Effects of the symmetry energy slope on the axial oscillations of neutron stars*

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The impact of symmetry energy slope L on the axial w-mode oscillations is explored, where the range of the constrained slope L of symmetry energy at saturation density is adopted from 25 MeV to 115 MeV while keeping the equation of state (EOS) of symmetric nuclear matter fixed. Based on the range of the symmetry energy slope, a constraint on the frequency and damping time of the $w_{\rm I}$-mode of the neutron star is given. It is found that there is a perfect linear relation between the frequency and the stellar mass for a fixed slope L, and the softer symmetry energy corresponds to a higher frequency. Moreover, it is confirmed that both the frequencies and damping times have a perfect universal scaling behavior for the EOSs with different symmetry energy slopes at saturation density.

Keywords: symmetry energy, neutron star, oscillation

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1. Introduction

Nowadays, determining the equation of state (EOS) of dense neutron-rich matter and detecting the gravitational waves are two of the greatest challenges in modern physics. Fortunately, the neutron star provides a perfect laboratory for studying both phenomena: its non-radial oscillations may radiate detectable gravitational waves, which provide potentially a probe to the EOS of super-dense neutron-rich matter as the gravitational waves carry important information about the neutron star structure.[1–6] Among the multitudinous oscillation modes, there is a type of mode that only exists in general relativity, that is, the so-called w-mode associated with space–time and for which the motion of the fluid is negligible.[3–6] The w-mode is very important for astrophysical applications since it is related to the space–time curvature and exists for all relativistic stars, including black holes. The standard axial w-mode is categorized as $w_{\rm I}$. According to the work of Chandrasekhar & Ferrari,[3,4] the axial w-mode is described by a unique second-order differential equation. Its simple form makes it possible to conduct detailed numerical studies of its spectrum. In addition, the study will help stimulate discussions about detecting the high-frequency gravitational waves in the future.

The gravitational wave frequency of the axial w-mode depends on the structure and properties of neutron stars,[7,8] which are determined by the EOS of neutron-rich stellar matter.[9–13] However, at the present time the EOS of matter (densities, pressures, and isospin asymmetries) under extreme conditions is still rather uncertain and theoretically controversial. One of the main sources of uncertainties in the EOS of neutron-rich matter is the poorly known density dependence of the nuclear symmetry energy, $E_{\rm sym}(\rho)$.[9,10] Significant progress has been made in constraining the density dependence of the symmetry energy in recent years, see, e.g., Refs. [11], [14]–[17], the slope of the symmetry energy L at nuclear saturation density is strongly correlated with EOS inside a neutron star at densities just above and below saturation, and the change in stiffness of the EOS for neutron-star matter is completely due to varying the symmetry energy slope parameter L. A conservative range of L (25 MeV $< L <$ 115 MeV) from model analysis of terrestrial nuclear laboratory experiments was taken recently,[11,15,16,18–24] so it is very interesting to investigate the effect and the constraint of the slope of the symmetry energy L on the gravitational radiation from the w-mode oscillation.

The rest of this paper is arranged as follows. In Section 2 we describe the EOSs used in this work. In Section 3 we review the formalism used to calculate the w-modes. In Section 4 we present the results before concluding in Section 5.

2. Equations of state

For describing the uniform nuclear matter EOS $E(\rho,\delta)$ mainly made up of neutrons and protons, where δ is a measure of the isospin asymmetry of the matter, it is useful to decompose it into the two extremes of isospin asymmetry:[11,25,26] the symmetric nuclear matter (SNM) $E_{\rm SNM} = E_0(\rho,\delta = 0)$ and the pure neutron matter (PNM) $E_{\rm PNM} = E(\rho,\delta = 1)$. Introducing the symmetry energy, defined as $S(\rho) =$

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$\partial^2 E(\rho,\delta)/2\partial\delta^2_{\delta=0}$, the energy per particle of an idealized system of matter at a uniform density ρ can be approximately described by $E(\rho) \approx E_0(\rho) + S(\rho)\delta^2$. The slope L of the symmetry energy is defined as $L = \partial S(\rho)/\partial\chi|_{\rho=\rho_s}$ where $\chi = (\rho-\rho_s)/(3\rho_s)$, and ρ_s is the baryon density at nuclear saturation density. The slope of the symmetry energy L at nuclear saturation density $n \approx 0.16$ fm^{-3} (1 fm = 10^{-15} m) is strongly correlated with the pressure inside a neutron star at densities just above and below saturation. In this work, the EOS of symmetric nuclear matter $E_{\rm SNM}$ is fixed with an incompressibility modulus of $K_0 = 240$ MeV.[25,26]

