A comparison of different entransy flow definitions and entropy generation in thermal radiation optimization*

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In thermal radiation, taking heat flow as an extensive quantity and defining the potential as temperature T or the blackbody emissive power U will lead to two different definitions of radiation entransy flow and the corresponding principles for thermal radiation optimization. The two definitions of radiation entransy flow and the corresponding optimization principles are compared in this paper. When the total heat flow is given, the optimization objectives of the extremum entransy dissipation principles (EEDPs) developed based on potentials T and U correspond to the minimum equivalent temperature difference and the minimum equivalent blackbody emissive power difference respectively. The physical meaning of the definition based on potential U is clearer than that based on potential T, but the latter one can be used for the coupled heat transfer optimization problem while the former one cannot. The extremum entropy generation principle (EEGP) for thermal radiation is also derived, which includes the minimum entropy generation principle for thermal radiation. When the radiation heat flow is prescribed, the EEGP reveals that the minimum entropy generation leads to the minimum equivalent thermodynamic potential difference, which is not the expected objective in heat transfer. Therefore, the minimum entropy generation is not always appropriate for thermal radiation optimization. Finally, three thermal radiation optimization examples are discussed, and the results show that the difference in optimization objective between the EEDPs and the EEGP leads to the difference between the optimization results. The EEDP based on potential T is more useful in practical application since its optimization objective is usually consistent with the expected one.

Keywords: thermal radiation, entransy flow, entropy generation, optimization

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1. Introduction

Great attention has been paid to the field of heat transfer enhancement under the background of the global energy shortage.[1] The traditional heat transfer enhancement techniques originated mainly from engineering practices rather than theories, and the extra costs (such as the power consumption, the heat transfer area, etc.) increase rapidly with the enhancement of heat transfer.[2] Therefore, the concept of heat transfer optimization has aroused the interest of researchers and the relevant theories for the optimization are urgently needed.

From the non-equilibrium thermodynamic viewpoint, Bejan[3] presented the entropy generation minimization to optimize heat transfer processes. This principle has been widely used in heat transfer optimizations.[4–8] However, when it was used to analyze balanced counter flow heat exchangers, the "entropy generation paradox" was noted.[3] The heat exchanger effectiveness does not always increase with entropy generation number decreasing.

Based on the energy equation of the convection heat transfer, Guo *et al.*[9] introduced a new viewpoint of the physical mechanism of heat convection, and found that the essence of heat convection is heat conduction with an inner heat source. With this perspective, they developed the field synergy principle, which has been well used in various convective heat transfer processes.[10–13]

In recent years, Guo *et al.*[14] introduced the concept of entransy based on the analogy between heat and electric conductions. For an object with internal energy E and temperature T, its entransy is

$$G = \frac{1}{2}ET. \tag{1}$$

Entransy has a nature of "energy", which is the potential of the internal energy, and represents the total ability for the object to release heat. In any heat transfer process, energy is conserved, but entransy is always dissipated because of the irreversibility of heat transfer.[14,15] The expression of the entransy dissipation rate per unit volume is[14]

$$\phi_{\mathrm{h}} = -\dot{q}\cdot\nabla T = k(\nabla T)^2, \tag{2}$$

where $\dot{q}$ is the heat flux, ∇T the temperature gradient, and k the thermal conductivity. Guo *et al.*[14] also proposed the extremum entransy dissipation principle (EEDP), which states that the extremum entransy dissipation corresponds to the optimal heat transfer performance when the total heat flow or the

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equivalent temperature difference is prescribed. Furthermore, the EEDP can be generalized into the minimum equivalent thermal resistance principle (METRP).[14,16] The entransy theory has been used in various heat transfer optimizations,[17–22] and relevant studies show that the entransy theory is more appropriate for optimizing the heat conduction and convection than the entropy generation minimization in many cases.[23–25] Research also shows that the entransy theory is superior to the field synergy principle in heat transfer optimization with a prescribed boundary heat flux.[26]

Thermal radiation is another kind of basic heat transfer process, and its mechanism is intrinsically different from heat conduction and convection. Some studies were carried out to develop the entransy theory for thermal radiation. For instance, Wu and Liang[27] and Wu[28] defined the radiation potential as temperature (T), and utilized the same definition of entransy flow as that for heat conduction and heat convection. They developed the radiation entransy theory, and applied it to the optimization of a radiation system composed of three infinite parallel-plates. Cheng *et al.*[29] found that the concept of "entransy" exists universally in nature. For a system in which a transfer process occurs and there is only one extensive quantity, entransy can be described as the product of the extensive quantity and the corresponding intensive quantity, where the intensive quantity is the potential of the extensive quantity, and then the entransy theory can be developed. According to the Stefan–Boltzmann law, the total heat flux from a radiation surface is in direct proportion to the fourth power of the temperature, so Cheng and Liang[30] thought that the potential of thermal radiation is not the temperature, but the blackbody emissive power,

$$U = \sigma T^4, \tag{3}$$

where σ is the Stefan–Boltzmann constant. With this consideration, Cheng and Liang[30] defined the entransy flow anew, developed the radiation entransy theory correspondingly, and applied it to some radiation optimization problems.

