Ecological optimization for an irreversible magnetic Ericsson refrigeration cycle*

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An irreversible Ericsson refrigeration cycle model is established, in which multi-irreversibilities such as finite-rate heat transfer, regenerative loss, heat leakage, and the efficiency of the regenerator are taken into account. Expressions for several important performance parameters, such as the cooling rate, coefficient of performance (COP), power input, exergy output rate, entropy generation rate, and ecological function are derived. The influences of the heat leakage and the time of the regenerative processes on the ecological performance of the refrigerator are analyzed. The optimal regions of the ecological function, cooling rate, and COP are determined and evaluated. Furthermore, some important parameter relations of the refrigerator are revealed and discussed in detail. The results obtained here have general significance and will be helpful in gaining a deep understanding of the magnetic Ericsson refrigeration cycle.

Keywords: paramagnetic salt, Ericsson refrigerator, irreversibility, ecological function, parameter relations

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1. Introduction

Finite-time thermodynamics came into existence after the novel work of Curzon–Ahlborn[1–3] in the investigation of the performance of thermodynamic cycles, and various objective functions have been adopted for optimization.[4–8] Angulo–Brown[9] proposed an ecological optimization criterion for the finite-time Carnot heat engines as $E = P - T_L\sigma$, where P is the power output, σ is the entropy generation rate, and T_L is the cold-reservoir temperature. Yan[10] proposed that it is more reasonable to use $E = P - T_0\sigma$ when the cold reservoir temperature is not equal to the environment temperature T_0. Recently, the ecological function was extended by Yan[11,12] to optimize the performance of some refrigeration cycles and is defined as $E = R - \varepsilon_C T_0\sigma$, where R is the rate of refrigeration, $\varepsilon_C = T_C/(T_H - T_C)$ is the coefficient of performance (COP) of a reversible Carnot refrigerator operating between the two heat reservoirs at temperatures T_H and T_C, and the second term $\varepsilon_C T_0\sigma$ represents the dissipated refrigeration rate. Chen *et al.*[13] provided a unified ecological optimization objective for all thermodynamic cycles, $E = A/\tau - T_0\sigma$, where A is the exergy output and τ the time period of the cycle. For the refrigerator, the ecological optimization function is $E = A/\tau - T_0\sigma$, which represents a compromise between the exergy output rate $A/\tau = [Q_C(T_0/T_C - 1) - Q_H(T_0/T_H - 1)]/\tau$ and the exergy loss rate $T_0\sigma$, where Q_C is the released heat, Q_H is the absorbed heat, and $T_0\sigma$ is produced by entropy generation in the system and its surrounding.

Ecological performance optimization has been carried out for endoreversible and irreversible Carnot, Brayton, Stirling, and Ericsson heat engines,[14–18] the irreversible Carnot refrigerator,[19,20] and irreversible Carnot, Stirling, and Ericsson heat pumps.[21–23] Magnetic refrigeration is an environmentally-safe refrigeration technology that has been studied for a long time[24–28] for its potential use in refrigeration cycles, as an alternative to gases, especially at low temperatures. As increasing attention is paid to environmental protection and investigations into room-temperature magnetic refrigeration materials, magnetic refrigeration theory and technology have developed greatly.[29–34] In recent years, the performance optimization of magnetic cycles (magnetic engines, magnetic refrigerators, and magnetic heat pumps) has become one of the most interesting research subjects. However, most of the literature has only researched the optimal performance characteristics of magnetic Carnot,[35] Brayton,[36–43] Stirling,[44–47] and Ericsson[48–52] refrigeration cycles.

In this paper, we use the ecological optimization criterion to investigate the optimal performance of an irreversible Ericsson refrigeration cycle working with paramagnetic material systems. The irreversibilities of finite-rate heat transfer, regenerative loss, heat leakage, and the efficiency of the regenerator are considered. The cooling rate (R), COP (ε), power input (P), exergy output rate (A/τ), entropy generation rate (σ), and ecological function (E) are deduced. The influence of the heat leakage and the time of the regenerative processes on the ecological performance of the refrigerator are discussed. The results obtained are general and can provide some guidelines for the optimum design of real magnetic refrigerators.

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2. An irreversible model of the magnetic Ericsson refrigeration cycle

An irreversible Ericsson refrigeration cycle employing paramagnetic material (the magnetic intensity is not very high or the temperature is not very low) as the working substance may be represented by a field-temperature diagram, as illustrated schematically in Fig. 1. Processes A–B and C–D are the two isothermal ones, in which the temperatures of the working substance are T_1 and T_2, respectively; processes D–A and B–C are the two isomagnetic field ones, in which the magnetic fields are H_1 and H_2, respectively; Q_{AB} and Q_{CD} are the amounts of heat released into the reservoir at temperature T_H and absorbed from the reservoir at temperature T_C; Q_{DA} and Q_{BC} are the amounts of heat exchanged between the working substance and the regenerator during isomagnetic field processes H_1 and H_2, respectively; $Q_{\rm L}$ is the amount of heat leakage between the heat reservoirs T_H and T_C; and $Q_{\rm r}$ is the amount of regeneration.

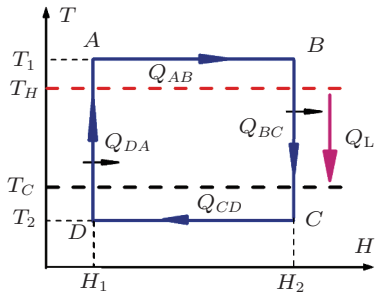


Fig. 1. (color online) The T–H diagram of an irreversible magnetic Ericsson refrigeration cycle.

