# Entropy resistance analyses of a two-stream parallel flow heat exchanger with viscous heating*

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Heat exchangers are widely used in industry, and analyses and optimizations of the performance of heat exchangers are important topics. In this paper, we define the concept of entropy resistance based on the entropy generation analyses of a one-dimensional heat transfer process. With this concept, a two-stream parallel flow heat exchanger with viscous heating is analyzed and discussed. It is found that the minimization of entropy resistance always leads to the maximum heat transfer rate for the discussed two-stream parallel flow heat exchanger, while the minimizations of entropy generation rate, entropy generation numbers, and revised entropy generation number do not always.

**Keywords:** entropy generation minimization, heat exchanger, entropy resistance, analyses

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## 1. Introduction

The analyses of heat transfer have been attracting more and more attention because heat transfer improvement is very important for industrial applications and the reduction of energy consumption.[1–6] Heat exchanger is a kind of industrial equipment that is used to heat or cool objective liquids, and is widely used in manned spascraft,[7] solar heat networks,[8] loop units in power plants,[9] etc. Therefore, the analysis of heat exchangers is very important in engineering.[7–16]

For heat exchanger analyses and design, the entropy generation minimization is often used.[10,11,13,15,16] It is thought that the minimization of entropy generation can lead to the best performance of heat exchangers. However, an entropy generation paradox was noted when this method was applied to the analyses of a counter flow heat exchanger.[10] It was found that the heat exchanger effectiveness does not always increase with the decrease of entropy generation number. Instead, it decreases for some conditions. Shah and Skiepko[13] found that the heat exchanger effectiveness can be the maximum, an intermediate value or the minimum at the maximum entropy generation. When analyzing a generlized two-stream heat exchanger, Cheng and Liang[14] also found that the entropy generation rate and the entropy generation number both first increase and then decrease with the increase of heat transfer rate and heat exchanger effectiveness when the inlet temperatures and the heat capacity flow rates of the streams are prescribed.

To remove the entropy generation paradox, the concept of revised entropy genration number was proposed.[15,16] It was shown that the heat exchanger effectiveness increases monotonically with the decrease of revised entropy generation number.[16] However, when the ratios of the heat capacity flow rates to the heat transfer rate are given, the revised entropy generation number is constant with the increase of heat transfer rate in two-stream heat exchanger.[14]

On the other hand, another concept, entransy,[17] was proposed and used in heat transfer analyses and optimizations. It was found that the entransy-dissipation-based thermal resistance always decreases with the improvement in performance of heat exhangers.[14,18,19] However, the theory was mainly applied to the heat exchanger in which the viscous heating can be ignored. When the viscous heating should be considered, there are few reports[20] and further investigations are needed.

As entropy generation can be used to analyze the viscous heating and the entropy generation paradox has not been completely removed with the concept of entropy generation, further investigations are also necessary on the analyses of heat exchangers with the concept of entropy generation. In this paper, we define the concept of entropy resistance based on the entropy generation analyses of a one-dimensional heat transfer process, and then apply it to a two-stream parallel flow heat exchanger. Numerical results are presented and discussed.

## 2. Entropy resistance

We first discuss a one-dimensional heat transfer process shown in Fig. 1. The heat flow rate $Q$ is transferred from a high temperature $T_{\mathrm{H}}$ to a low one $T_{\mathrm{L}}$. It can be seen that the entropy generation rate of this process is

$$S_{\mathrm{g}} = Q\left(\frac{1}{T_{\mathrm{L}}} - \frac{1}{T_{\mathrm{H}}}\right). \tag{1}$$

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As the reciprocal of temperature $(1/T)$ is known as the thermodynamic force,[21–23] we have

$$S_g = Q(F_L - F_H) = Q\Delta F, \tag{2}$$

where the thermodynamic forces $F_L = 1/T_L$, $F_H = 1/T_H$, and $\Delta F$ is the thermodynamic force difference. Therefore, the entropy generation rate is the product of the driving force difference $\Delta F$ and the corresponding flow rate $Q$. With the thermodynamic force difference fixed, a large entropy generation rate leads to a larger heat transfer rate, while smaller entropy generation rate leads to a smaller thermodynamic force difference with the heat transfer rate fixed.