There is no definite conclusion on the entransy theory for thermal radiation at the moment. In the present paper, the radiation entransy theory under two different entransy flow definitions and the entropy generation optimization principle are analyzed for optimization of the thermal radiation problem. The differences between the optimization principles are discussed.

2. Optimization principles for thermal radiation and discussion

2.1. Definitions of entransy flow and the corresponding extremum entransy dissipation principles (EEDPs)

In heat conduction and heat convection, the entransy flow is defined as[14]

$$\dot{G}_{\mathrm{f}} = \dot{Q}_{\mathrm{c}} T, \tag{4}$$

where $\dot{Q}_{\mathrm{c}}$ is the heat flow.

Currently, there are two different definitions of entransy flow for thermal radiation. The first definition was proposed by Wu and Liang.[27] They continued to use the definition of Eq. (4) for thermal radiation. According to the total net radiation heat flow $\dot{Q}$ and the equivalent temperature difference ΔT for an enclosed radiation heat transfer system composed of multiple surfaces, they defined the entransy dissipation rate, $\dot{G}_{\mathrm{dis}-T}$, as[27]

$$\dot{G}_{\mathrm{dis}-T} = \dot{Q}\Delta T. \tag{5}$$

It can be seen that the minimum equivalent temperature difference corresponds to the minimum entransy dissipation rate for prescribed total net radiation heat flow, while the maximum total net radiation heat flow corresponds to the maximum entransy dissipation rate for prescribed equivalent temperature difference. This is the EEDP for thermal radiation. Furthermore, they defined the equivalent radiative thermal resistance R_T as[28]

$$R_T = \Delta T/\dot{Q} = \dot{G}_{\mathrm{dis}-T}/\dot{Q}^2. \tag{6}$$

The EEDP for thermal radiation can be generalized into the METRP for thermal radiation based on Eq. (6).

The second definition was proposed by Cheng and Liang.[30] They considered that the essence of the entransy flow is the product of the heat flow and the driving potential. In heat conduction and convection, the driving potential is temperature. However, the driving potential of the thermal radiation is the blackbody emissive power. Hence, the entransy flow was redefined as[30]

$$\dot{G}_{\mathrm{f}} = \dot{Q}U. \tag{7}$$

For an enclosure composed of multiple diffuse grey surfaces as shown in Fig. 1, the total net radiation heat flow $\dot{Q}$ and the equivalent blackbody emissive power difference ΔU are defined respectively as[30]

$$\dot{Q} = -\sum \dot{Q}_{\mathrm{net-in}} = \sum \dot{Q}_{\mathrm{net-out}}, \tag{8}$$

$$\Delta U = \sum_i (\dot{Q}_{\mathrm{net-in}})_i U_i/\dot{Q} + \sum_j (\dot{Q}_{\mathrm{net-out}})_j U_j/\dot{Q}, \tag{9}$$

where the subscript i and j are the indexes of surfaces that absorb heat flow $\dot{Q}_{\mathrm{net-in}}$ from the system and release heat flow $\dot{Q}_{\mathrm{net-out}}$ to the system respectively. The radiation entransy dissipation rate, $\dot{G}_{\mathrm{dis}-U}$, is[30]

$$\dot{G}_{\mathrm{dis}-U} = \dot{Q}\Delta U. \tag{10}$$

Then the EEDP for thermal radiation based on Eq. (7) was obtained, which states that the minimum equivalent blackbody emissive power difference corresponds to the minimum entransy dissipation rate for prescribed total net radiation heat

flow, while the maximum total net radiation heat flow corresponds to the maximum entransy dissipation rate for prescribed equivalent blackbody emissive power difference. Similarly, they defined the equivalent radiative thermal resistance R_U as[30]

$$R_U = \Delta U/\dot{Q} = \dot{G}_{\mathrm{dis}-U}/\dot{Q}^2. \tag{11}$$

Based on Eq. (11), this new EEDP for thermal radiation can also be generalized into the corresponding METRP for thermal radiation.

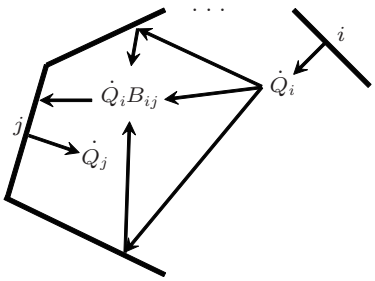


Fig. 1. An enclosed radiation heat transfer system.[30]

2.2. Comparison between the two entransy flow definitions for thermal radiation

The basic difference between the two definitions is the radiation potential. One is the temperature T, and the other is the blackbody emissive power U.

For the physical mechanism, heat conduction is due to the collisions and diffusion in gases and liquids, or the vibrations of the lattices in solids, and heat convection involves the combined effects of conduction and fluid motion.[31] The heat fluxes of heat conduction and convection are linearly related to the temperature difference in most cases. However, thermal radiation is due to the electromagnetic radiation. According to the Stefan–Boltzmann law, the radiation heat flux is linearly related to the difference of the fourth power of temperature, but not to the temperature difference. Thus, in essence, the physical meaning is clear when U is defined as the radiation potential. However, the units of the radiation entransy flow and the entransy dissipation rate both are $\mathrm{W}^2/\mathrm{m}^2$ in such a case, while those for heat conduction, and heat convection processes both are W·K. Therefore, the optimization principle expressed in Eq. (10) cannot be applied to the coupled problem which involves thermal radiation, heat conduction as well as heat convection because of their unit inconsistency, i.e., dimension inconsistency. If the definition of Eq. (4) is adopted, the dimensions of entransy flow and entransy dissipation for thermal radiation, heat conduction, and heat convection will be the same, so that the coupled problem can be analyzed.

