Dynamics of measurement-induced non-locality and geometric discord with initial system–environment correlations*

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We analyze the dynamics of geometric measure of discord (GMOD) and measurement-induced non-locality (MIN) in the presence of initial system–reservoir correlations without Born and Markov approximation. Although the initial system–environment states have the same reduced density matrices for both the system and environment, the effects of different initial system–environment correlations have been shown to fundamentally alter the time evolution of GMOD and MIN between two quantum systems in both Markovian and non-Markovian regimes. In general, both GMOD and MIN experience a sudden increase for initially quantum-correlated states, and a sudden decrease for classical-correlated states before they reach the same stationary values with initially factorized states.

Keywords: geometric measure of discord, measurement-induced non-locality, initial correlations

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1. Introduction

There are many surprising phenomena in the quantum world.[1] Non-locality is a significant concept in the quantum world without the classical counterpart.[2] Quantum non-locality usually refers to the fact that the properties of one particle can be affected by another particle that is sufficiently far away and is widely studied by the violation of Bell inequalities.[3–7]

Recently, Luo and Fu[8] studied the measurement-induced non-locality (MIN) from a geometric perspective based on von Neumann measurements, which is quite different from the violation of Bell inequalities. The key idea they used is that any disturbance in the system's overall state can be attributed to the genuine nonlocal features of the system when local measurements do not disturb the states of the subsystems. The MIN is intimately related to, but different from the geometric measure of quantum discord (GMOD).[9,10] However, both MIN and GMOD provide novel classification schemes for bipartite states, and are quantum resources quite different from entanglement. They may be useful in quantitative study of quantum state steering,[11] remote state control,[12,13] general quantum dense coding,[14] and cryptography.[15]

Unfortunately, no matter what kind of quantum systems, the loss of quantum coherence caused by the interactions of the system with the surrounding environment is unavoidable.[16,17] Many researchers have studied the behaviors of quantum entanglement, quantum discord, and MIN under the environments of both Markovian and non-Markovian regimes.[18–24] However, most of them have neglected the initial correlations between the system and its surrounding environment at the beginning. In a complete theory, both the system and the environment are quantum-mechanical. It is therefore possible, and often inevitable in experiment,[25] to create coherent superpositions of system and environment states (namely, initial correlations between system and environment), and such superpositions will play an important role in the time evolution of the qubit system.[26–28]

In this paper, we study the dynamics of GMOD and MIN of two non-interacting two-level atoms (A and B) coupling with a zero-temperature environment without the Born and Markov approximations. In our calculations, the initial system–environment correlations are considered at the beginning. The main purpose of this paper is to reveal the influences of different initially correlated states of system and environment on the dynamics of GMOD and MIN. We consider three typically initial system–environment states which have the same reduced density matrices for the system and the environment. The first case is that both quantum correlation and classical correlation are contained initially (i.e., quantum-correlated states); the second case only has classical correlation (i.e., classical-correlated states); the third case is that neither quantum correlation nor classical correlation is involved between the system and environment (i.e., factorized states). The effects of initial system–environment correlations have been shown to fundamentally alter the time evolution of GMOD and MIN between two quantum systems.

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2. Basic theories

2.1. MIN and GMOD

The concept of MIN was first introduced by Luo and Fu.[8] Let us review their key result in the following. Considering a bipartite state ρ from a and b components with the density matrices ρ^a and ρ^b, the the MIN can be written as

$$N(\rho) = \max_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2, \tag{1}$$

where the max is taken over the von Neumann measurements $\Pi^a = \{\Pi_k^a\}$ which do not disturb ρ^a locally and satisfy $\sum_k \Pi_k^a \rho^a \Pi_k^a = \rho^a$. $||\cdot||$ is the Hilbert–Schmidt norm with $||X||^2 = \mathrm{tr}(X^+X)$. From this definition we naturally associate MIN with the definition of GMOD[9]

$$D(\rho) = \min_{\Pi^a} ||\rho - \Pi^a(\rho)||^2. \tag{2}$$

Here the min is over all local von Neumann measurements Π^a which is sharply different from the max over locally invariant ones used in defining the MIN $N(\rho)$ in Eq. (1). Generally speaking, $m \times n$ bipartite state ρ can be expressed as

