Control of sudden transition between classical and quantum correlations of two strongly driven atoms in dissipative cavities*

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(Received 29 November 2012; revised manuscript received 7 February 2013)

We investigate analytically the dynamics of classical and quantum correlations between two strongly driven atoms, each of which is trapped inside a dissipative cavity. It is found that there exists a finite time interval during which the quantum discord initially prepared in the X-type states is not destroyed by the decay of the cavities. The sudden transition between classical correlation and quantum discord is sensitive to the initial-state parameter, the cavity decay rate, and the cavity mode-driving field detuning. Interestingly, we show that the transition time can be prolonged significantly by increasing the degree of the detuning.

Keywords: quantum discord, sudden transition, classical correlations

PACS: 03.67.Mn, 42.50.Dv **DOI:** 10.1088/1674-1056/22/8/080305

1. Introduction

It is quite well known that the quantum entanglement, which characterizes the nonclassical correlations between the constituents of the system, has played and continues to play an important role in the field of quantum information and quantum communication.[1–3] It is a fundamental issue in quantum mechanics to distinguish between the quantum and classical aspects of correlations. However, recent studies show that entanglement is not the only type of the quantum correlations useful for quantum information technology,[4,5] and there are other nonclassical correlations apart from entanglement that can also be of certain importance to these fields. Ollivier and Zurek introduced the so-called quantum discord,[6] which is used to account for all nonclassical correlations in a composite system. Quantum discord is a more general measure than entanglement since there exist separable mixed states with nonzero quantum discord. It is observed that such states can be responsible for the efficiency of a quantum computer.[7,8] Consequently, quantum discord might have a significant role in quantum computation tasks and has been studied in many contexts.[9–13]

However, in realistic quantum information processing, quantum systems are not closed and their inevitable interaction with the environment may lead to the destruction of all quantum properties, which is recognized as the main obstacle in the development of quantum technology. Therefore, it is rather important to understand the influence of the environment on quantum correlations. In recent years, the investigation of quantum correlations in open quantum systems has received great attention,[14–22] and the results indicate that the quantum discord resists sudden death in Markovian environments[14] and vanishes only at discrete instants in non-Markovian environments.[17–19] Interestingly, it is observed that for some special initial states, there exists a sudden transition from classical to quantum decoherence, which implies that the quantum discord can be completed unaffected by a noisy environment during a finite time interval.[23] More recently, Luo *et al.* investigated the quantum discord for two qubits in XY spin baths and observed that the transition time can be prolonged by applying the bang–bang pulses.[24] This phenomenon of sudden transition has also been verified by the recent experiment.[25,26] That the quantum correlations do not decay during a finite time interval in the presence of decoherence opens a series of interesting topics. Thus it is essential to investigate the sudden transition phenomenon in different open quantum systems.

In this paper, we consider a system consisting of two strongly driven atoms, each of which is trapped inside a dissipative cavity. With a strong classical field detuned from the cavity eigenmode, we derive the analytical solution to the master equation. We show that for initial X-type states, a sudden transition is found to exist between classical and quantum decoherence. During a finite time interval, the quantum discord does not decay under the dissipation of the cavity modes. We observed that the sudden transition depends on the initial-state parameter, the cavity decay rate, and the detuning between the cavity mode and the driving field. It is worth noting that the transition time can be prolonged largely by increasing the value of the detuning. Compared with Ref. [24], the QED system is more convenient to manipulate and the parameters are easier to control in the experiment implementation than the spin chain system.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11205056 and 11247308) and the Special Funds for the Co-construction Project of Beijing.

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2. Description of the model

We investigate a quantum system consisting of two non-interacting atoms each coupled to a single-mode cavity and additionally driven by an external classical field, as shown in Fig. 1.

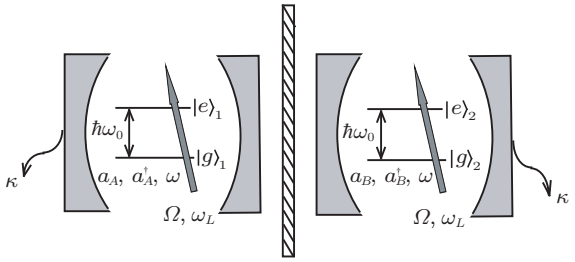


Fig. 1. Two driven two-level atoms each coupled to a single-mode cavity.