In addition, when the radiation potential is defined as U, we can obtain the minimum principle of entransy loss for thermal radiation by derivation,[30] which means that the realistic radiation heat transport makes the radiation entransy loss minimized. However, the same conclusion cannot be obtained with T defined as the potential. This also illustrates that there will be more abundant physical connation when U is defined as the radiation potential.

The optimization objectives of the two potential definitions are different, which can be seen from the comparison of the EEDPs for thermal radiation under different entransy flow definitions for prescribed total net radiation heat flow. The minimum entransy dissipation based on T corresponds to the minimum equivalent temperature difference, while the minimum entransy dissipation based on U corresponds to the minimum equivalent blackbody emissive power difference. The optimization results of the two principles are the same if the variation tendencies for the equivalent temperature difference and the blackbody emissive power difference are the same. Alternatively, the optimization results are different from each other if the variation tendencies are not the same.

Finally, the equivalent radiative thermal resistance defined in Eq. (11) has the same dimension as the radiative thermal resistance in a general sense, but the equivalent radiative thermal resistance defined in Eq. (6) does not.

Overall, the physical meaning of the entransy flow definition based on U is clearer than that based on T. However, the entransy flow definition based on T is considered to be more useful in practical applications, because it can be used for optimizing coupled heat transfer problems, and the minimum equivalent temperature difference is usually the expected optimization objective for prescribed heat flow, which is consistent with the objective of the EEDP based on T.

2.3. Extremum entropy generation principle (EEGP) for thermal radiation

Entropy generation is a measure for the irreversibility of physical processes, and it is widely applied to heat transfer optimization.[3] We briefly discuss the entropy generation of surface radiation heat exchange in this subsection.

The energy balance equation of surface j in Fig. 1 is[30]

$$\dot{Q}_{\mathrm{net-j}} = \dot{Q}_j - \sum_{i=1}^{n} \dot{Q}_i B_{ij}, \tag{12}$$

where $\dot{Q}_j$ is the rate of emitted thermal energy from surface j, $\dot{Q}_{\mathrm{net-j}}$ the net heat flow leaving from the surface, B_{ij} the absorption factor which is the fraction of total heat absorbed by surface j from the emitted energy of surface i though direct incidence and various paths of reflection. The energy balance requires[32]

$$\sum_{j=1}^{n} B_{ij} = 1. \tag{13}$$

Dividing Eq. (12) by T_j, the temperature of surface j, results in

$$\dot{Q}_{\text{net-j}}/T_j = \dot{Q}_j/T_j - \sum_{i=1}^{n} \dot{Q}_i B_{ij}/T_j. \tag{14}$$

Equation (14) can be changed into

$$\dot{Q}_{\text{net-j}}/T_j = \dot{Q}_j/T_j - \left[\sum_{i=1}^{n} \dot{Q}_i B_{ij}(1/T_j - 1/T_i) + \sum_{i=1}^{n} \dot{Q}_i B_{ij}/T_i\right]. \tag{15}$$

Summing Eq. (15) over all the surfaces gives

$$\sum_{j=1}^{n} \dot{Q}_{\text{net-j}}/T_j = \sum_{j=1}^{n} \dot{Q}_j/T_j - \sum_{j=1}^{n}\left[\sum_{i=1}^{n} \dot{Q}_i B_{ij}(1/T_j - 1/T_i) + \sum_{i=1}^{n} \dot{Q}_i B_{ij}/T_i\right]. \tag{16}$$

According to Eq. (13), equation (16) can be simplified into

$$-\sum_{j=1}^{n} \dot{Q}_{\text{net-j}}/T_j = \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{Q}_i B_{ij}(1/T_j - 1/T_i), \tag{17}$$

where the left term is the total net entropy flow out of all the surfaces, and the right term is the total entropy generation rate due to the irreversible heat transfer in the enclosure. So equation (17) is the entropy balance equation for the radiation heat transfer in the enclosure.

Denoting the total entropy generation rate by $\dot{S}_{\text{gen}}$, we have

$$\dot{S}_{\text{gen}} = \sum_{j=1}^{n}\sum_{i=1}^{n} \dot{Q}_i B_{ij}(1/T_j - 1/T_i). \tag{18}$$

The total net radiation heat flow $\dot{Q}$ is the same as that in Eq. (8), and the equivalent thermodynamic potential difference ΔP is defined as

$$\Delta P = -\left[\sum_i (\dot{Q}_{\text{net-in}})_i/T_i + \sum_j (\dot{Q}_{\text{net-out}})_j/T_j\right] \Big/ \dot{Q}. \tag{19}$$

Combining Eqs. (17)–(19) leads to

$$\dot{S}_{\text{gen}} = \dot{Q}\Delta P. \tag{20}$$

It can be seen that the minimum equivalent thermodynamic potential difference corresponds to the minimum radiation entropy generation rate when the total net radiation heat flow is prescribed, while the maximum total net radiation heat flow corresponds to the maximum entropy generation rate when the equivalent thermodynamic potential difference is prescribed. This is the extremum entropy generation principle (EEGP) for thermal radiation.