$$\begin{aligned}\rho \;=\; & \frac{1}{\sqrt{mn}} \frac{\mathbf{1}^a}{\sqrt{m}} \otimes \frac{\mathbf{1}^b}{\sqrt{n}} + \sum_{i=1}^{m^2-1} x_i X_i \otimes \frac{\mathbf{1}^b}{\sqrt{n}} \\ & + \frac{\mathbf{1}^a}{\sqrt{m}} \otimes \sum_{j=1}^{n^2-1} y_j Y_j + \sum_{i=1}^{m^2-1} \sum_{j=1}^{n^2-1} r_{ij} X_i \otimes Y_j,\end{aligned} \tag{3}$$

where X_j and Y_j are the Hermitian operators which are traceless. The $\langle X_i | X_j \rangle = \delta_{ij}$ ($\langle Y_i | Y_j \rangle = \delta_{ij}$) is also satisfied. Luo and Fu[8] have figured out an analytical formula of MIN for the $(2 \times n)$-dimensional states

$$N(\rho) = \begin{cases} ||R||^2 - \dfrac{1}{||\boldsymbol{x}||} \boldsymbol{x}^{\mathrm{T}} R R^{\mathrm{T}} \boldsymbol{x}, & \text{if } \boldsymbol{x} \neq 0, \\ ||R||^2 - \lambda_{\min}, & \text{if } \boldsymbol{x} = 0, \end{cases} \tag{4}$$

where the $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$ with $||\boldsymbol{x}|| = \sum_{i=1}^3 x_i^2$ and the R is the real 3×3 matrix with elements r_{ij}. $\lambda_{\min}$ is the smallest eigenvalue of RR^{T}, and the expression of GMOD can be written as[9]

$$D(\rho) = ||\boldsymbol{x}||^2 + ||R||^2 - k_{\max}, \tag{5}$$

where the $k_{\max}$ is the largest eigenvalue of the matrix $xx^{\mathrm{T}} + RR^{\mathrm{T}}$.

For a two-qubit X state, the density matrix can be written as

$$\rho(t) = \begin{pmatrix} a(t) & 0 & 0 & w(t) \\ 0 & b(t) & z(t) & 0 \\ 0 & z^*(t) & c(t) & 0 \\ w^*(t) & 0 & 0 & d(t) \end{pmatrix}. \tag{6}$$

According to Eq. (4), we can obtain the analytical expression of MIN as[8]

$$\begin{aligned} N[\rho(t)] \;=\; & \frac{1}{4}\Big\{ [a(t) - b(t) - c(t) + d(t)]^2 + 8[|w(t)| + |z(t)|]^2 \\ & - 4\min\Big[\frac{1}{4}[a(t) - b(t) - c(t) + d(t)]^2, \\ & [|w(t)| + |z(t)|]^2, [|w(t)| - |z(t)|]^2\Big]\Big\}, \end{aligned} \tag{7}$$

and according Eq. (5), the GMOD can be written as[9]

$$\begin{aligned} D[\rho(t)] \;=\; & \frac{1}{4}\{2[a(t) - c(t)]^2 + 2[b(t) - d(t)]^2 \\ & + 8[|w(t)| + |z(t)|]^2 - \max[2[a(t) - c(t)]^2 \\ & + 2[b(t) - d(t)]^2, 4[|w(t)| + |z(t)|]^2, \\ & 4[|w(t)| - |z(t)|]^2]\}. \end{aligned} \tag{8}$$

2.2. Initial system–environment correlations

In the theory of open quantum systems, we usually concentrate on the dynamics of quantum systems. As we know, the reduced density matrix of the open quantum system is derived by eliminating variables of the environment from a time-evolution of the whole system. However, for a given reduced density matrix, we can obtain it from different states of the whole system. Thus, we are not able to determine how this state was initially prepared. For instance, if we know the reduced density matrices of the system and environment which have the following expressions

$$\rho_{AB} = \begin{pmatrix} |\gamma|^2 & 0 & 0 & 0 \\ 0 & |\alpha|^2 & \alpha\beta^* & 0 \\ 0 & \alpha^*\beta & |\beta|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

$$\rho_E = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & 0 \\ 0 & |\gamma|^2 \end{pmatrix}. \tag{10}$$

We can construct three different whole states of system and environment.