According to the irreversible magnetic Ericsson refrigeration cycle model mentioned above, Q_{AB}, Q_{BC} , Q_{CD}, and Q_{DA} may be expressed as[51–54]

$$Q_{AB} = T_1 (S_A - S_B) = \frac{C\mu_0}{2T_1}\left(H_2^2 - H_1^2\right) = \Delta S/T_1, \quad (1)$$

$$Q_{BC} = \int_{T_2}^{T_1} C_{H_2}\,{\rm d}T = C\mu_0 H_2^2\left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad (2)$$

$$Q_{CD} = T_2 (S_D - S_C) = \frac{C\mu_0}{2T_2}\left(H_2^2 - H_1^2\right) = \Delta S/T_2, \quad (3)$$

and

$$Q_{DA} = \int_{T_2}^{T_1} C_{H_1}\,{\rm d}T = C\mu_0 H_1^2\left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad (4)$$

where $C = Ng^2\mu_{\rm B}^2 J(J+1)/3k_{\rm B}$ is the Curie constant, $\Delta S = C\mu_0 (H_2 - H_1)/2$. The amount of heat transfer between the working substance and the regenerator during the two regenerative processes is determined by

$$Q_{\rm r} = Q_{BC} - Q_{DA} = C\mu_0\left(H_2^2 - H_1^2\right)\left(\frac{1}{T_2} - \frac{1}{T_1}\right). \quad (5)$$

Taking into account the finite rate heat-transfer, there is an additional regenerative loss besides the inherent regenerative loss $Q_{\rm r}$. The additional regenerative loss may be expressed as

$$\Delta Q_{\rm r} = (1 - \eta_{\rm r})\, Q_{\rm r}, \quad (6)$$

where $\eta_{\rm r}$ is the efficiency of the regenerator.

When the influence of thermal resistance on the performance of a magnetic Ericsson refrigeration cycle is taken into account, the temperatures T_1 and T_2 of the working substance in the two isothermal processes are different from T_H and T_C. When heat transfer obeys the Newtonian heat-transfer law,[41,46–50,55] one has

$$Q_{AB} = k_1 (T_1 - T_H)\, t_1 \quad (7)$$

and

$$Q_{CD} = k_2 (T_C - T_2)\, t_2, \quad (8)$$

where k_1, k_2 are the thermal conductances between the working substance and the reservoirs at temperatures T_H and T_C, respectively, and t_1 and t_2 are the corresponding heat exchange times.

Due to the existence of non-perfect regeneration and finite rate heat-transfer, the times of the regenerative processes may be written into[49,50]

$$t_3 = t_4 = \gamma\left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad (9)$$

where γ is a proportional constant that is independent of temperature but depends on the properties of the regenerative material.

In addition, heat leakage per cycle is also inevitable for magnetic refrigerators. It is supposed that heat leakage exists between reservoirs T_H and T_C and is given by[49,50]

$$Q_{\rm L} = k_{\rm L} (T_H - T_C)\, \tau, \quad (10)$$

where $k_{\rm L}$ is the heat leakage coefficient. Thus, the cycle period is given by

$$\tau = t_1 + t_2 + t_3 + t_4. \quad (11)$$

Based on the above analysis, the amounts of heat absorbed from the cold source Q_C and released to the heat reservoir Q_H are, respectively, given below

$$\begin{aligned} Q_C &= Q_{CD} - Q_{\rm L} - (Q_{\rm r} + \Delta Q_{\rm r}) \\ &= \left[1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)\right]\Delta S \\ &\quad - k_{\rm L}(T_H - T_C)\left[\frac{1}{k_2 T_2 (T_C - T_2)}\right. \\ &\quad \left. + \frac{1}{k_1 T_1 (T_1 - T_H)} + \frac{2\gamma}{\Delta S}(1/T_2 - 1/T_1)\right]\Delta S \end{aligned} \quad (12)$$

and

$$\begin{aligned} Q_H &= Q_{AB} - Q_{\rm L} - \Delta Q_{\rm r} \\ &= [1/T_1 - 2(1-\eta_{\rm r})(1/T_2 - 1/T_1)]\Delta S \\ &\quad - k_{\rm L}(T_H - T_C)\left[\frac{1}{k_2 T_2 (T_C - T_2)}\right. \\ &\quad \left. + \frac{1}{k_1 T_1 (T_1 - T_H)} + \frac{2\gamma}{\Delta S}(1/T_2 - 1/T_1)\right]\Delta S. \end{aligned} \quad (13)$$

Using Eqs. (12) and (13), one can give the total work input per cycle

$$W = Q_H - Q_C = (1/T_2 - 1/T_1)\Delta S. \quad (14)$$

Using Eqs. (11) and (12), one can give the cooling rate (R)

$$R = \frac{Q_C}{\tau}$$

$$= \frac{1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)}{\dfrac{1}{k_2 T_2 (T_C - T_2)} + \dfrac{1}{k_1 T_1 (T_1 - T_H)} + \dfrac{2\gamma}{\Delta S}(1/T_2 - 1/T_1)} - k_{\rm L}(T_H - T_C). \quad (15)$$