The EEGP for thermal radiation includes the minimum entropy generation principle for thermal radiation and it also illustrates the physical essence of the entropy-generation-based optimization, that is, the minimum entropy generation corresponds to the minimum equivalent thermodynamic potential difference when the total net radiation heat flow is given.

The objectives of the two different EEDPs are the minimum equivalent temperature difference and the minimum equivalent blackbody emissive power difference respectively when the total net radiation heat flow is given. Hence, the objectives of the entropy-generation-based optimization and the entransy-dissipation-based optimization under two different entransy flow definitions are not the same, and the optimization results of the EEDPs and the EEGP may be different. This problem will be discussed and analyzed in the next section.

3. Examples of the thermal radiation optimization

3.1. Thermal radiation system composed of three infinite parallel-plates

Now we come to consider a steady thermal radiation system composed of three infinite parallel-plates as shown in Fig. 2, where plate 3 has a higher thermal potential, and plate 1 and plate 2 each have a lower potential. Heat radiates from plate 3, and finally transports into the environment through plate 1 and plate 2. The temperatures of plate 1 and plate 2, T_1 and T_2, the emissivity of plate 3, ε_3, and the environmental temperature, T_0, are given. In order to enhance the heat transfer, a certain quantity of the high emissivity material is to be coated on plate 1 and plate 2, and the limiting condition is that the sum of the emissivities of plate 1 and plate 2 is constant, that is, $\varepsilon_1 + \varepsilon_2 = C$. The objective of this problem is to find the optimal distribution of the emissivity between plate 1 and plate 2 which can lead to the best heat transfer performance. This problem has been considered in Refs. [27] and [30]. In the present report, the EEGP is also applied to the optimizing process, and the optimization results of the EEDPs and the EEGP are compared.

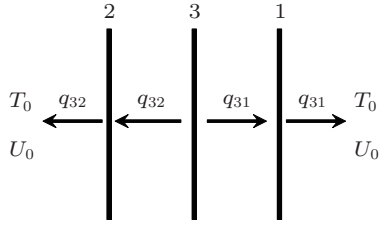


Fig. 2. Thermal radiation system composed of three infinite parallel-plates.[30]

Assume that $\dot{q}$ is the total heat flux that is released from plate 3, and $\dot{q}_{31}$ and $\dot{q}_{32}$ are the heat fluxes from plate 3 to plate

1 and plate 2, respectively. According to the theory of thermal radiation and the energy conservation, there are

$$\dot{q}_{31} = (U_3 - U_1)/(1/\varepsilon_3 + 1/\varepsilon_1 - 1), \tag{21}$$

$$\dot{q}_{32} = (U_3 - U_2)/(1/\varepsilon_3 + 1/\varepsilon_2 - 1), \tag{22}$$

$$\dot{q} = \dot{q}_{31} + \dot{q}_{32}, \tag{23}$$

where U_i is the blackbody emissive power of plate i and ε_i is the emissivity of plate i. Then the entransy dissipation rates per unit area under two different entransy flow definitions and the entropy generation rate per unit area can be calculated by

$$\dot{g}_{\mathrm{dis}-T} = \dot{q}(T_1 - T_0), \tag{24}$$

$$\dot{g}_{\mathrm{dis}-U} = \dot{q}(U_1 - U_0), \tag{25}$$

$$\dot{s}_{\mathrm{gen}} = \dot{q}(1/T_0 - 1/T_1), \tag{26}$$

where T_i is the temperature of plate i .

In the first case, it is considered that the net radiation heat flux is prescribed. Let $\dot{q} = 12$ kW/m^2, $T_1 = 680$ K, $T_2 = 400$ K, $T_0 = 300$ K, $\varepsilon_3 = 0.4$, and $\varepsilon_1 + \varepsilon_2 = 1$. The variations of the equivalent temperature difference (ΔT), the blackbody emissive power difference (ΔU), the thermodynamic potential difference (ΔP), the entransy dissipation rates per unit area ($\dot{g}_{\mathrm{dis}-T}$ and $\dot{g}_{\mathrm{dis}-U}$) under two definitions, and the entropy generation rate per unit area ($\dot{s}_{\mathrm{gen}}$) with the emissivity of plate 1 (ε_1) are obtained and shown in Fig. 3. It can be seen that $\dot{g}_{\mathrm{dis}-T}$ and ΔT are minimized simultaneously when $\varepsilon_1 = 0.35$, and $\dot{g}_{\mathrm{dis}-U}$ and ΔU are also minimized simultaneously. The results illustrate that the minimum entransy dissipation principles based on the two different entransy flow definitions both can be applied to the optimization of this system. Moreover, $\dot{s}_{\mathrm{gen}}$ and ΔP also reach their minimum values at the same ε_1, so the minimum entropy generation principle can also be used for optimizing this case.