1) The system has both quantum correlation and classical correlation with the environment (i.e., quantum-correlated state)

$$\rho_{ABE}^{(1)} = |\Psi\rangle_{ABE} \langle\Psi|, \tag{11}$$

with $|\Psi\rangle_{ABE} = \alpha |ge\bar{0}\rangle + \beta |eg\bar{0}\rangle + \gamma |gg\bar{1}\rangle$.

2) The system is classically correlated with the environment (i.e., classical-correlated state)

$$\begin{aligned} \rho_{ABE}^{(2)} \;=\; & (\alpha |ge\rangle + \beta |eg\rangle)(\alpha^* \langle ge| + \beta^* \langle eg|) \otimes |\bar{0}\rangle \langle\bar{0}| \\ & + |\gamma|^2 |gg\rangle \langle gg| \otimes |\bar{1}\rangle \langle\bar{1}|. \end{aligned} \tag{12}$$

3) The system is without any correlation with the environment (i.e., factorized states)

$$\rho_{ABE}^{(3)} = \rho_{AB} \otimes \rho_E, \tag{13}$$

where $|e\rangle$ and $|g\rangle$ are the excited state and ground state of the two-level atoms, respectively, and $|\bar{0}\rangle = \prod_{k=1}^N |0_k\rangle$ and $|\bar{1}\rangle = (1/C)\sum_{k=1}^M C_k |1_k\rangle$ are the environment states. α, β, and γ satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ ($\alpha, \beta, \gamma \neq 0$).

In all the states mentioned above, tracing over the environment variables (or the system variables), we end up with the same density operators given in Eqs. (9) and (10). As the information about the environment is not directly accessible, any measurement carried out on the qubit state cannot distinguish whether the system was initially correlated to the environment or not. In the next section, we will illustrate the role of different initial correlations between the system and its environment on the dynamics of GMOD and MIN.

2.3. Physical model

The physical model examined in this paper is a composite system of two two-level atoms coupling with a zero-temperature environment. Under the rotating wave approximation (RWA), the Hamiltonian of the composite system can be written as ($\hbar = 1$)

$$H = H_0 + H_{\rm I}, \tag{14}$$

with

$$H_0 = \omega_0(\sigma_+^A\sigma_-^A + \sigma_+^B\sigma_-^B) + \sum_k \omega_k a_k^\dagger a_k, \tag{15}$$

$$H_{\rm I} = (\sigma_+^A + \sigma_+^B)\sum_k g_k a_k + {\rm h.c.}, \tag{16}$$

where ω_0 is the transition frequency and $\sigma_\pm^j$ is the inversion operators ($j = A, B$) of the atom. ω_k is the k-th frequency of the environment, and $a_k^\dagger$ (a_k) is the creation (annihilation) operator of the environment. g_k is the coupling coefficient between the atom and the k-th mode.

Using the pseudomode theory[29–32] and assuming the two atoms interact resonantly with a Lorentzian structured reservoir, the spectral distribution of the pseudomode can be written as

$$J(\omega) = \frac{\Omega^2}{2\pi}\frac{\Gamma}{(\omega-\omega_0)^2 - (\Gamma/2)^2}, \tag{17}$$

where Γ is the pseudomode decay rate and Ω is the coupling constant between the pseudomode and the two atoms. Typically, there are two regimes:[33] weak-coupling regime $\Gamma > 2\Omega$, where the behavior of the system is Markovian and irreversible decay occurs, and strong-coupling regime $\Gamma < 2\Omega$, where non-Markovian dynamics occur accompanied by an oscillatory reversible decay.