Under the rotating wave approximation, the associated Hamiltonian reads

$$H = \hbar\omega_0 \sum_{j=A,B} \sigma_j^\dagger \sigma_j + \hbar\omega \sum_{j=A,B} a_j^\dagger a_j + \hbar g \sum_{j=A,B} (\sigma_j^\dagger a_j + \sigma_j a_j^\dagger) + \hbar\Omega \sum_{j=A,B} (\mathrm{e}^{-\mathrm{i}\omega_L t}\sigma_j^\dagger + \mathrm{e}^{\mathrm{i}\omega_L t}\sigma_j), \quad (1)$$

where $\sigma_j = |g_j\rangle\langle e_j|$ and $\sigma_j^\dagger = |e_j\rangle\langle g_j|$ are the atomic spin-flip operators associated with the upper level $|e_j\rangle$ and the lower level $|g_j\rangle$ of atom j; $a_j^\dagger$ and a_j are the creation and annihilation operators of the single cavity mode; ω_0 and ω are the atomic transition frequency and cavity mode frequency; g is the atom–cavity mode coupling constant; Ω and ω_L are the amplitude and the frequency of the classical field. The Hamiltonian (1) can be transformed to a reference frame rotating with the classical field frequency

$$H = \hbar\Delta \sum_{j=A,B} \sigma_j^\dagger \sigma_j + \hbar\delta \sum_{j=A,B} a_j^\dagger a_j + \hbar g \sum_{j=A,B} (\sigma_j^\dagger a_j + \sigma_j a_j^\dagger) + \hbar\Omega \sum_{j=A,B} (\sigma_j^\dagger + \sigma_j), \quad (2)$$

with $\Delta = \omega_0 - \omega_L$ and $\delta = \omega - \omega_L$. We take the classical field to be sufficiently off-resonant, e.g., $\delta \gg \{g, \kappa\}$ (κ stands for the cavity decay rate), so that the classical field–cavity coupling can be ignored. For simplicity, we assume $\Delta = 0$. By making the unitary transformation $U = \mathrm{e}^{-\mathrm{i}At}$, with $A = \hbar\Omega \sum_{j=A,B}(\sigma_j^\dagger + \sigma_j)$, the resulting Hamiltonian now takes the form

$$H' = \hbar\delta \sum_{j=A,B} a_j^\dagger a_j + \frac{\hbar g}{2} \sum_{j=A,B} (|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + \mathrm{e}^{2\mathrm{i}\Omega t}|+_j\rangle\langle -_j| - \mathrm{e}^{-2\mathrm{i}\Omega t}|-_j\rangle\langle +_j|)a_j + \mathrm{H.c.}, \quad (3)$$

where $|\pm_j\rangle = (|g_j\rangle \pm |e_j\rangle)/\sqrt{2}$ are the eigenstates of $\sigma_j^\dagger + \sigma_j$ with eigenvalues ± 1, respectively. In the strong driving limit $\Omega \gg \{g, \delta\}$, we ignore the terms that oscillate with high frequencies and redefine $g/2 \to g$, the Hamiltonian reads

$$H_{\mathrm{eff}} = \hbar\delta \sum_{j=A,B} a_j^\dagger a_j + \hbar g \sum_{j=A,B} (\sigma_j^\dagger + \sigma_j)(a_j + a_j^\dagger). \quad (4)$$

Considering the photon dissipation of the cavity, the evolution for the whole system can be described by the Lindblad-type master equation (ME)

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H_{\mathrm{eff}}, \rho] + \frac{\kappa}{2} \sum_{j=A,B} (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \quad (5)$$

where κ is the cavity decay rate.

We now consider the solution of ME (5) in the case of a cavity field initially in vacuum state. We assume that the two atoms are initially prepared in a class of state which can be described by the three-parameter X-type density matrix in the basis $\{|m\rangle\}_{m=1}^4 = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ as

$$\rho_{AB}(0) = \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1+c_3 \end{pmatrix}, \quad (6)$$

where c_i $(0 \le |c_i| \le 1)$ are real numbers which satisfy the unit trace and positivity conditions of the density matrix. The above states include the Werner states ($|c_1| = |c_2| = |c_3| = c$) and the Bell states ($|c_1| = |c_2| = |c_3| = 1$). It has been shown that for Bell states, there exist both quantum discord and entanglement. For Werner states, when $c < 1/3$, there exists quantum discord but without entanglement. It has been pointed out that,[15] depending on the geometry (encoding in the relations between c_i) of the state Eq. (6), the evolution of the state correlations can show a peculiar behavior.