In our model, the EOS of the nuclear matter is obtained from the modified Skyrme-like (MSL) model.[27] The MSL interaction contains parameters directly related to the properties of nuclear matter at saturation density, such as the slope of the symmetry energy L, allowing them to be independently adjusted and giving us the ability to scan over the experimentally constrained values. In this work, we take a range of 25 MeV $< L <$ 115 MeV from model analysis of terrestrial nuclear laboratory experiments.[11,15,16,18–24,26] It is worth noting that for the most recent constraints,[16,23,24,28–30] coupled with inference from theoretical calculations of pure neutron matter,[31,32] the value of L is placed in a lower range of 25 MeV–75 MeV. For each value of L, the magnitude of the symmetry energy at saturation density is adjusted so that the EOS fits the most recent calculations of the EOS of PNM[31,32] as detailed in Ref. [25]. The core EOS is calculated by using the MSL model as the calculation for the EOS of the crust. At high densities $\geq 3\rho_s$, it is expected that the description of the matter in terms of just neutron and protons to break down, as the exact composition of the inner core is still uncertain. Due to our lack of knowledge in this regime, and also to ensure that our complete EOS is always sufficiently stiff to produce $2M_\odot$ neutron stars as demanded by observations,[33] we smoothly join the MSL EOS to two polytropic EOSs of the form $P = K\varepsilon^{(1+1/n)}$ in a similar way to Ref. [34]. The joins are made at energy densities of 300 MeV·fm^{-3} and 600 MeV·fm^{-3} by adjusting the constant K to keep the pressure continuous at the join. The lower density polytrope has an index set at $n = 0.5$, while the second index takes a range $n = 0.5 \sim 1.5$ for values of L from 25 MeV–115 MeV respectively. Note that although the additions of the polytropes substantially changes the maximum neutron star mass, especially for small (soft) values of L, it does not substantially affect the radius of a neutron star with a given mass, as such properties are determined dominantly by the pressure of the star in the range $\geq (1\sim 2)\rho_s$ through the symmetry energy slope parameter L.[9,10]

The EOSs are displayed in Fig. 1. It is shown that at density below about 8 times of the saturation density, a larger value of the slope parameter L gives a stiffer EOS. The mass-radius relation and the radius of the core are displayed in Fig. 2. It is easy to see that at a fixed stellar mass, a larger slope L corresponds to a larger size of the neutron star.

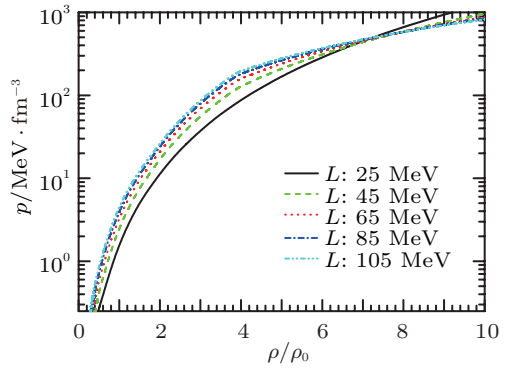


Fig. 1. (color online) The EOS of neutron-star matter versus the reduced baryon number density ρ/ρ_0, where the range of the slope L of symmetry energy at saturation density is constrained in a range of 25 MeV–115 MeV.

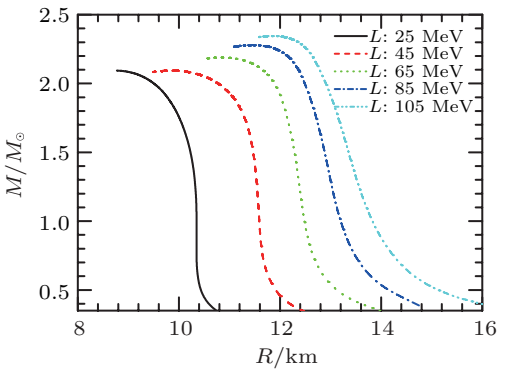


Fig. 2. (color online) The mass-radius relations.