If the whole system contains only one excitation, for instance, $\rho_{ABE}^{(1)}$ and $\rho_{ABE}^{(2)}$, the Hamiltonian is given by

$$\begin{aligned} H =\ & \omega_0(\sigma_+^A\sigma_-^A + \sigma_+^B\sigma_-^B) + \omega_c a^\dagger a \\ & + [\Omega(\sigma_+^A + \sigma_+^B)a + {\rm h.c.}]. \end{aligned} \tag{18}$$

So the master equation in this case can be written as

$$\frac{{\rm d}\rho}{{\rm d}t} = -{\rm i}[H,\rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a]. \tag{19}$$

For $\rho_{ABE}^{(3)}$, the single excitation approximation is invalid, as the quantum system and environment contain one excitation, respectively. However, the two atoms can be equivalently described by a four-level system. In the basis $\{|0\rangle = |gg\rangle, |+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}, |-\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}, |2\rangle = |ee\rangle\}$, the Hamiltonian can be written as

$$\begin{aligned} H =\ & 2\omega_0 |2\rangle\langle 2| + \omega_0(|+\rangle\langle +| + |-\rangle\langle -|) + \omega_c a^\dagger a \\ & + \sqrt{2}\Omega(|+\rangle\langle 0|a + |2\rangle\langle +|a) + {\rm h.c.} \end{aligned} \tag{20}$$

We can also write the master equation as Eq. (19)

$$\frac{{\rm d}\rho}{{\rm d}t} = -{\rm i}[H,\rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a]. \tag{21}$$

Whether one or two excitations are involved, we can obtain the time-evolution reduced density matrix of the system by solving the master equation in Eqs. (19) and (21) by tracing out the pseudomode degree of freedom. According to Eqs. (7) and (8), it is easy to calculate MIN and GMOD, respectively. As the analytical results are too complicated to present, we just show the numerical results in the following discussions.

3. Numerical results and discussions

First, we focus on the dynamics of GMOD and MIN in Markovian environment and explore the influence of initial correlations between the system and its environment on GMOD and MIN. Here, we choose the parameters $\alpha = \sin\theta\cos\varphi$, $\beta = \sin\theta\sin\varphi$, and $\gamma = \cos\theta$ ($\theta \in [0, \pi/2]$ and $\varphi \in [0, \pi]$). In Fig. 1, the GMOD and MIN are plotted as a function of time, starting from different initial system–environment correlated states in which $\theta = \pi/3$ and $\varphi = 2\pi/3$ are set.

As the physical model examined in this paper is two qubits coupling with a common environment, thus the GMOD and MIN inevitably reach equilibrium values no matter what kind of correlations is involved. Despite all this, it is clear that the presence of the system–environment correlation in the initial state changes the dynamics dramatically. The solid line shows the results of the initially quantum-correlated state. We note that both GMOD and MIN experience a sudden increase before reaching the same stationary values with the initially factorized state. However, the results from the initially classical-correlated state (shown as a solid line) are markedly different from the initially quantum-correlated state. Before reaching the same stationary values as in the initially factorized state, it shows a sudden decrease of both GMOD and MIN. Moreover, it is worth noting that the stationary value of MIN is larger than its initial value while the stationary value of GMOD is smaller than its initial value.

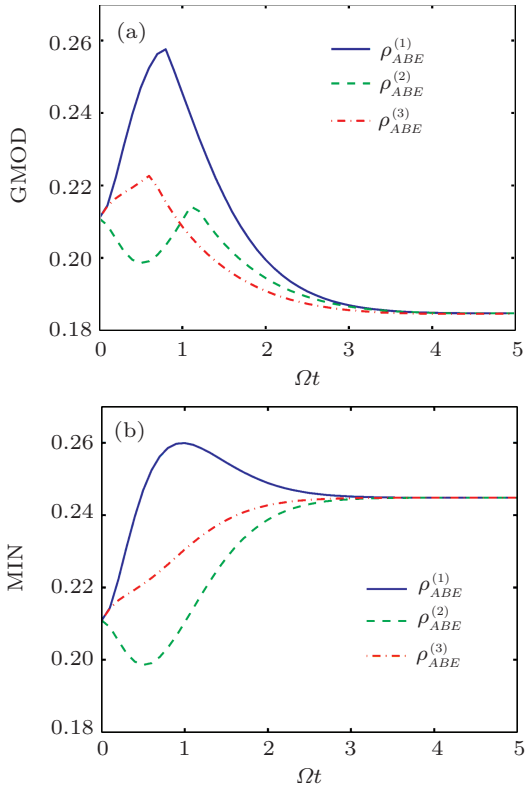


Fig. 1. (color online) GMOD (panel (a)) and MIN (panel (b)) as a function of Ωt in Markovian regime ($\Gamma = 5\Omega$) with $\theta = \pi/3$ and $\varphi = 2\pi/3$ in different initial states. For the cases of (i) $\rho_{ABE}^{(1)}$ (solid line), (ii) $\rho_{ABE}^{(2)}$ (dashed line), (iii) $\rho_{ABE}^{(3)}$ (dash–dotted line).