In order to solve ME (5), we introduce the following decomposition

$$\rho(t) = \sum_{m,n=1}^{4} \langle m|\rho(t)|n\rangle |m\rangle\langle n| = \sum_{m,n=1}^{4} \rho_{mn}(t)|m\rangle\langle n|. \quad (7)$$

Therefore, the ME (5) is equivalent to the following set of equations for the field operators

$$\begin{aligned}
\dot{\rho}_{11}(t) &= -\mathrm{i}g\left[a_A^\dagger + a_A, \rho_{11}(t)\right] - \mathrm{i}g\left[a_B^\dagger + a_B, \rho_{11}(t)\right] + L_A\rho_{11}(t) + L_B\rho_{11}(t), \\
\dot{\rho}_{14}(t) &= -\mathrm{i}g\left\{a_A^\dagger + a_A, \rho_{14}(t)\right\} - \mathrm{i}g\left\{a_B^\dagger + a_B, \rho_{14}(t)\right\} + L_A\rho_{14}(t) + L_B\rho_{14}(t), \\
\dot{\rho}_{22}(t) &= -\mathrm{i}g\left[a_A^\dagger + a_A, \rho_{22}(t)\right] + \mathrm{i}g\left[a_B^\dagger + a_B, \rho_{22}(t)\right] + L_A\rho_{22}(t) + L_B\rho_{22}(t), \\
\dot{\rho}_{23}(t) &= -\mathrm{i}g\left\{a_A^\dagger + a_A, \rho_{23}(t)\right\} + \mathrm{i}g\left\{a_B^\dagger + a_B, \rho_{23}(t)\right\} + L_A\rho_{23}(t) + L_B\rho_{23}(t), \\
\dot{\rho}_{33}(t) &= \mathrm{i}g\left[a_A^\dagger + a_A, \rho_{33}(t)\right] - \mathrm{i}g\left[a_B^\dagger + a_B, \rho_{33}(t)\right] + L_A\rho_{33}(t) + L_B\rho_{33}(t), \\
\dot{\rho}_{44}(t) &= \mathrm{i}g\left[a_A^\dagger + a_A, \rho_{44}(t)\right] + \mathrm{i}g\left[a_B^\dagger + a_B, \rho_{44}(t)\right] + L_A\rho_{44}(t) + L_B\rho_{44}(t), \quad (8)
\end{aligned}$$

with

$$L\rho_{mn} = \frac{\kappa}{2}\left[2a\rho_{mn}a^{\dagger} - \left(1+\frac{2\mathrm{i}\delta}{k}\right)a^{\dagger}a\rho_{mn} - \left(1-\frac{2\mathrm{i}\delta}{k}\right)\rho_{mn}a^{\dagger}a\right]$$

and $[,]$ $(\{,\})$ denotes the commutation (anticommutation) brackets. The appearance of commutator and anticommutator brackets for different elements of density operator is because $|\pm_j\rangle$ are the eigenstates of $\sigma_j^{\dagger}+\sigma_j$ with eigenvalues ± 1. The detailed calculation is described in Appendix A.

Using superoperator technique,[27] these decoupled equations can be derived as

$$\begin{aligned}
\rho_{11}(t) &= \frac{1+c_3}{4}|\alpha,\alpha\rangle\langle\alpha,\alpha|,\\
\rho_{14}(t) &= \frac{(c_1-c_2)f^2(t)}{4\mathrm{e}^{-4|\alpha|^2}}|\alpha,\alpha\rangle\langle-\alpha,-\alpha|,\\
\rho_{22}(t) &= \frac{1-c_3}{4}|\alpha,-\alpha\rangle\langle\alpha,-\alpha|,\\
\rho_{23}(t) &= \frac{(c_1+c_2)f^2(t)}{4\mathrm{e}^{-4|\alpha|^2}}|\alpha,-\alpha\rangle\langle-\alpha,\alpha|,\\
\rho_{33}(t) &= \frac{1-c_3}{4}|-\alpha,\alpha\rangle\langle-\alpha,\alpha|,\\
\rho_{44}(t) &= \frac{1+c_3}{4}|-\alpha,-\alpha\rangle\langle-\alpha,-\alpha|,
\end{aligned} \tag{9}$$