Using Eqs. (12) and (14), one can give the COP (ε)

$$\begin{aligned} \varepsilon &= \frac{Q_C}{W} = \frac{1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)}{1/T_2 - 1/T_1} \\ &\quad - \frac{k_{\rm L}(T_H - T_C)}{1/T_2 - 1/T_1}\left[\frac{1}{k_2 T_2 (T_C - T_2)} + \frac{1}{k_1 T_1 (T_1 - T_H)}\right. \\ &\quad \left. + \frac{2\gamma}{\Delta S}(1/T_2 - 1/T_1)\right]. \end{aligned} \quad (16)$$

Using Eqs. (11), (12), and (13), one can give the exergy output rate (A/τ) and the entropy generation rate (σ), respectively

$$\begin{aligned} A/\tau &= [Q_C(T_0/T_C - 1) - Q_H(T_0/T_H - 1)]/\tau \\ &= \frac{\{[1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)](T_0/T_C - 1) - [1/T_1 - 2(1-\eta_{\rm r})(1/T_2 - 1/T_1)](T_0/T_H - 1)\}}{1/k_2T_2(T_C - T_2) + 1/k_1T_1(T_1 - T_H) + (2\gamma/\Delta S)(1/T_2 - 1/T_1)} \\ &\quad - k_{\rm L}(T_H - T_C)T_0(1/T_C - 1/T_H) \end{aligned} \quad (17)$$

and

$$\begin{aligned} \sigma &= \frac{1}{\tau}\left(\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right) = \frac{[1/T_1 - 2(1-\eta_{\rm r})(1/T_2 - 1/T_1)/T_H - 1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)/T_C]}{1/k_2T_2(T_C - T_2) + 1/k_1T_1(T_1 - T_H) + (2\gamma/\Delta S)(1/T_2 - 1/T_1)} \\ &\quad + k_{\rm L}(T_H - T_C)(1/T_C - 1/T_H). \end{aligned} \quad (18)$$

Using Eqs. (17) and (18), the ecological function (E) is given by

$$\begin{aligned} E &= A/\tau - T_0\sigma = [Q_C(2T_0/T_C - 1) - Q_H(2T_0/T_H - 1)]/T_C - 1) - Q_H(2T_0/\tau \\ &= \frac{\{[1/T_2 - 2(2-\eta_{\rm r})(1/T_2 - 1/T_1)](2T_0/T_C - 1) - [1/T_1 - 2(1-\eta_{\rm r})(1/T_2 - 1/T_1)](2T_0/T_H - 1)\}}{1/k_2T_2(T_C - T_2) + 1/k_1T_1(T_1 - T_H) + (2\gamma/\Delta S)(1/T_2 - 1/T_1)} \\ &\quad - 2k_{\rm L}T_0(T_H - T_C)(1/T_C - 1/T_H). \end{aligned} \quad (19)$$

It is clearly seen from Eqs. (15)–(19) that the cooling rate (R), COP (ε), power input (P), exergy output rate (A/τ), entropy generation rate (σ), and ecological function (E) are all functions of T_1 and T_2 for given T_H, T_C, k_1, k_2, $k_{\rm L}$, γ. From Eq. (19), we can obtain the three-dimensional performance characteristic curves of the ecological function (E) varying with temperatures T_1 and T_2 for the parameters $T_C = 260$ K, $T_H = 300$ K, $T_0 = 290$ K, $\eta_{\rm r} = 0.98$, $k_{\rm B} = 1.38 \times 10^{-23}$ J/K, $k_1 = k_2 = 1.0 \times 10^{-12}$ W/K, $\gamma = 3.0 \times 10^{-14}$, and $k_{\rm L} = 0.5 \times 10^{-15}$ W/K, as shown in Fig. 2. $E^* = E/kT_C$ is the dimensionless ecological function, $k = k_1k_2/(\sqrt{k_1} + \sqrt{k_2})^2$. Figure 2 shows the existence of the optimal temperatures T_1 and T_2 of the working medium in the isothermal processes, which leads to the maximum dimensionless ecological function for the irreversible magnetic Ericsson refrigeration cycle.

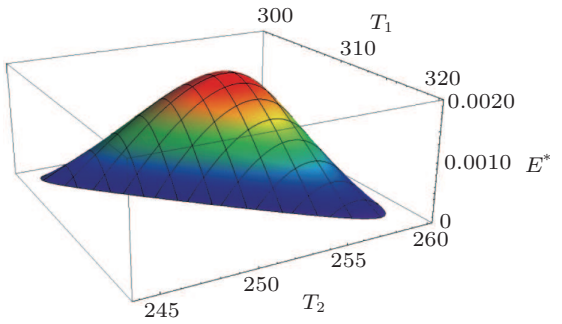


Fig. 2. (color online) The dimensionless ecological function $E^* = E/kT_C$ versus temperatures T_1 and T_2 for parameters $T_C = 260$ K, $T_H = 300$ K, $T_0 = 290$ K, $\eta_{\rm r} = 0.98$, $\gamma = 3.0 \times 10^{-14}$, $k_1 = k_2 = 1.0 \times 10^{-12}$, $k_{\rm L} = 0.5 \times 10^{-15}$ W/K, $g = 10$, $J = 7/2$, $H_1 = 2$ T, and $H_2 = 5$ T.