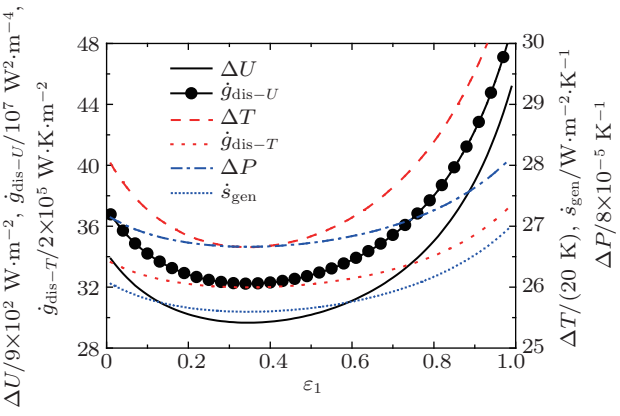


Fig. 3. (color online) Variations of ΔT, ΔU, ΔP between plate 3 and the environment, and $\dot{g}_{\mathrm{dis}-T}$, $\dot{g}_{\mathrm{dis}-U}$, $\dot{s}_{\mathrm{gen}}$ with ε_1.

Another case is with a prescribed temperature condition in which $T_3 = 850$ K, $T_0 = 300$ K, $T_1 = 680$ K, $T_2 = 400$ K, $\varepsilon_3 = 0.4$, $C = 1$. The relevant parameters are calculated, and the results are shown in Fig. 4. It can be seen that $\dot{g}_{\mathrm{dis}-T}$, $\dot{g}_{\mathrm{dis}-U}$, and $\dot{s}_{\mathrm{gen}}$ are maximized simultaneously when $\varepsilon_1 = 0.36$, and the radiation heat flux released from plate 3 reaches its maximum value at the same time. Therefore, when the boundary temperatures are prescribed, the maximum entransy dissipation principles based on the two definitions and the maximum entropy generation principle both can be used to optimize this system.

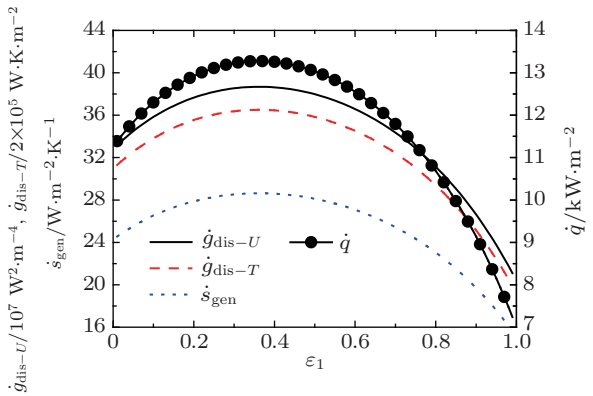


Fig. 4. (color online) The variations of $\dot{q}$ and $\dot{g}_{\mathrm{dis}-T}$, $\dot{g}_{\mathrm{dis}-U}$, $\dot{s}_{\mathrm{gen}}$ with ε_1.

For the system shown in Fig. 2, the optimization results based on the EEDPs and EEGP are the same because ΔT, ΔU, and ΔP always have the same variation tendencies. Hence, the objectives of the minimum ΔT, ΔU, and ΔP become the same when the heat transfer rate is given. When the boundary temperatures are given, ΔT, ΔU, and ΔP are fixed too, the optimization objectives of these theories are all the maximum heat transfer rate.

3.2. Thermal radiation within a two-dimensional triangular enclosure

Figure 5 shows, the radiation heat exchange in an enclosure composed of three surfaces which are diffusive and gray. The depth of the enclosure in the direction normal to the paper is L, and the lengths of the three sides are l_1, l_2, and l_3, respectively. Assume that $L \gg l_1$, l_2, l_3, and the end effects on the heat transfer can be neglected. Surface 1 and surface 2 each has a higher thermal potential, and release heat fluxes $\dot{q}_1$ and $\dot{q}_2$ respectively. Surface 3 has a lower thermal potential, and absorbs the heat from the other surfaces. The temperature and the emissivity of surface 3, T_3 and ε_3, are given. Similarly, the limiting condition is that the sum of the emissivities of the surface 1 and surface 2 is constant, that is, $\varepsilon_1 + \varepsilon_2 = C$. The objective of this problem is to find out the optimal distribution of the emissivity between surface 1 and surface 2, which corresponds to the optimal heat transfer performance.

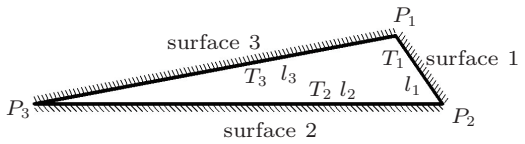


Fig. 5. Sketch of the enclosed thermal radiation system composed of three surfaces.

For the above radiation heat transfer process, the total net radiation heat flow of the system is

$$\dot{Q}_{\rm h} = \dot{q}_1 l_1 L + \dot{q}_2 l_2 L. \tag{27}$$

The entransy dissipation rates under the two entransy flow definitions and the entropy generation rate can be calculated by

$$\dot{G}_{{\rm dis}-T} = \dot{q}_1 l_1 L T_1 + \dot{q}_2 l_2 L T_2 - \dot{Q}_{\rm h} T_3, \tag{28}$$

$$\dot{G}_{{\rm dis}-U} = \dot{q}_1 l_1 L U_1 + \dot{q}_2 l_2 L U_2 - \dot{Q}_{\rm h} U_3, \tag{29}$$

$$\dot{S}_{\rm gen} = \dot{Q}_{\rm h}/T_3 - \dot{q}_1 l_1 L/T_1 - \dot{q}_2 l_2 L/T_2, \tag{30}$$

where T_i is the temperature of surface i, and U_i is the blackbody emissive power of surface i.