where we define

$$\begin{aligned}
\alpha &= -\frac{2\mathrm{i}g(1-\mathrm{e}^{-t(\kappa+2\mathrm{i}\delta)/2})}{\kappa+2\mathrm{i}\delta},\\
f(t) &= -\frac{8g^2\kappa t}{\kappa^2+4\delta^2}+\frac{8g^2}{(\kappa+2\mathrm{i}\delta)^2}\left(1-\mathrm{e}^{-t(\kappa+2i\delta)/2}\right)\\
&\quad+\frac{8g^2}{(\kappa-2\mathrm{i}\delta)^2}(1-\mathrm{e}^{-t(\kappa-2i\delta)/2}).
\end{aligned} \tag{10}$$

3. Dynamics of the quantum discord for two atoms

In the following study, we analyze the time-dependent behavior of the quantum discord for the two noninteracting atoms. The quantum discord is defined as the difference of the total amount of correlations and the classical correlations,

$$Q(\rho_{AB}) = I(\rho_{AB}) - C(\rho_{AB}). \tag{11}$$

Here, the total correlations are expressed by quantum mutual information,

$$I(\rho_{AB}) = S(\rho_A)+S(\rho_B)-S(\rho_{AB}), \tag{12}$$

where $\rho_{A(B)}$ is the reduced density matrix of ρ_{AB} by tracing out $B(A)$ and $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy. The classical correlations between two subsystems are defined as

$$C(\rho_{AB}) = S(\rho_A) - \min_{\{P_k^B\}}\sum_k p_k S(\rho_k^A). \tag{13}$$

Here $\{P_k^B\}$ is a set of projectors performed locally only on the subsystem B and the remaining state of A associated with the measurement result k changes to $\rho_k^A = [(I\otimes P_k^B)\rho_{AB}(I\otimes P_k^B)]/p_k$, where I denotes the density operator performed on A and $p_k = \mathrm{Tr}[(I\otimes P_k^B)\rho_{AB}(I\otimes P_k^B)]$ is the probability with respect to the outcome k.

Using Eq. (12), we have quantum mutual information as follows:

$$I(\rho_{AB}) = 2+\sum_{i=1}^{4}\lambda_i\log_2\lambda_i \tag{14}$$

with

$$\begin{aligned}
\lambda_{1,2} &= \frac{1}{4}[(1-c_3)\pm f^2(t)(c_1+c_2)],\\
\lambda_{3,4} &= \frac{1}{4}[(1+c_3)\pm f^2(t)(c_1-c_2)].
\end{aligned} \tag{15}$$

According to the method used in Ref. [9], the analytical expression for the classical correlations can be calculated as

$$C(\rho_{AB}) = \sum_{n=1}^{2}\frac{1+(-1)^n\Lambda(t)}{2}\log_2[1+(-1)^n\Lambda(t)], \tag{16}$$

where we have defined

$$\Lambda(t) = \max\left\{|c_3|, f^2(t)\sqrt{\frac{c_1^2+c_2^2+|c_1^2-c_2^2|}{2}}\right\}.$$

Consequently, we have the quantum discord

$$\begin{aligned}
Q(\rho_{AB}) &= I(\rho_{AB})-C(\rho_{AB})\\
&= 2+\sum_{i=1}^{4}\log_2\lambda_i - \sum_{n=1}^{2}\frac{1+(-1)^n\Lambda(t)}{2}\\
&\quad\times\log_2[1+(-1)^n\Lambda(t)].
\end{aligned} \tag{17}$$

We consider the initial parameters $c_1 = 1$, $c_2 = -c_3$. Then it is straightforward to obtain the explicit expression of quantum discord as follows:

$$\begin{aligned}
Q(\rho_{AB}) &= \frac{1}{2}(1-W)\log_2(1-W)\\
&\quad+\frac{1}{2}(1+W)\log_2(1+W)
\end{aligned} \tag{18}$$

with

$$W = \min\left\{|c_3|, f^2(t)\right\}. \tag{19}$$

In Fig. 2 we plot the dynamics of the quantum discord, the classical correlations and the mutual information as a function of the scaled time gt for two initial-state parameters c_3 with $\kappa = 2g$, $\delta = 6g$. As is apparent from the plot, the system has a sudden transition from classical to quantum decoherence at time $t = t_0$. During the time regime ($t \le t_0$), the quantum discord remains constant while classical correlations decay strongly under the decoherence. After the sudden-transition point ($t \ge t_0$), the classical correlations become more robust