We further analyze the process in Fig. 1 with the concepts of entropy generation numbers and revised entropy generation number. There are[10,11,15,16]

$$N_{S1} = S_g/C_{min}, \tag{3}$$

$$N_{S2} = S_g/S_{g-min}, \tag{4}$$

$$N_{RS} = T_L S_g/Q = 1 - T_L/T_H, \tag{5}$$

where $C_{min}$ is the minimum heat capacity flow rate of the system, and $S_{g-min}$ is the minimum entropy generation rate. It can be seen that the first entropy generation number $N_{S1}$ cannot be defined because there is no fluid flow in the system, and the variation tendency of the second entropy generation number $N_{S2}$ is the same as that of the entropy generation rate because the minimum entropy generation rate of the system should be a definite value. For the revised entropy generation number $N_{RS}$, it is constant when the thermodynamic forces are fixed.

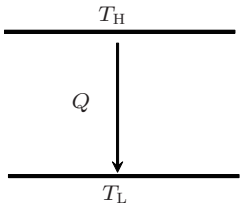



**Fig. 1.** A one-dimensional heat transfer process.

According to the above analyses, it can be seen that the entropy generation minimization does not always lead to the best performance of the process.

The analogy between electrical conduction and heat conduction has been often used to solve complex steady-state or transient heat transfer problems since the last century.[24] In electrical conduction, the driving force is the electrical potential difference, the corresponding flow is the electrical current, and the electrical resistance is defined as the ratio of the electrical potential difference to the electrical current. Considering Eq. (2), we can define the entropy resistance as

$$R_S = \Delta F/Q = S_g/Q^2. \tag{6}$$

It can be seen that a smaller entropy resistance leads to a smaller thermodynamic force difference with fixing heat transfer rate, and a larger heat transfer rate with fixing thermodynamic force difference. This is the minimum entropy resistance principle for the heat transfer process in Fig. 1. Therefore, the performance of the process in Fig. 1 can be described by the concept of entropy resistance. In the following, we will apply this concept to the analyses of a two-stream parallel flow heat exchanger with viscous heating.

## 3. Analyses of a two-stream parallel flow heat exchanger

The sketch of the two-stream parallel flow heat exchanger is shown in Fig. 2. The inlet temperatures of the high and low temperature streams are $T_{H-in}$ and $T_{L-in}$, while the outlet temperatures are $T_{H-out}$ and $T_{L-out}$ respectively. The heat capacity flow rates of the streams are $C_H$ and $C_L$, respectively. For the heat exchanger, assume that the entrance effect of the flow paths and the heat transfer between the heat exchanger and the environment can be ignored, and there is no phase change. The energy conservation equation of the streams with the consideration of the viscous heating can be expressed as[20]

$$C_H \frac{dT_H}{dx} = kW(T_L - T_H) + \frac{d\phi_H}{dx}, \tag{7}$$

$$C_L \frac{dT_L}{dx} = kW(T_H - T_L) + \frac{d\phi_L}{dx}, \tag{8}$$

where $k$ is the heat transfer coefficient between the streams, $T_H$ and $T_L$ are the local temperatures of the high and low temperature streams, $W$ is the width of the heat exchanger in the $z$ direction, and $\phi_H$ and $\phi_L$ are the viscous dissipation functions.

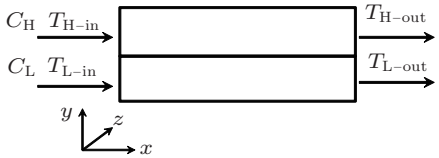



**Fig. 2.** Sketch of a two-stream parallel flow heat exchanger.

The viscous dissipation function is[20]

$$\frac{d\phi}{dx} = \frac{1}{2} f \frac{mu^2}{d}, \tag{9}$$

where $m$ is the mass flow rate of the fluid, $u$ is the velocity, $d$ is the hydraulic diameter, and $f$ is the Darcy friction factor,

$$f = (1.82 \lg Re - 1.64)^{-2}, \tag{10}$$

where $Re$ is the Reynolds number of the fluid.

For the heat transfer coefficient $k$, there is

$$k = (1/h_H + \delta/\lambda + 1/h_L)^{-1}, \tag{11}$$

where $\delta$ is the thickness of the solid separator between the streams, $\lambda$ is the thermal conductivity of the separator, $h_H$

is the heat transfer coefficient between the high temperature stream and the separator, while $h_L$ is that between the low temperature stream and the separator.[25] The heat transfer coefficients for the high and low temperature stream sides can be calculated by

$$h = \frac{\lambda_f (f/8)(Re - 1000)Pr}{d\left[1 + 12.7\sqrt{f/8}\left(Pr^{2/3} - 1\right)\right]}, \tag{12}$$

according to their thermal conductivity $\lambda_f$ and Prandtl number $Pr$, respectively.