Let $l_1 = 0.02$ m, $l_2 = 0.10$ m, $l_3 = 0.09$ m, $L = 1$ m, $\dot{q}_1 = 12$ kW/m^2, $\dot{q}_2 = 8$ kW/m^2, $T_3 = 300$ K, $\varepsilon_3 = 0.3$, and $C = 1$. According to the radiation network method and the above formulas, the variations of ΔT, ΔU, ΔP, $\dot{G}_{{\rm dis}-T}$, $\dot{G}_{{\rm dis}-U}$, and $\dot{S}_{\rm gen}$ with ε_1 are obtained and shown in Fig. 6. It can be seen that ΔT and $\dot{G}_{{\rm dis}-T}$ have the same variation tendencies, and ΔT and $\dot{G}_{{\rm dis}-T}$ are minimized simultaneously when $\varepsilon_1 = 0.36$. ΔU and $\dot{G}_{{\rm dis}-U}$ have the same variation tendencies, and ΔU and $\dot{G}_{{\rm dis}-U}$ are minimized simultaneously when $\varepsilon_1 = 0.40$. ΔP and $\dot{S}_{\rm gen}$ have the same variation tendencies, and ΔP and $\dot{S}_{\rm gen}$ are also minimized simultaneously when $\varepsilon_1 = 0.32$.

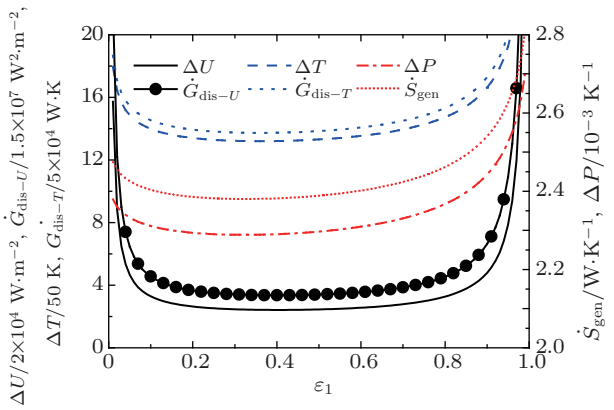


Fig. 6. (color online) Variations of ΔT, ΔU, ΔP, and $\dot{G}_{{\rm dis}-T}$, $\dot{G}_{{\rm dis}-U}$, $\dot{S}_{\rm gen}$ with ε_1.

Since the optimization results from the numerical method based on the EEDPs and EEGP are close to each other, theoretical analysis is carried out to verify the numerical results. In this problem, T_1 and T_2 each are a function of ε_1, and U is a function of T as shown in Eq. (3). Therefore, the first order derivatives of Eqs. (28)–(30) with respect to ε_1 are

$$\dot{G}'_{{\rm dis}-T}(\varepsilon_1) = \dot{q}_1 l_1 L T'_1(\varepsilon_1) + \dot{q}_2 l_2 L T'_2(\varepsilon_1), \tag{31}$$

$$\dot{G}'_{{\rm dis}-U}(\varepsilon_1) = 4\sigma \dot{q}_1 l_1 L T_1^3(\varepsilon_1) T'_1(\varepsilon_1) + 4\sigma \dot{q}_2 l_2 L T_2^3(\varepsilon_1) T'_2(\varepsilon_1), \tag{32}$$

$$\dot{S}'_{\rm gen}(\varepsilon_1) = \dot{q}_1 l_1 L \frac{T'_1(\varepsilon_1)}{T_1^2(\varepsilon_1)} + \dot{q}_2 l_2 L \frac{T'_2(\varepsilon_1)}{T_2^2(\varepsilon_1)}. \tag{33}$$

Assume that $\dot{G}_{{\rm dis}-T}$ reaches its minimum value when $\varepsilon_1 = \varepsilon$, at this point there must be

$$\dot{G}'_{{\rm dis}-T} = \dot{q}_1 l_1 L T'_1(\varepsilon) + \dot{q}_2 l_2 L T'_2(\varepsilon) = 0. \tag{34}$$

If the optimization results of the EEDPs and EEGP are the same, $\dot{G}'_{{\rm dis}-U}$ and $\dot{S}'_{\rm gen}$ should also be equal to zero when $\varepsilon_1 = \varepsilon$. Hence, combining with Eq. (34), the following equations should be satisfied

$$\dot{G}'_{{\rm dis}-U}(\varepsilon) = 4\sigma \dot{q}_1 l_1 L T'_1(\varepsilon)[T_1^3(\varepsilon) - T_2^3(\varepsilon)] = 0, \tag{35}$$

$$\dot{S}'_{\rm gen}(\varepsilon) = \dot{q}_1 l_1 L T'_1(\varepsilon) \left[\frac{1}{T_1^2(\varepsilon)} - \frac{1}{T_2^2(\varepsilon)}\right] = 0. \tag{36}$$