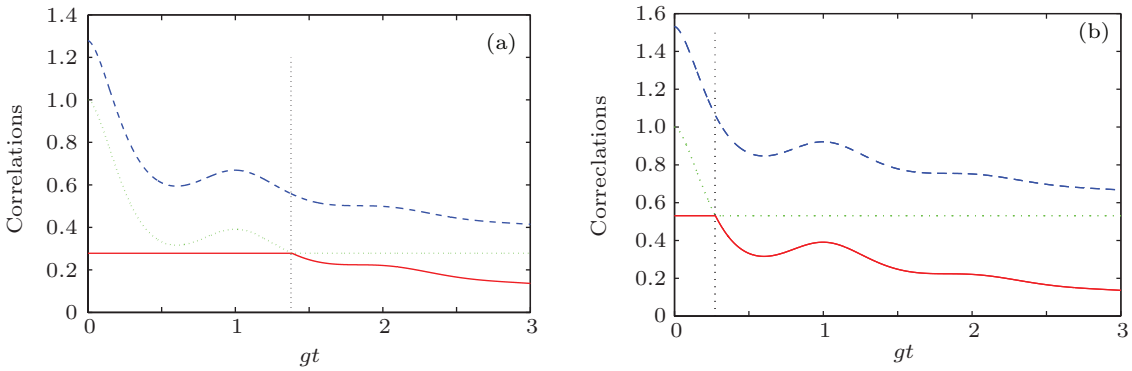


Fig. 2. (color online) Dynamics of quantum mutual information (blue dashed line), classical correlations (green dotted line), and the quantum discord (red solid line) versus dimensionless scaled time gt with $\kappa = 2g$, $\delta = 6g$ for (a) $c_3 = 0.6$, (b) $c_3 = 0.8$. The vertical dotted line denotes the transition time t_0.

against decoherence than the quantum discord. Comparing with Figs. 2(a) and 2(b), one sees that the sudden transition time depends on initial-state parameter c_3 strongly. In Fig. 3, we plot explicitly the sudden transition time t_0 versus c_3, where the inset shows the dynamics of the quantum discord for different values of c_3. It can be clearly seen that the stable value of the quantum discord decreases and the transition time increases with the decreasing of c_3. This result coincides with the conclusion in Ref. [24], where they investigated the quantum discord for two qubits in XY spin baths.

We see that the parameters c_i play different roles during the evolution. This can be explained from the evolution of the density matrix. Tracing over the degrees of the cavity fields in Eq. (9), the reduced density matrix for the two atoms can be derived as

$$\rho_{AB}(t) = \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & (c_1-c_2)f^2(t) \\ 0 & 1-c_3 & (c_1+c_2)f^2(t) & 0 \\ 0 & (c_1+c_2)f^2(t) & 1-c_3 & 0 \\ (c_1-c_2)f^2(t) & 0 & 0 & 1+c_3 \end{pmatrix}. \tag{20}$$

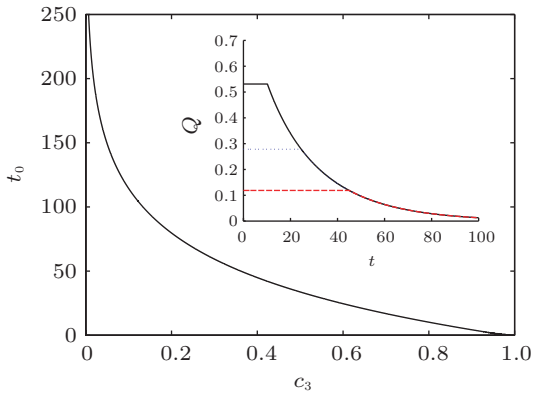


Fig. 3. (color online) A plot of sudden transition time t_0 versus c_3 with $\kappa = 2g$ and $\delta = 20g$. Inset: dynamics of the quantum discord for different values of c_3: $c_3 = 0.4$ (red dotted line), $c_3 = 0.6$ (blue dashed line), and $c_3 = 0.8$ (black solid line).