3. Ecological optimization performance of the refrigerator

According to Eq. (19) and the extreme condition $(\mathrm{d}E/\mathrm{d}T_1)_x = 0$, we can obtain the following equation

$$T_1 = T_C + xT_H\sqrt{k_1/k_2}/x + x\sqrt{k_1/k_2}, \tag{20}$$

where $x = T_2/T_1$. Substituting Eq. (20) into Eqs. (15)–(19), we can obtain the optimal cooling rate (R), optimal COP (ε), optimal exergy output rate (A/τ), optimal entropy generation rate (σ), and optimal ecological function (E) of an irreversible magnetic Ericsson refrigerator, respectively, expressed as follows:

$$R = \frac{Q_C}{t} = \frac{k_1^{3/2}k_2^{3/2}[3+2x(\eta_\mathrm{r}-2)-2\eta_\mathrm{r}](xT_H-T_C)\Delta S}{x^2\sqrt{k_1k_2}\Delta S + k_1[\Delta S(1+x^2+\sqrt{k_1/k_2})+2\gamma\sqrt{k_1k_2}(x-1)(xT_H-T_C)]} - k_\mathrm{L}(T_H-T_C), \tag{21}$$

$$\varepsilon = \frac{Q_C}{W} = \frac{x^2\sqrt{k_1k_2}k_\mathrm{L}(T_C-T_H)\Delta S + k_1\left\{\begin{array}{l}\Delta S(1+x^2+\sqrt{k_1/k_2})k_\mathrm{L}(T_C-T_H)\\ -\sqrt{k_1k_2}\left[\begin{array}{l}\Delta S(\,3-4x+2(x-1)\eta_\mathrm{r}\,)\\ +2(x-1)\gamma k_\mathrm{L}(T_C-T_H)\end{array}\right]\end{array}\right\}}{k_1^{3/2}k_2^{3/2}(x-1)(xT_H-T_C)\Delta S}, \tag{22}$$

$$\begin{aligned} A/\tau &= [Q_C(T_0/T_C-1)-Q_H(T_0/T_C-1)-Q_H(T_0/T_H-1)]/\tau \\ &= \frac{\Delta S k_1^{3/2}k_2^{3/2}(xT_H-T_C)\left\{(x-1)T_CT_H+T_0\left[\begin{array}{l}(-2+3x+2\eta_\mathrm{r}-2x\eta_\mathrm{r})T_C\\ +(3-4x-2\eta_\mathrm{r}+2x\eta_\mathrm{r})T_H\end{array}\right]\right\}}{T_CT_H\left\{\Delta Sx^2\sqrt{k_1k_2}+k_1\left[\begin{array}{l}\Delta S(1+x^2+\sqrt{k_1/k_2})\\ +2(x-1)\gamma\sqrt{k_1k_2}(xT_H-T_C)\end{array}\right]\right\}} \\ &\quad -k_\mathrm{L}T_0(T_H-T_C)(1/T_C-1/T_H), \end{aligned} \tag{23}$$

$$\begin{aligned} \sigma &= \frac{1}{\tau}\left(\frac{Q_H}{T_H}-\frac{Q_C}{T_C}\right) = \frac{\Delta S k_1^{3/2}k_2^{3/2}(xT_H-T_C)\left[\begin{array}{l}(2-3x+2\eta_\mathrm{r}+2x\eta_\mathrm{r})T_C\\ +(-3+4x+2\eta_\mathrm{r}-2x\eta_\mathrm{r})T_H\end{array}\right]}{T_CT_H\left\{\Delta Sx^2\sqrt{k_1k_2}+k_1\left[\begin{array}{l}\Delta S(1+x^2+\sqrt{k_1/k_2})\\ +2(x-1)\gamma\sqrt{k_1k_2}(xT_H-T_C)\end{array}\right]\right\}} \\ &\quad +k_\mathrm{L}(T_H-T_C)(1/T_C-1/T_H), \end{aligned} \tag{24}$$

and

$$\begin{aligned} E &= A/\tau - T_0\sigma = \frac{\Delta S k_1^{3/2}k_2^{3/2}(xT_H-T_C)\left\{(x-1)T_CT_H+2T_0\left[\begin{array}{l}(-2+3x+2\eta_\mathrm{r}-2x\eta_\mathrm{r})T_C\\ +(3-4x-2\eta_\mathrm{r}+2x\eta_\mathrm{r})T_H\end{array}\right]\right\}}{T_CT_H\left\{\Delta Sx^2\sqrt{k_1k_2}+k_1\left[\begin{array}{l}\Delta S(1+x^2+\sqrt{k_1/k_2})\\ +2(x-1)\gamma\sqrt{k_1k_2}(xT_H-T_C)\end{array}\right]\right\}} \\ &\quad -2k_\mathrm{L}T_0(T_H-T_C)(1/T_C-1/T_H). \end{aligned} \tag{25}$$

Using Eqs. (21)–(25), one can reveal the general performance characteristics of the irreversible magnetic Ericsson refrigeration cycle and give the optimal criteria of some important performance parameters.

It is clearly seen from Eqs. (21)–(25) that the optimal cooling rate (R), optimal coefficient of performance (ε), optimal exergy output rate (A/τ), optimal entropy generation rate (σ), and optimal ecological function (E) of an irreversible magnetic Ericsson refrigerator are all functions of the temperature ratio $x = T_2/T_1$ of the working medium and the efficiency η_r of the regenerator, for given T_C, T_H, T_0, γ, η_r, k_1, k_2, k_L, g, J, H_1, and H_2.