From the thermal radiation theory, it can be found that $T_1(\varepsilon_1)$ always decreases with ε_1 increasing under the prescribed heat flow condition. So we have $T'_1(\varepsilon) < 0$. Then, equations (35) and (36) lead to $T_1(\varepsilon) = T_2(\varepsilon)$. However, $T_1(\varepsilon) = T_2(\varepsilon)$ and equaltion (34) cannot be satisfied at the same time for most cases. In the present case, equation (34) can be satisfied when $\varepsilon = 0.36$, and the corresponding temperatures T_1 and T_2 are 1050.4 K and 933.0 K respectively, i.e., $T_1 = 1050.4$ K and $T_2 = 933.0$ K. It means that the optimization result from the EEDP based on potential T is different from the results from the EEDP based on potential U and the EEGP.

Furthermore, assume that $\dot{G}_{{\rm dis}-U}$ reaches its minimum value when $\varepsilon_1 = \varepsilon_0$, at this point there must be

$$\dot{G}'_{{\rm dis}-U}(\varepsilon_0) = 4\dot{q}_1 l_1 L T_1^3(\varepsilon_0) T'_1(\varepsilon_0) + \dot{q}_2 l_2 L T_2^3(\varepsilon_0) T'_2(\varepsilon_0) = 0. \tag{37}$$

If the optimization results from the EEDP are based on the definition of potential U, the following equation must be satisfied

$$\dot{S}'_{\rm gen}(\varepsilon_0) = \dot{q}_1 l_1 L T'_1(\varepsilon_0) \left[\frac{1}{T_1^2(\varepsilon_0)} - \frac{T_1^3(\varepsilon_0)}{T_2^5(\varepsilon_0)}\right] = 0. \tag{38}$$

Since $T'_1(\varepsilon_0) < 0$, equation (38) leads to $T_1(\varepsilon_0) = T_2(\varepsilon_0)$. However, $T_1(\varepsilon_0) = T_2(\varepsilon_0)$ and equation (37) cannot always be satisfied simultaneously for most cases, either. In the present case, equation (37) can be satisfied when $\varepsilon_0 = 0.40$, while the corresponding temperatures T_1 and T_2 are 1037.5 K and 937.5 K

respectively, i.e., $T_1 = 1037.5$ K and $T_2 = 937.5$ K. It means that the optimization results from the EEDP based on potential U and the EEGP are not the same.

From the above analysis, the optimization results from the EEDPs and the EEGP for this problem are different. This is because the optimization objectives of these principles are inconsistent.

3.3. Thermal radiation system composed of four infinite parallel-plates

Shown in Fig. 7 is another case in which the thermal radiation system is composed of four infinite parallel-plates. Plate 1 and plate 2 each have a higher potential, and the plates on both sides each have a lower potential. Heat fluxes are released from plate 1 and plate 2, and finally absorbed by the outside plates of blackbody. The quantity of the high emissivity material coated on plate 1 and plate 2 is limited, so the limiting condition is that the sum of emissivities of plate 1 and plate 2 is constant, i.e., $\varepsilon_1 + \varepsilon_2 = C$. The optimization objective is to find out the optimal emissivity distribution between plate 1 and plate 2 which leads to the best heat transfer performance.

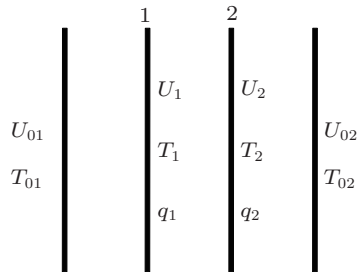


Fig. 7. Thermal radiation system composed of four infinite parallel-plates.[30]

Assume that the heat fluxes released from plate 1 and plate 2 are $\dot{q}_1$ and $\dot{q}_2$ respectively. Based on the thermal radiation theory, the heat transfer equations of the above problem are[30]

$$\varepsilon_1(U_1 - U_{01}) + (U_1 - U_2)/(1/\varepsilon_1 + 1/\varepsilon_2 - 1) = \dot{q}_1, \quad (39)$$

$$\varepsilon_2(U_2 - U_{02}) + (U_2 - U_1)/(1/\varepsilon_1 + 1/\varepsilon_2 - 1) = \dot{q}_2, \quad (40)$$

where U_1 and U_2 are the blackbody emissive powers of plate 1 and plate 2, U_{01} and U_{02} are the blackbody emissive powers of the outside plates, respectively. The entransy dissipation rates per unit area under two different entransy flow definitions and the entropy generation rate per unit area can be calculated by

$$\dot{g}_{\mathrm{dis}-T} = \varepsilon_1(U_1 - U_{01})(T_1 - T_{01}) + \frac{(U_1 - U_2)(T_1 - T_2)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} + \varepsilon_2(U_2 - U_{02})(T_2 - T_{02}), \quad (41)$$

$$\dot{g}_{\mathrm{dis}-U} = \varepsilon_1(U_1 - U_{01})^2 + \frac{(U_1 - U_2)^2}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} + \varepsilon_2(U_2 - U_{02})^2, \quad (42)$$

$$\dot{s}_{\mathrm{gen}} = \varepsilon_1(U_1 - U_{01})(1/T_{01} - 1/T_1) + \frac{(U_1 - U_2)(1/T_2 - 1/T_1)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} + \varepsilon_2(U_2 - U_{02})(1/T_{02} - 1/T_2). \quad (43)$$