Comparing with Eqs. (6) and (20), we find that after the evolution, the diagonal elements (relevant with c_3) do not change, while all the off-diagonal elements (relevant with c_1 and c_2) have one more time factor $f^2(t)$. This is why the sudden transition happens and why c_i play different roles during the evolution.

Let us turn to study the effects of the decay rate κ on the quantum correlations. As shown in Fig. 4, we plot the dynamics of the quantum discord, the classical correlations, and the mutual information for two initial-state parameters κ with $c_3 = 0.6$, $\delta = 10g$. We observe that the change of κ does not affect the stable value of quantum discord, however, the transition time decreases with the larger value of κ.

In fact, from Eq. (27), we can see that the stable value of quantum discord only depends on the initial-state parameter c_3, however, the transition time is decided by c_3 and $\Lambda(t)$ jointly. Figure 3 shows that with the increasing of c_3, the stable value of quantum discord increases but the transition time decreases. We also observe from Fig. 4 that the larger value of κ will shorten the transition time. Next we will investigate whether the sudden transition time can be prolonged when c_3 and κ are settled. In Fig. 5, we show the evolution of the quantum discord as a function of dimensionless time gt for different values of δ with $c_3 = 0.6$, $k = 2g$. It is found that the transition time can be largely prolonged over which the quantum discord does not decay. To gain further insight, we plot the sudden transition time t_0 versus the initial-state parameter c_3 for different values of δ with $k = 2g$ in Fig. 6. Obviously, large values of δ can enhance the transition time significantly. This property can be useful in performing quantum information tasks without any influence by the noise environment for long enough intervals of time.

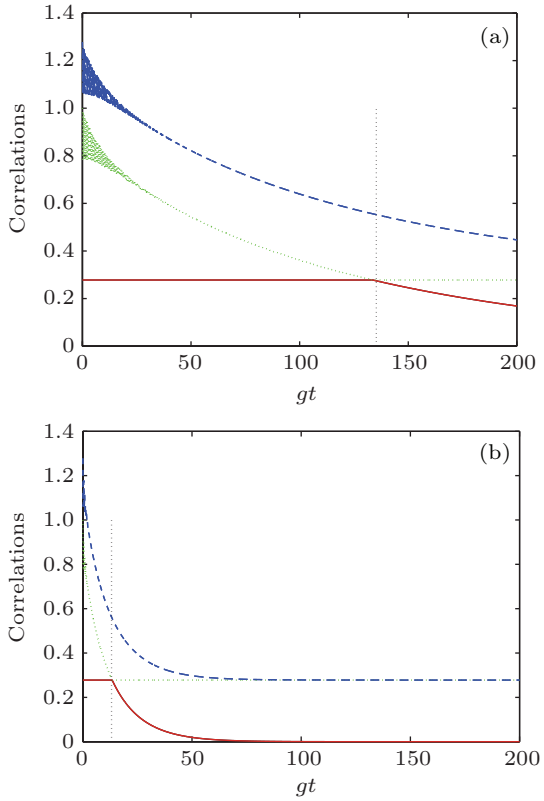


Fig. 4. (color online) Dynamics of quantum mutual information (blue dashed line), classical correlations (green dotted line), and the quantum discord (red solid line) versus dimensionless scaled time gt with $c_3 = 0.6$, $\delta = 20g$ for (a) $\kappa = 0.2g$, (b) $\kappa = 2g$. The vertical dotted line denotes the transition time t_0.

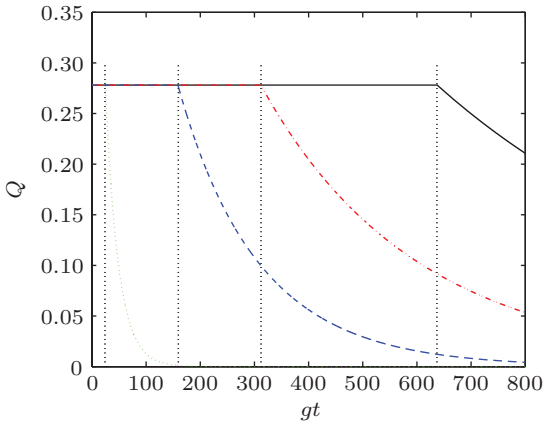


Fig. 5. (color online) Dynamics of the quantum discord versus dimensionless scaled time gt with $c_3 = 0.6$, $\kappa = 2g$ for $\delta = 20g$ (green dotted line), $\delta = 50g$ (blue dashed line), $\delta = 70g$ (red dash-dotted line), $\delta = 100g$ (black solid line). The vertical dotted lines denote the transition time t_0.