Using Eqs. (21), (22), and (25), one can generate R^*–ε, E^*–ε, and E^*–R^* curves of the irreversible magnetic Ericsson refrigeration cycle for given $T_C = 260$ K, $T_H = 300$ K, $T_0 = 290$ K, $\eta_\mathrm{r} = 0.98$, $\gamma = 2.0\times10^{-14}$, $k_1 = k_2 = 1.0\times10^{-12}$, $k_\mathrm{L} = 3.0\times10^{-15}$, $g = 10$, $J = 7/2$, $H_1 = 2$ T, and $H_2 = 5$ T values of the parameters, as shown in Figs. 3–5. Figure 3 indicates that for the irreversible magnetic Ericsson refrigeration cycle, when the heat leakage coefficient (k_L) is considered, there exists a maximum cooling rate (R_max) with corresponding COP (ε_R) and a maximum COP (ε_max) with corresponding cooling rate (R_ε). Considering the cooling rate (R), the optimal operating region of the irreversible magnetic Ericsson refrigeration cycle should be situated in

$$R_\varepsilon \le R \le R_\mathrm{max}, \tag{26}$$

and

$$\varepsilon_R \le \varepsilon \le \varepsilon_\mathrm{max}, \tag{27}$$

where R_ε, R_max, ε_R, and ε_max are four important performance

bounds of the irreversible magnetic Ericsson refrigeration cycle.

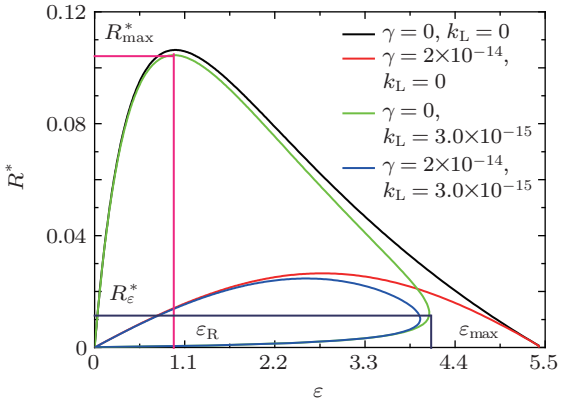


Fig. 3. (color online) The variation in dimensionless ecological function $E^* = E/kT_C$ with respect to the COP ε. The black, red, green, and blue curves are $\gamma = 0$, $k_L = 0$; $\gamma = 2.0\times10^{-14}$, $k_L = 0$; $\gamma = 0$, $k_L = 3.0\times10^{-15}$, and $\gamma = 2.0\times10^{-14}$, $k_L = 3.0\times10^{-15}$, respectively. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 2.

Figure 4 shows that the ecological function (E) is also a hyperbolic function of the COP (ε) for the magnetic Ericsson refrigeration cycle. The increase in the times of the regenerative processes (a proportional constant (γ)), decreases the ecological function, while the ecological function (E) versus COP (ε) characteristic is a parabolic-like one. However, the heat leakage coefficient (k_L) makes the ecological function (E) versus COP (ε) characteristic become loop-shaped. There exists a maximum ecological function $(E_{\max})$ with the corresponding COP (ε_E) and a maximum COP $(\varepsilon_{\max})$ with the corresponding ecological function (E_ε). Similarly, considering the ecological function (E), the optimal operating region of the irreversible magnetic Ericsson refrigeration cycle should be situated in

$$E_\varepsilon \le E \le E_{\max} \tag{28}$$

and

$$\varepsilon_E \le \varepsilon \le \varepsilon_{\max}, \tag{29}$$

where E_ε and ε_E are two other important performance bounds of the irreversible magnetic Ericsson refrigeration cycle. These two ranges provide a region with a negative slope, which is referred to as the optimal region of the refrigerator. E_ε and $E_{\max}$, as well as ε_E and $\varepsilon_{\max}$, are four important performance parameters that, respectively, determine the lower and upper bounds of the optimal ecological function (E) and the optimal COP (ε).

Figure 5 illustrates that the ecological function (E) is a hyperbolic function of the cooling rate (R). The corresponding cooling rate (R) has two values when the ecological function (E) is fixed, except at the maximum point. The ecological function (E) decreases with the increasing heat leakage coefficient (k_L) and the times of the regenerative processes (a proportional constant (γ)). The irreversible magnetic Ericsson refrigeration cycle should work at the point where the cooling rate (R) value is larger

$$R_E \le R \le R_{\max}, \tag{30}$$

where the cooling rate (R_E) is another important performance bound of the irreversible magnetic Ericsson refrigeration cycle.

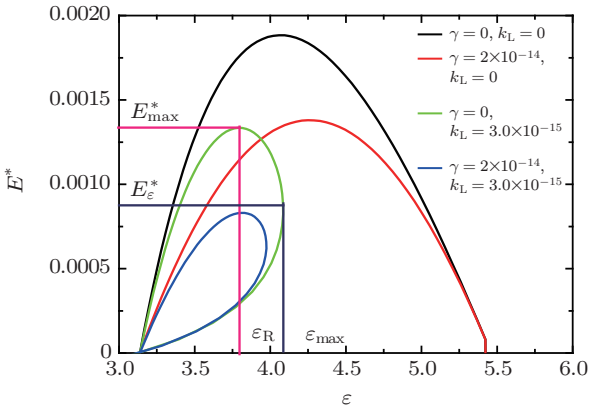


Fig. 4. (color online) The variation in the dimensionless ecological function $E^* = E/kT_C$ with respect to the COP ε. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