The temperature distribution of the heat exchanger can be calculated with Eqs. (7)–(12). The heat transfer rate of the heat exchanger can be expressed as

$$Q = \int_0^L kW\,(T_H - T_L)\mathrm{d}x. \tag{13}$$

The local entropy generation rate is[20]

$$\sigma = -\frac{C_H}{T_H}\frac{\mathrm{d}T_H}{\mathrm{d}x} + \frac{C_L}{T_L}\frac{\mathrm{d}T_L}{\mathrm{d}x}, \tag{14}$$

so the total entropy generation rate is

$$S_g = \int_0^L \sigma \mathrm{d}x. \tag{15}$$

The entropy generation numbers, the revised entropy generation number and the entropy resistance of the heat exchanger can be obtained with Eqs. (3)–(6), and (13).

Assume that the width $W = 0.5$ m and the height of each channel $H = 0.1$ m. Then the across areas of the streams are $A_H = A_L = WH = 0.05$ m$^2$, the circumferences of the across areas are $P_H = P_L = 2(W + H) = 1.2$ m, and the hydraulic diameters are $d_H = d_L = 4A_H/P_H = 4A_L/P_L = 1/6$ m. For the separator, assume that it is made of aluminum whose thermal conductivity $\lambda$ is 217.7 W/(m·K), and the thickness $\delta$ is $3\times10^{-3}$ m. For the fluid in the heat exchanger, we discuss two cases below. In the first case, the fluid in the heat exchanger is water, while it is olive oil in the second case. The physical parameters of the fluids are assumed to be constant and shown in Table 1.

**Table 1.** Parameters of the fluids.

| Parameters | Olive oil | Water |
|---|---|---|
| Thermal conductivity/W·m$^{-1}$·K$^{-1}$ | 0.169 | 0.6 |
| Density/kg·m$^{-3}$ | 914 | 1000 |
| Specific heat capacity/J·kg$^{-1}$·K$^{-1}$ | 1633 | 4200 |
| Dynamic viscosity/kg·m$^{-1}$·s$^{-1}$ | 0.0808 | 0.001005 |
| Velocity/m·s$^{-1}$ | 2 | 2 |
| $T_{L-in}$/K | 300 | 300 |

As the minimum entropy generation rate of the heat exchanger is a definite value and the heat capacity flow rates of the streams are fixed, the variation tendencies of the entropy generation numbers are always the same as that of the entropy generation rate. Hence, we do not calculate the entropy generation number below. For the first case, if we assume that the inlet temperature of the high temperature stream is 320 K, the variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the length of the parallel flow heat exchanger $L$ are shown in Fig. 3. If we assume the length of the heat exchanger to be 3 m, the variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the inlet temperature of the high temperature stream are shown in Fig. 4. For the second case, the numerical results are shown in Figs. 5 and 6. When the inlet temperature of the high temperature stream is fixed, it can be seen from Figs. 3 and 5 that both the revised entropy generation number and the entropy resistance decrease with heat transfer rate increasing, while the entropy generation rate increases. When the inlet temperature of the high temperature stream is not fixed, it can be seen from Figs. 4 and 6 that only the entropy resistance decreases with heat transfer rate increasing, while the other parameters all increase.

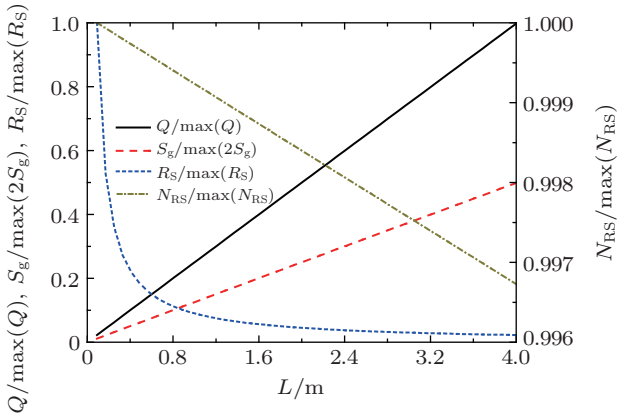



**Fig. 3.** (color online) Variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the length of the heat exchanger when the fluid is water.

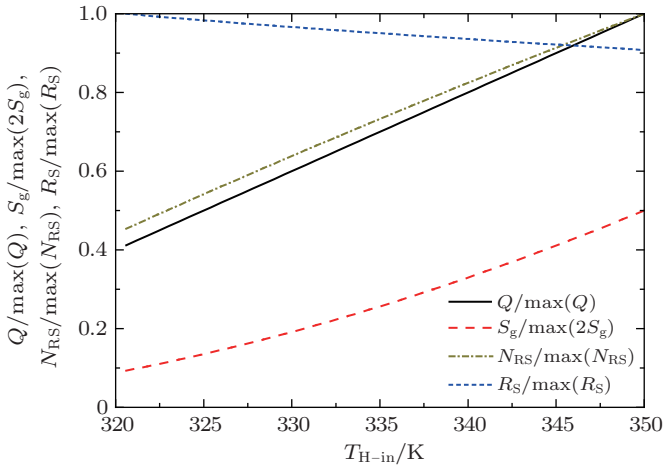



**Fig. 4.** (color online) Variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the inlet temperature of the high temperature stream when the fluid is water.