3. Equations for the axial *w*-mode oscillation

In this section, the formalism for numerically calculating the eigen-frequency of the axial w-mode oscillation is briefly introduced, where the geometrical unit ($G = c = 1$) is adopted. According to Chandrasekhar & Ferrari,[3,4] the axial w-mode perturbation equations for a static neutron star can be simplified by introducing a function $z(r)$, constructed from the radial part of the perturbed axial metric components. The axial w-mode perturbation satisfies the following differential equation

$$\frac{\mathrm{d}^2 z}{\mathrm{d}r_*^2} + [\omega^2 - V(r)]z = 0, \tag{1}$$

where $\omega(=\omega_0 + \mathrm{i}\omega_i)$ is the complex eigen-frequency of the axial w-mode, r_* is the tortoise coordinate, which is defined inside the star as

$$r_* = \int_0^r \mathrm{e}^{\lambda-\nu}\,\mathrm{d}r, \tag{2}$$

and the potential function V is defined inside the star by

$$V = \frac{\mathrm{e}^{2\nu}}{r^3}[l(l+1)r + 4\pi r^3(\rho - p) - 6m], \tag{3}$$

in which l is the spherical harmonics index (used in describing the perturbed metric, here only the case $l = 2$ is considered), ρ and p are the density and pressure, and m is the mass inside radius r, respectively. Outside the neutron star, equations (2) and (3) reduce to

$$r_* = 2M\ln(r-2M) \tag{4}$$

and

$$V = \frac{r-2M}{r^4}[l(l+1)r-6M], \tag{5}$$

where M is the total gravitational mass of the neutron star. The functions e^{ν} and e^{λ} are given by the line element

$$-ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{6}$$

of a static neutron star.

The solutions to this problem are subject to a set of boundary conditions constructed by Chandrasekhar & Ferrari,[3] i.e., regular boundary conditions at the neutron star center, continuous boundary conditions at the surface and behaving as a purely outgoing wave at infinity.

As discussed in Refs. [5], [7], [8], [35], and [36], the w-modes have larger imaginary parts and this introduces considerable difficulties to dealing with the boundary conditions at infinity. For a larger radius, the term representing the incoming wave, $z^{\rm in} \sim e^{-r_*\omega_i}$, vanishes and therefore the problem no longer satisfies the necessary boundary conditions. A practical way to overcome these difficulties is to utilize the continued fraction method,[7,35] which is applicable to both the axial and polar w-modes. The details of the numerical procedure are referred to in Refs. [5], [7], [8], [35], and [36].

4. Results and discussion

In this section, we present our numerical results. Shown in Fig. 3 are the frequency (a) and the damping time (b) of $w_{\rm I}$-mode as a function of the stellar mass M, where the slope of symmetry energy at saturation density for the EOSs of neutron stars takes a range of 25 MeV $< L <$ 115 MeV.[25,26] It is clear that the frequencies and damping times of the $w_{\rm I}$-modes should be constrained between the lines of $L = 25$ MeV and $L = 115$ MeV. For a canonical neutron star ($1.4M_\odot$), its frequency and damping time of $w_{\rm I}$-mode will be constrained in 7.302 kHz$< \nu <$ 8.768 kHz and 28.61 μs$< \tau <$ 30.68 μs, respectively. It is easy to find in Fig. 3(a) that there is a perfect linear relation between the frequency and the stellar mass for a fixed slope L. Moreover, these lines have a similar slope except for the massive stars of $L = 25$ MeV. It is worth pointing out that this linear relation does not always exist for other EOS models.[8,36] For a fixed stellar mass, the soft symmetry energy ($L = 25$ MeV) has a higher frequency while the stiff symmetry energy corresponds with a lower frequency. One most recent work constrains the value of L in a narrow range of 43 MeV–52 MeV.[30] If this is true, as an example, for a canonical neutron star ($1.4M_\odot$) with $L = 45$ MeV, its frequency and damping time of $w_{\rm I}$-mode are 8.111 kHz and 29.37 μs, respectively. For the damping time of $w_{\rm I}$-mode, it is shown in Fig. 3(b) that for the neutron stars with lower mass (smaller than $1.4M_\odot$), the effect of the slope of symmetry energy is not so distinct, while for neutron stars with higher mass (larger than $1.6M_\odot$), the effect of the slope is obvious, and the softer symmetry energy (smaller slope) corresponds to a longer damping time for a fixed mass neutron star.