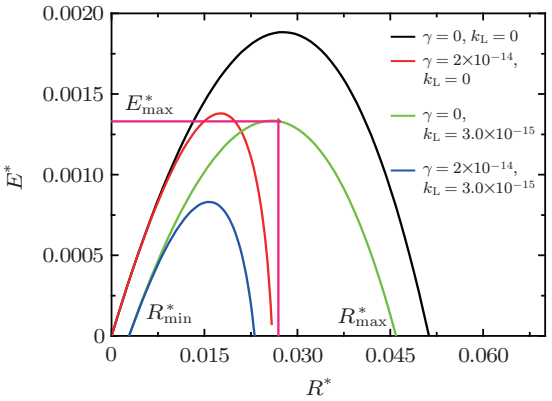


Fig. 5. (color online) The variation in the dimensionless ecological function $E^* = E/kT_C$ with respect to the dimensionless cooling rate $R^* = R/kT_C$. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

4. Some important parameter relations

According to Eq. (21) and the extreme condition $\mathrm{d}R/\mathrm{d}x = 0$, we can obtain the following equation

$$k_1\left[\Delta S\begin{pmatrix}4x(-2+\eta_r)+3\\+4y-2(1+y)\eta_r\end{pmatrix}+2(x-y)^2\gamma k_2 T_H\right] + k_2[\alpha_2 x^2+\alpha_1 x+\alpha_0]\Delta S = 0. \tag{31}$$

Solving Eq. (31), we get the result

$$x_R = \frac{\left\{\begin{array}{l} 8\Delta S k_1 - 4\Delta S \eta_r k_1 - \alpha_1 \Delta S k_2 + 4y\gamma k_1 k_2 T_H \\ -\sqrt{\left\{\begin{array}{l}[\alpha_1 \Delta S k_2 + 4k_1(\Delta S(-2+\eta_r) - y\gamma k_2 T_H)]^2 \\ -4k_2(\,\alpha_2 \Delta S + 2\gamma k_1 T_H\,)[\,\alpha_0 \Delta S k_2 + k_1(\,\alpha_3 \Delta S + 2y^2 \gamma k_2 T_H\,)\,]\end{array}\right\}}\end{array}\right\}}{2k_2(\alpha_2 S + 2\gamma k_1 T_H)}, \tag{32}$$

where $y = T_C/T_H$ and $\alpha_0 = (3+4y-2(1+y)\eta_r)\sqrt{k_1/k_2}$, $\alpha_1 = 4(\eta_r - 2)\sqrt{k_1/k_2} - 2y(2\eta_r - 3)(1+\sqrt{k_1/k_2})$, $\alpha_2 = (2y(\eta_r - 2) + 2\eta_r - 3)(1+\sqrt{k_1/k_2})$, and $\alpha_3 = 3 + 4y - 2(1+y)\eta_r$. Substituting Eq. (32) into Eqs. (21)–(25), we can obtain the maximum cooling rate($R_{\max}$), the corresponding COP (ε_R), the corresponding entropy generation rate (σ_R), and the corresponding ecological function (E_R). However, they are too complicated to yield simple analytical solutions.

According to Eq. (22) and the extreme condition $\mathrm{d}\varepsilon/\mathrm{d}x = 0$, we can obtain the following equation

$$\begin{aligned} &k_1[(x-y)^2 k_2 + (y-1)(1-2x+y)k_L] \\ &-k_L(y-1)[x(x+(x-2)y)k_2 + 2(x^2-y)\sqrt{k_1 k_2}] = 0. \end{aligned} \tag{33}$$

Solving Eq. (33), we get the result

$$x_\varepsilon = \frac{\left\{\begin{array}{l} yk_1k_2 + (y-1)(k_1 - yk_2)k_L \\ -(1-y)\sqrt{k_L}\sqrt{\begin{array}{l} yk_2k_L[yk_2 + 2(1+y)\sqrt{k_1k_2}] + k_1^2(k_2+k_L) \\ +k_1\left\{\begin{array}{l} y^2k_2^2 + 2(1+y)\sqrt{k_1k_2}k_L \\ +k_2[\,2y\sqrt{k_1k_2} + (\,1+4y+y^2\,)k_L\,]\end{array}\right\}\end{array}}\end{array}\right\}}{k_1k_2 - (y-1)[(1+y)k_2 + 2\sqrt{k_1k_2}]k_L}. \tag{34}$$

Substituting Eq. (34) into Eqs. (21)–(25), we can obtain the maximum COP ($\varepsilon_{\max}$), the corresponding cooling rate (R_ε), the corresponding entropy generation rate (σ_ε), and the corresponding ecological function (E_ε). We can gain the maximum COP ($\varepsilon_{\max}$) and the corresponding x_ε, but these are also too complicated to yield simple analytical solutions.