Then we turn to discuss the non-Markovian situations. The influence of non-Markovian effect has been shown to fundamentally alter the time evolution of entanglement and quantum discord between two quantum systems at zero temperature even without considering the initial system–environment correlations.[18,19,22,23] In Fig. 2, we show the difference of non-Markovian GMOD and MIN dynamics in the presence of initial system–environment coherence. For the initially quantum-correlated state, both the two-qubit GMOD and MIN can first increase to a maximum, then periodically decrease to a long-time asymptotic value in the non-Markovian environment (gradually reaching the asymptotic value in the Markovian regime). However, for the initially classical-correlated state, the GMOD and MIN initially reduce to a minimum, and eventually periodically increase to the same stationary values. Similarly, the stationary value of MIN is larger than its initial value while the stationary value of GMOD is smaller than its initial value. These calculations show that the initial system–environment correlations cannot be ignored in order to completely describe the dynamics of quantum systems.

One may ask why we do not consider the entanglement simultaneously. The reason is that if we want to compare something, we must put them in the same starting point. In this paper, both MIN and GMOD are defined from the geometric perspective of Hilbert–Schmidt norm, while the definition of entanglement from Hilbert–Schmidt norm[34] is not a good definition as shown in Ref. [35]. Therefore, we restrict our discussions to the dynamics of GMOD and MIN only.

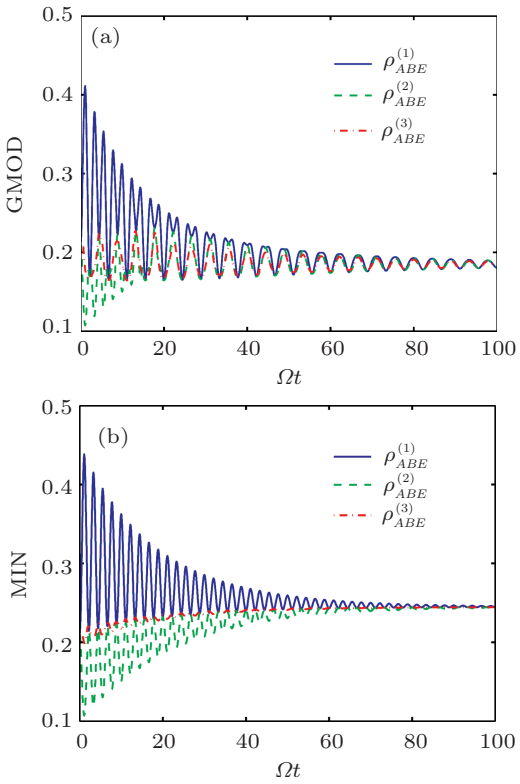


Fig. 2. (color online) GMOD (panel (a)) and MIN (panel (b)) as a function of Ωt in non-Markovian regime ($\Gamma = 0.1\Omega$) with $\theta = \pi/3$ and $\varphi = 2\pi/3$ in different initial states. For the cases of (i) $\rho_{ABE}^{(1)}$ (solid line), (ii) $\rho_{ABE}^{(2)}$ (dashed line), (iii) $\rho_{ABE}^{(3)}$ (dash–dotted line).

4. Conclusions

In conclusion, we have investigated the GMOD and MIN of two non-interacting qubits in the presence of a zero-temperature environment. The initial correlations between system and environment are found to have an important effect on the time evolution of the GMOD and MIN in both the non-Markovian regime and Markovian regime. The behaviors of the GMOD and MIN are quite different: For the initially quantum-correlated states, GMOD and MIN experience a sudden increase before approaching the same stationary values as initially factorized states, while for the initially classical-correlated states, they suffer a sudden decrease. These findings show that, even in calculating properties such as the GMOD and MIN that depend only on the system degrees of freedom, the initial system–environment correlations must be properly taken into account.

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