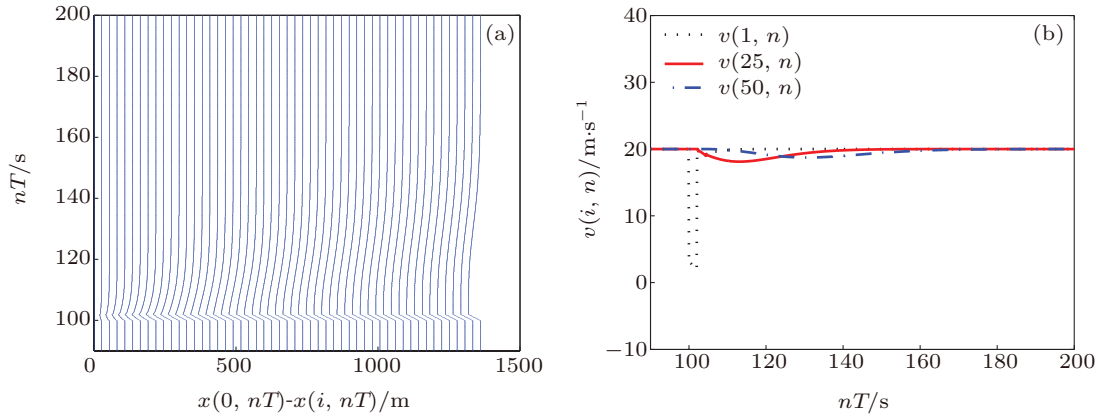


Fig. 6. (color online) Numerical simulations for the system controlled by the scheme of Zhao *et al.* The leading vehicle stops at $nT = 100$–102 s. (a) Space–time plot of the system controlled by the control signal of Zhao *et al.* (b) Temporal velocity behaviors of the first, 25-th, and 50-th vehicles.

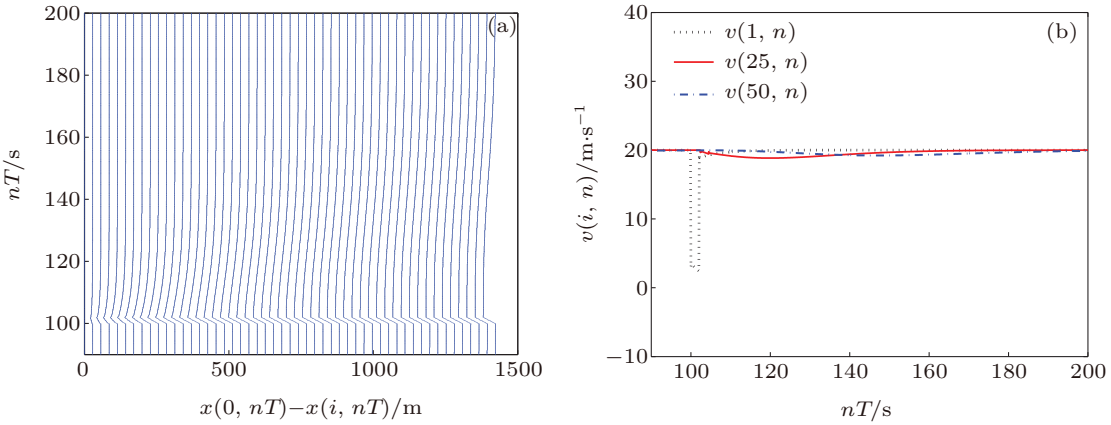


Fig. 7. (color online) Numerical simulations for the system controlled by our scheme. The leading vehicle stops at $nT = 100$–102 s. (a) Space–time plot of the system controlled by our control signal. (b) Temporal velocity behaviors of the first, 25-th, and 50-th vehicles.

5. Conclusions

In the paper, a new control scheme for suppressing the traffic jam is proposed, based on the pioneering work of Konishi *et al.*[14] The control effect of our control scheme on traffic congestions is investigated with the CM car-following model.

Compared with the previous schemes adopting the same parameters, our scheme has the following three advantages:

(i) traffic jams can be suppressed by introducing the feedback control signal;

(ii) the stability of a traffic system becomes better by considering the influence of safe headway;

(iii) the numerical simulations are in good agreement with theoretical analyses.

Additionally, our control scheme is simple and feasible for real traffic and would be applicable to any size of traffic system.

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