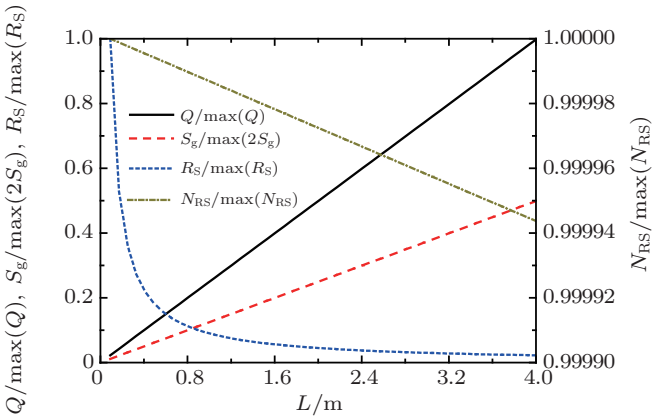



**Fig. 5.** (color online) Variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the length of the heat exchanger when the fluid is olive oil.

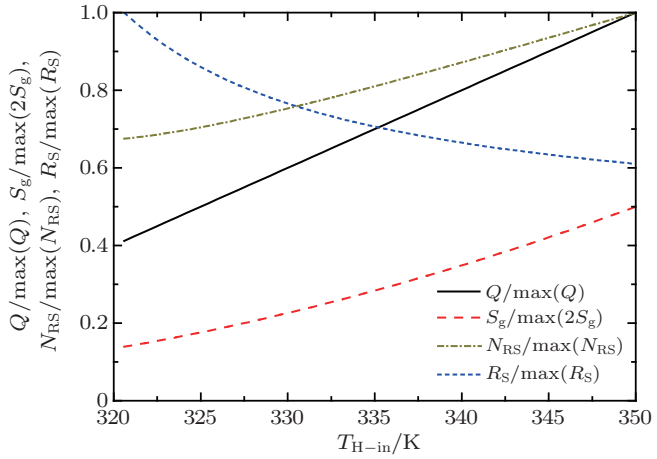



**Fig. 6.** (color online) Variations of the heat transfer rate, the entropy generation rate, the revised entropy generation number, and the entropy resistance with the inlet temperature of the high temperature stream when the fluid is olive oil.

According to the analyses and numerical results mentioned above, it can be found that the concept of entropy resistance is the most appropriate parameter for analyzing the performance of the discussed heat exchanger because it always decreases with the increase of heat transfer rate. On the other hand, the concept of entransy-dissipation-based thermal resistance is also used to analyze heat exchangers,[18,19] and the results show that the heat transfer rate of heat exchanger also increases with the decrease of entransy-dissipation-based thermal resistance. What is the difference between the two concepts? In this paper, the concept of entropy resistance is proposed from the viewpoint of thermodynamics, while the entransy-dissipation-based thermal resistance is proposed from the viewpoint of entransy which is the potential energy of heat.[17] This is the essential difference between the two resistances. Furthermore, the concept of entropy resistance can be used to analyze the irreversibility due to viscous heating, while the applicability of the concept of entransy-dissipation-based thermal resistance to the heat exchangers with viscous heating needs further discussion. This is another difference between the two concepts.

## 4. Conclusions

In this paper, the concept of entropy resistance is introduced based on the analyses of a one-dimensional heat transfer process. Based on this concept, a two-stream parallel flow heat exchanger with viscous heating is analyzed and discussed. We assume that the structural parameters of the cross section, inlet velocities, physical parameters of the fluids, and inlet temperature of the low temperature stream are fixed. If the inlet temperature of the high temperature is given, the heat transfer rate, entropy generation rate, and entropy generation numbers all increase with the length of the heat exchanger increasing, while the revised entropy generation number and entropy resistance decrease. If the inlet temperature of the high temperature is not given, only the entropy resistance decreases with the increase of heat transfer rate, while the other parameters increase. Therefore, the entropy resistance minimization is always appropriate for analyzing the heat exchanger discussed in this paper, whereas the entropy generation minimization is not.

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