A lot of the previous works tried to find the universal behaviors of the w-modes, which are independent of the EOSs.[7,8,36,37] This has the benefit that if the mass and radius of a neutron star are accurately measured, the frequency and damping time of the gravitational waves from the w-modes can be precisely determined by the universal scaling behavior. In Fig. 4 we show the frequency (a) and the damping time (b) of the $w_{\rm I}$-mode scaled by the stellar mass M as a function of the compactness M/R. It is shown in this figure that both the frequency and damping time have a perfect universal scaling behavior for the EOSs with different symmetry energy slope at saturation density.

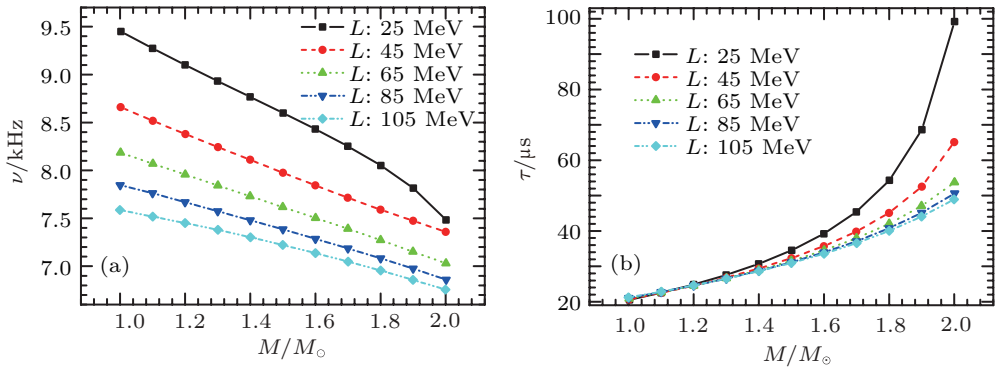


Fig. 3. (color online) Frequency (a) and damping time (b) of $w_{\rm I}$-mode as a function of the neutron star mass M.

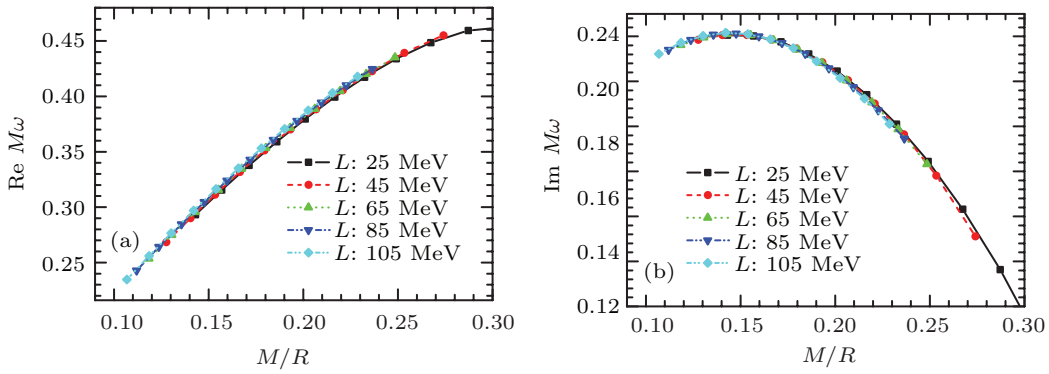


Fig. 4. (color online) Frequency (a) and damping time (b) of $w_{\rm I}$-mode scaled by the stellar mass M as a function of compactness M/R.

5. Conclusions

In this work, we have explored the impact of symmetry energy slope L on the w-mode oscillations, where the values of the symmetry energy slope are constrained by the model analysis of the terrestrial nuclear laboratory experiments. The EOSs are calculated by using a simple model, where the dependence of the symmetry energy on the density is probed by varying the slope of the symmetry energy at saturation density L from 25 MeV (soft symmetry energy and EOS) to 115 MeV (stiff symmetry energy and EOS) while keeping the EOS of symmetric nuclear matter fixed. Based on the range of the symmetry energy slope, a constraint on the frequency and damping time of the $w_{\rm I}$-mode of neutron stars is obtained. It is found that there is a perfect linear relation between the frequency and the stellar mass for a fixed slope L, and these lines have a similar slope except for the massive stars of $L = 25$ MeV. Moreover, the softer symmetry energy corresponds to a higher frequency. Finally, we also confirm that both the frequency and damping time have a perfect universal scaling behavior for the EOSs with different symmetry energy slopes at saturation density.

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