Let $T_{01} = 300$ K, $T_{02} = 200$ K, $\dot{q}_1 = 80$ W/m^2, $\dot{q}_2 = 100$ W/m^2, and $C = 1$. The variations of ΔT, ΔU, ΔP, $\dot{g}_{\mathrm{dis}-T}$, $\dot{g}_{\mathrm{dis}-U}$, and $\dot{s}_{\mathrm{gen}}$ with ε_1 are shown in Fig. 8. It can be seen that ΔT and $\dot{g}_{\mathrm{dis}-T}$ are minimized ($\Delta T = 70.71$ K) when $\varepsilon_1 = 0.81$, while ΔU and $\dot{g}_{\mathrm{dis}-U}$ are minimized ($\Delta U = 274.28$ W/m^2) when $\varepsilon_1 = 0.24$. ΔP and $\dot{s}_{\mathrm{gen}}$ are minimized simultaneously when $\varepsilon_1 = 0.89$. Therefore, when the EEDPs under the two different entransy flow definitions and the EEGP are used to optimize this problem, the three optimization results are totally different from each other, which results from the different objectives of these principles.

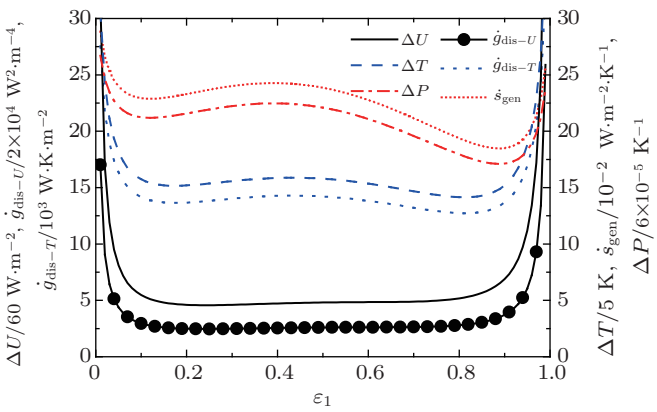


Fig. 8. (color online) Variations of ΔT, ΔU, ΔP, and $\dot{g}_{\mathrm{dis}-T}$, $\dot{g}_{\mathrm{dis}-U}$, $\dot{s}_{\mathrm{gen}}$ with ε_1.

Figure 8 also shows that there are two extremum points if the EEDP based on potential T is applied, while there is only one extremum point with the EEDP based on potential U applied. For a complex optimization problem, it might be hard to find the global optimal result because the numerical results may be not globally optimal, but are locally optimal points when some numerical optimization methods are used.

For a practical heat transfer problem, whether a principle can be used for optimizing an objective is mainly determined by whether the optimization objective of the principle is consistent with the expected one. For the case with prescribed heat flow, the expected objective is usually the minimum temperature difference for the heat conduction and heat convection problem, so the EEDP is effective to be used in this case. For a thermal radiation problem, owing to the fact that the optimizing objectives of the EEDPs under two different definitions are not the same, the EEDP based on potential U

is expected to be used if the anticipated objective is the minimum equivalent blackbody emissive power difference, while the EEDP based on potential T is expected to be used if the anticipated objective is the minimum equivalent temperature difference. However, the EEDP based on potential T would always be more recommendable than the other one in general application, since the expected objective is usually the minimum equivalent temperature difference. Moreover, when the heat flow is prescribed, the EEGP usually is not suitable to the optimization of the radiation heat transfer because the objective of the entropy optimization principle is always different from the expected one.

4. Conclusions

The two definitions of the radiation entransy flow and the corresponding optimization principles are compared, and the applications of the entransy dissipation and the entropy generation to the thermal radiation optimization are analyzed and discussed with specific examples. The major findings are summarized below.

(i) When the radiation potential is defined as the blackbody emissive power U, the physical meaning of entransy flow definition is clearer than the potential definition of temperature T. However, the latter one can be used to optimize the coupled heat transfer problem while the former one cannot. The objectives of the EEDPs for thermal radiation developed based on potentials T and U are different when the radiation heat flow is prescribed, which correspond to the minimum equivalent temperature difference and the minimum equivalent blackbody emissive power difference, respectively.

(ii) The EEGP for thermal radiation is derived. The EEGP includes the minimum entropy generation principle for thermal radiation and illustrates the essence of the minimum entropy generation principle, that is, the minimum entropy generation leads to the minimum equivalent thermodynamic potential difference with prescribed heat flow. The result reveals that the minimum entropy generation principle may not be suitable for the optimization of the thermal radiation, and clarifies the physical mechanism of the fact that the thermodynamic optimization method is not always appropriate for the heat transfer optimization.

(iii) The optimization results of the EEDPs under two different entransy flow definitions and EEGP could be different because the optimization objectives of these principles are different. For the cases with prescribed heat flow, the EEDP based on potential T would be more recommendable in the practical optimization, since its objective, i.e. the minimum equivalent temperature difference, is usually consistent with the expected one.

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