In order to control the sudden transition time, we have chosen the range $\delta \gg g \gg \gamma$ (γ is the atom decay rate, which in our analytical study has been assumed to be zero). These conditions can be realized in two types of solid-state qubits.[28,29] One system is a superconducting qubit coupled to a microwave cavity,[28] where people can achieve $g \approx 100$ MHz, $\gamma \approx 1$ MHz. Thus the qubit decay can be ignored. Moreover in this system, the detuning δ is much larger than the coupling strength g and then can be chosen to be $100g$. Therefore, all of the approximation conditions are fulfilled for the parameters in our figures. Another system is a single nitrogen atom strongly coupled to an optical cavity,[29] in which the detuning can reach 2π GHZ. The large value of δ is a critical factor to prolong the transition time, which is beneficial to provide plenty of time to perform quantum information tasks without any disturbance from the noisy environment.

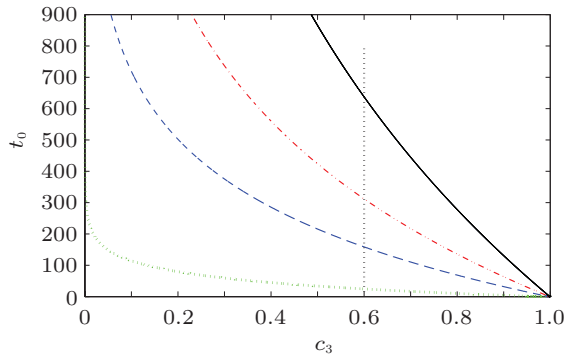


Fig. 6. (color online) Dynamics of transition time t_0 versus initial-state parameter c_3 with $\kappa = 2g$ for $\delta = 20g$ (green dotted line), $\delta = 50g$ (blue dashed line), $\delta = 70g$ (red dash-dotted line), $\delta = 100g$ (black solid line). The vertical dotted line denotes the transition time when $c_3 = 0.6$.

4. Conclusion

In this paper, we have investigated the dynamical behavior of the quantum discord in the system of two strongly driven atoms, each of which is trapped in a dissipative cavity. It is observed that for a class of initial state, there exists a sudden transition at critical time t_0 between classical and quantum loss of correlations. During a finite time interval $0 < t < t_0$, the quantum discord does not decay in the presence of the cavity dissipation. On the other hand, for $t > t_0$, the quantum discord begins to decay but the classical correlations keep constant with time. We show that the constant value of the quantum discord only depends on the initial-state parameter, however, the transition time is determined collectively by the initial-state parameter, the cavity decay rate, and the cavity mode-driving field detuning. We find that the transition time can be prolonged largely by increasing the value of detuning. This suggests an interesting research on performing quantum information tasks without any disturbance from noise for a sufficiently long interval of time.

Appendix

We illustrate how to derive Eq. (8). By using ME (5), we obtain the following equations for the field operator $\rho_{11}(t) = \langle ++|\rho(t)|++\rangle$,

$$\begin{aligned}
&\dot{\rho}_{11}(t) \\
&= -\frac{\mathrm{i}}{\hbar}\langle ++|H_{\mathrm{eff}}\rho(t)|++\rangle + \frac{\mathrm{i}}{\hbar}\langle ++|\rho(t)H_{\mathrm{eff}}|++\rangle \\
&\quad + \frac{\kappa}{2}\sum_{j=A,B}\langle ++|2a_j\rho(t)a_j^{\dagger} - a_j^{\dagger}a_j\rho(t) - \rho(t)a_j^{\dagger}a_j|++\rangle
\end{aligned}$$