Similarly, according to Eq. (23) and the extreme condition $\mathrm{d}E/\mathrm{d}x = 0$, we can obtain the following equation

$$\begin{aligned} &\Delta S k_2\{[\phi_2 x^2 - \phi_1 x - \phi_0]yT_H - 2(\beta_2 x^2 + \beta_1 x + \beta_0)T_0\} \\ &+k_1\{2\Delta S[3 - 2\eta_r + x(6y - 8 + 4\eta_r - 4y\eta_r) \\ &+y(2 - 3y + 2y\eta_r)]T_0 \\ &-[\Delta S y(1 - 2x + y) + 4(x-y)^2 \\ &\times (y-1)\gamma k_2 T_0]T_H\} = 0, \end{aligned} \tag{35}$$

where

$$\begin{aligned} \phi_0 &= (1+y)\sqrt{k_1/k_2}, \\ \phi_1 &= 2(y - \sqrt{k_1/k_2} + y\sqrt{k_1/k_2}), \\ \phi_2 &= (1+y)(1+\sqrt{k_1/k_2}), \\ \beta_0 &= (y(-2 + y(3 - 2\eta_r)) + 2\eta_r - 3)\sqrt{k_1/k_2}, \\ \beta_1 &= [4(2-\eta_r)\sqrt{k_1/k_2} + 2y(2\eta_r - 3)(1 + 2\sqrt{k_1/k_2}) \\ &\quad -4y^2(\eta_r - 1)(1+\sqrt{k_1/k_2}), \\ \beta_2 &= (3 - 2\eta_r + y(2 + y(2\eta_r - 3)))(1+\sqrt{k_1/k_2}). \end{aligned}$$

Solving Eq. (35), we get the result

$$x_E = \frac{\left\{\begin{array}{l} \left\{\begin{array}{l} 2\Delta S[(8 - 4\eta_r + y(4\eta_r - 6))k_1 + \beta_1 k_2]k_2 \\ +\Delta S y(\phi_1 k_2 - 2k_1)T_H - 8(y-1)y\gamma k_1 k_2 T_0 T_H \end{array}\right\} \\ +\sqrt{\{[\Delta S k_2(2\beta_1 T_0 + y\phi_1 T_H) + 2k_1\varphi_0]^2 - 4k_2[2\Delta S\varphi_1 T_0 + y\varphi_2\varphi_3 T_H]\}}\end{array}\right\}}{2k_2\{-2\beta_2\Delta S T_0 + [\phi_2 \Delta S y - 4(-1+y)\gamma k_1 T_0]T_H\}}, \tag{36}$$

where

$$\begin{aligned} \varphi_0 &= 2\Delta S(4 - 3y + 2(y-1)\eta_r)T_0 - y(\Delta S + 4(y-1)\gamma k_2 T_0)T_H, \\ \varphi_1 &= (y(y(3 - 2\eta_r) - 2) + 2\eta_r - 3)k_1 + \beta_0 k_2, \\ \varphi_2 &= \Delta S\phi_0 k_2 + k_1(\Delta S(1+y) + 4(y-1)y\gamma k_2 T_0), \\ \varphi_3 &= 2T_0(\beta_2 \Delta S + 2(y-1)\gamma k_1 T_H) - \phi_2 \Delta S y T_H. \end{aligned}$$

Substituting Eq. (36) into Eqs. (21)–(25), we obtain the maximum ecological function ($E_{\max}$), the corresponding cooling rate (R_E), the corresponding COP (ε_E), and the corresponding entropy generation rate (σ_E). However, they are still

too complicated to yield simple analytical solutions.

Using Eqs. (21)–(25), (32), (34), and (36), by means of a numerical approach, one can reveal the general performance characteristics of the irreversible magnetic Ericsson refrigeration cycle and give the relation of some important parameters. It can be seen from Fig. 6 that both the cooling rate R_ε and R_E first increase and then decrease as the temperature ratio T_C/T_H increases, while the maximum cooling rate $R_{\max}$ increases. Three cooling rates, $R_{\max}$, R_ε, and R_E, have the relations

$$R_\varepsilon \leq R_E \leq R_{\max}. \tag{37}$$

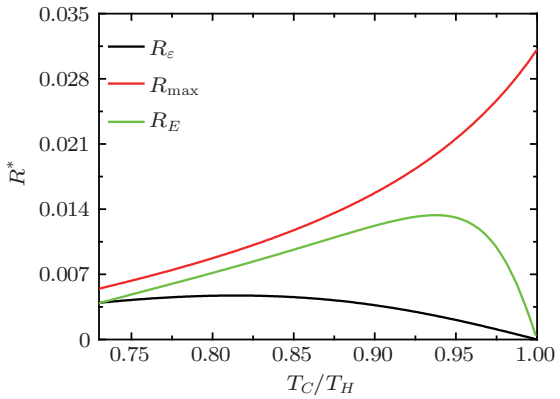


Fig. 6. (color online) Three dimensionless cooling rates, $R^*_{\max}$, R^*_ε, and R^*_E, versus the temperature ratio T_C/T_H. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

It can be seen from Fig. 7 that three COP, $\varepsilon_{\max}$, ε_E, and ε_R, increase as the temperature ratio T_C/T_H increases. Three COP, $\varepsilon_{\max}$, ε_E, and ε_R, have the relations

$$\varepsilon_R \leq \varepsilon_E \leq \varepsilon_{\max}. \tag{38}$$

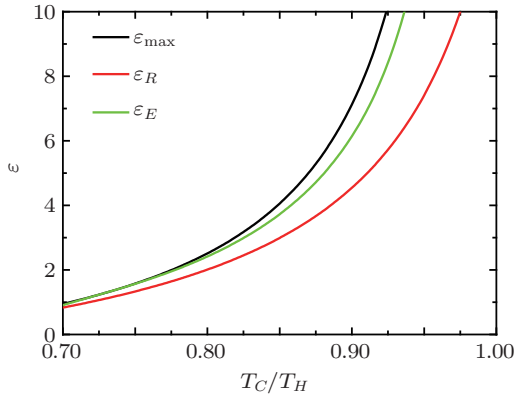


Fig. 7. (color online) Three COP, $\varepsilon_{\max}$, ε_E, and ε_R, versus temperature ratio T_C/T_H. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