$$
\begin{aligned}
&= -\mathrm{i}\sum_{j=A,B}\langle{+}{+}|[\delta a_j^\dagger a_j + g(\sigma_j^\dagger+\sigma_j)(a_j+a_j^\dagger)]\rho(t)|{+}{+}\rangle \\
&\quad +\mathrm{i}\sum_{j=A,B}\langle{+}{+}|\rho(t)[\delta a_j^\dagger a_j + g(\sigma_j^\dagger+\sigma_j)(a_j+a_j^\dagger)]|{+}{+}\rangle \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}[2a_j\rho_{11}(t)a_j^\dagger - a_j^\dagger a_j\rho_{11}(t) - \rho_{11}(t)a_j^\dagger a_j] \\
&= \sum_{j=A,B}(-\mathrm{i}\delta a_j^\dagger a_j\rho_{11}(t) + \mathrm{i}\delta\rho_{11}(t)a_j^\dagger a_j) \\
&\quad +\sum_{j=A,B}[-\mathrm{i}g(a_j^\dagger+a_j)\rho_{11}(t) + \mathrm{i}g\rho_{11}(t)(a_j^\dagger+a_j)] \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}[2a_j\rho_{11}(t)a_j^\dagger - a_j^\dagger a_j\rho_{11}(t) - \rho_{11}(t)a_j^\dagger a_j] \\
&= -\mathrm{i}g\sum_{j=A,B}[(a_j^\dagger+a_j),\rho_{11}(t)] \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}\left[2a_j\rho_{11}(t)a_j^\dagger - \left(1+\frac{2\mathrm{i}\delta}{k}\right)a_j^\dagger a_j\rho_{11}(t)\right. \\
&\quad \left. -\left(1-\frac{2\mathrm{i}\delta}{k}\right)\rho_{11}(t)a_j^\dagger a_j\right], \qquad \text{(A1)}
\end{aligned}
$$

where $[,]$ denotes the commutator symbol. For the field operator $\rho_{14}(t) = \langle{+}{+}|\rho(t)|{-}{-}\rangle$, thus we have

$$
\begin{aligned}
&\dot{\rho}_{14}(t) \\
&= -\frac{\mathrm{i}}{\hbar}\langle{+}{+}|H_{\mathrm{eff}}\rho(t)|{-}{-}\rangle + \frac{\mathrm{i}}{\hbar}\langle{+}{+}|\rho(t)H_{\mathrm{eff}}|{-}{-}\rangle \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}\langle{+}{+}|2a_j\rho(t)a_j^\dagger - a_j^\dagger a_j\rho(t) - \rho(t)a_j^\dagger a_j|{-}{-}\rangle \\
&= -\mathrm{i}\sum_{j=A,B}\langle{+}{+}|[\delta a_j^\dagger a_j + g(\sigma_j^\dagger+\sigma_j)(a_j+a_j^\dagger)]\rho(t)|{-}{-}\rangle \\
&\quad +\mathrm{i}\sum_{j=A,B}\langle{+}{+}|\rho(t)[\delta a_j^\dagger a_j + g(\sigma_j^\dagger+\sigma_j)(a_j+a_j^\dagger)]|{-}{-}\rangle \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}[2a_j\rho_{14}(t)a_j^\dagger - a_j^\dagger a_j\rho_{14}(t) - \rho_{14}(t)a_j^\dagger a_j] \\
&= \sum_{j=A,B}(-\mathrm{i}\delta a_j^\dagger a_j\rho_{14}(t) + \mathrm{i}\delta\rho_{14}(t)a_j^\dagger a_j) \\
&\quad +\sum_{j=A,B}[-\mathrm{i}g(a_j^\dagger+a_j)\rho_{14}(t) - \mathrm{i}g\rho_{14}(t)(a_j^\dagger+a_j)] \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}[2a_j\rho_{14}(t)a_j^\dagger - a_j^\dagger a_j\rho_{14}(t) - \rho_{14}(t)a_j^\dagger a_j] \\
&= -\mathrm{i}g\sum_{j=A,B}\{(a_j^\dagger+a_j),\rho_{14}(t)\} \\
&\quad +\frac{\kappa}{2}\sum_{j=A,B}\left[2a_j\rho_{14}(t)a_j^\dagger - \left(1+\frac{2\mathrm{i}\delta}{k}\right)a_j^\dagger a_j\rho_{14}(t)\right. \\
&\quad \left. -\left(1-\frac{2\mathrm{i}\delta}{k}\right)\rho_{14}(t)a_j^\dagger a_j\right], \qquad \text{(A2)}
\end{aligned}
$$

where $\{,\}$ denotes the anticommutator symbol. The other equations in Eq. (8) can also be derived by the same method.

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