It can be seen from Fig. 8 that both the entropy generation rate σ_E and σ_ε decrease as the temperature ratio T_C/T_H increases, while the entropy generation rate σ_R first decreases and then increases. Three entropy generation rates, σ_E, σ_ε, and σ_R, have the relations

$$\sigma_\varepsilon \leq \sigma_E \leq \sigma_R. \tag{39}$$

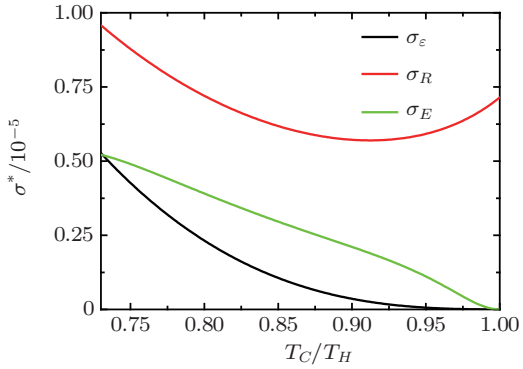


Fig. 8. (color online) Three dimensionless entropy generation rates, σ^*_R, σ^*_E, and σ^*_ε, versus the temperature ratio T_C/T_H. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

It can be seen from Fig. 9 that, as the temperature ratio T_C/T_H increases, three ecological functions, $E_{\max}$, E_ε, and E_R, first increase and then decrease. The three ecological functions, $E_{\max}$, E_ε, and E_R, have the relations

$$E_R \leq E_\varepsilon \leq E_{\max}. \tag{40}$$

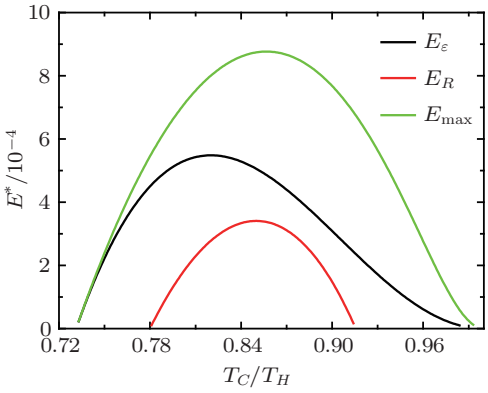


Fig. 9. (color online) Three dimensionless ecological functions, $E^*_{\max}$, E^*_ε, and E^*_R, versus the temperature ratio T_C/T_H. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

Similarly, using Eqs. (21)–(25), Eqs. (32), (34), and (36), by means of a numerical approach we can also obtain the ratio of the cooling rate $R^*_E/R^*_{\max}$, the COP $\varepsilon_E/\varepsilon_{\max}$, and the entropy generation rate σ^*_E/σ^*_R versus the temperature ratio T_C/T_H, as shown in Fig. 10, respectively. The ratio of the cooling rate $R^*_E/R^*_{\max}$ can reach more than 80% and the COP $\varepsilon_E/\varepsilon_{\max}$ can reach more than 60%, while the entropy generation rate σ^*_E/σ^*_R is less than 60%. Taking the maximum ecological optimization criterion as the objective provides a larger improvement in the cooling rate (R), the COP (ε), and the reduction in the entropy generation rate (σ). Therefore, using the ecological optimization criterion to investigate the optimal performance of an irreversible Ericsson refrigerator is effective and advantageous. It can reduce the amount of energy needed for the refrigerator.

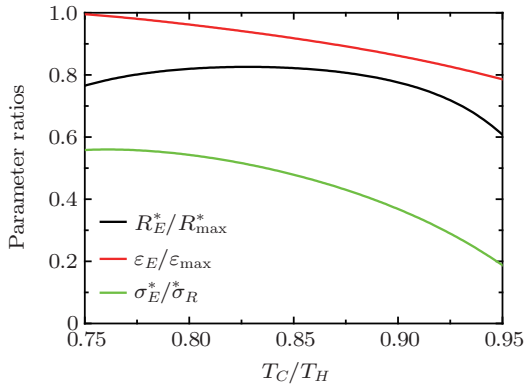


Fig. 10. (color online) The parameter ratios of the cooling rate $R_E^*/R_{\max}^*$, the COP $\varepsilon_E/\varepsilon_{\max}$, and the entropy generation rate σ_E^*/σ_R^* versus the temperature ratio T_C/T_H. The values of parameters T_C, T_H, η_r, k_1, k_2, k_L, γ, g, J, H_1, and H_2 are the same as those used in Fig. 3.

5. Conclusion

An irreversible Ericsson refrigeration cycle model is established, in which multi-irreversibilities such as finite-rate heat transfer, regenerative loss, heat leakage, and the efficiency of the regenerator are taken into account. Expressions for several important performance parameters, such as the cooling rate (R), COP (ε), power input (P), exergy output rate (A/τ), entropy generation rate (σ), and ecological function (E) are derived. The influence of the heat leakage and the time of the regenerative processes, (γ), on the ecological performance of the refrigerator are discussed. By means of a numerical approach, the optimal regions of the ecological function, cooling rate, and COP are determined and evaluated. Furthermore, some important parameter relations of the refrigerator are revealed and discussed in detail. The conclusions obtained here are of importance in the optimal design and reasonable use of the energy of real refrigerators.

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