Stochastic asymptotical synchronization of chaotic Markovian jumping fuzzy cellular neural networks with mixed delays and the Wiener process based on sampled-data control*

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We investigate the stochastic asymptotical synchronization of chaotic Markovian jumping fuzzy cellular neural networks (MJFCNNs) with discrete, unbounded distributed delays, and the Wiener process based on sampled-data control using the linear matrix inequality (LMI) approach. The Lyapunov–Krasovskii functional combined with the input delay approach as well as the free-weighting matrix approach is employed to derive several sufficient criteria in terms of LMIs to ensure that the delayed MJFCNNs with the Wiener process is stochastic asymptotical synchronous. Restrictions (e.g., time derivative is smaller than one) are removed to obtain a proposed sampled-data controller. Finally, a numerical example is provided to demonstrate the reliability of the derived results.

Keywords: stochastic asymptotical synchronization, fuzzy cellular neural networks, chaotic Markovian jumping parameters, sampled-data control

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1. Introduction

Chua and Yang first introduced cellular neural networks (CNNs) in 1988.[1,2] A set of CNNs in parallel local connectivity is widely utilized in both practical applications and biology to achieve higher-level information processing and reasoning functions. This integrated CNN system helps to solve more complex intelligence problems.[3,4] However, in every phase of image processing, there exist many uncertainties. Fuzzy set theory provides the mathematical strength to capture these uncertainties. Fuzzy cellular neural networks (FCNNs) were introduced by Yang *et al.*,[5,6] and have been used in Ref. [7] as an application tool in image processing and pattern recognition.

Chaos theory is a further development of dynamical system theory, which focuses on highly complex chaotic motions and explains the real world problem in suitable sense. Chaos theory explains the real world. We cannot predict certain processes over an extended period of time. Recently, the search for synchronization has moved to chaotic systems. A dynamical system is considered to be chaotic whenever its evolution sensitively depends on the initial conditions. As a result, chaotic systems essentially hold synchronization. Pecora and Carroll[8] introduced chaos synchronization, and since then it has become an important aspect in dynamical systems. Since the early 1990s, researchers have realized that chaotic systems can be synchronized. The recognized potential for communication systems has driven this phenomenon to become a distinct subfield of nonlinear dynamics, with the need to understand the phenomenon in its most fundamental form. All forms of identical synchronization, where two or more dynamical systems execute the same behavior at the same time, are real manifestations of dynamical behavior restricted to a flat hyperplane in the phase space. This is true regardless of whether the behavior is chaotic, periodic, or a fixed point. This leads to two fundamental considerations in studying synchronization:[9–18] finding the hyperplane; determining its stability.

Chaotic systems have various applications, including secure communication. To use a chaotic signal in communications, the receiver (response system) must have a duplicate of the transmitter's (drive system) chaotic signal. Since chaos is sensitive to initial conditions, the identical chaotic systems start almost (but not exactly) at initial conditions. The systems soon diverge from each other, but retain the same attractor pattern. In order to synchronize the transmitter's chaotic signal, the receiver (response system) needs a suitable digital secure communication scheme that utilizes the instantaneous phase as the signal transmitted from the drive system to the response system. However, due to this synchronization, the error between the drive system and the response system converges to zero.

Many results on synchronization of FCNNs with time delays can be found.[19–23] Using the linear matrix inequality (LMI) technique, Balasubramaniam *et al.*[19] have derived the synchronization control of FCNNs with mixed time delays. However, synchronization of FCNNs has been investigated with various control approaches such as periodically intermittent control,[22] sliding mode control,[23] and so on. In order to

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take advantage of modern high-speed computers, microelectronics, and communication networks, it is more preferable to use digital controllers instead of analog circuits.[24] It allows synchronization of chaotic systems only using the samples of the state variables of the drive (master) system and the response (slave) chaotic system at discrete instants. There are some studies dealing with the synchronization problem using the sampled-data control.[25–30]

Meanwhile, in real systems stochastic effects are usually unavoidable due to modeling errors and environmental noises, as Haykin[31] pointed out that in real nervous systems synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. So far, very little attention has been paid to synchronization of stochastic FCNNs.[32,33] On the other hand, FCNNs often exhibit a special characteristic of network mode switching. In other words, an FCNN sometimes has finite modes that switch from one to another at different times.[34] To the best of authors' knowledge, studies on synchronization of chaotic MJFCNNs with mixed delays and the Wiener process based on sampled-data control have not been discussed in any of the existing literatures. Thus, the problem is new and challenging for deriving sufficient conditions for the stability criterion.

Motivated by the aforementioned discussion, in this paper we derive new criteria for the stochastic asymptotical synchronization of chaotic delayed MJFCNNs with a Wiener process based on sampled-data control. By removing the restrictive condition on the time derivative (i.e., to be smaller than one), suitable Lyapunov–Krasovskii functional (LKF) has been constructed to derive sufficient conditions for delay-dependent stability criterion (for MJFCNNs) in terms of free-weighting matrices and LMIs. Finally, a numerical example is used to show the effectiveness of our proposed method.

Notations $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; for any matrix $\boldsymbol{A}=[a_{ij}]_{n\times n}$, let $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^{-1}$ denote the transpose and the inverse of $\boldsymbol{A}$, respectively; $|\boldsymbol{A}|=[|a_{ij}|]_{n\times n}$; $\boldsymbol{A}>0$ ($\boldsymbol{A}<0$) denotes the positive-definite (negative-definite) symmetric matrix; I denotes the identity matrix of appropriate dimension; $\Lambda=\{1,2,\ldots,n\}$; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively; $\|\boldsymbol{x}\|$ denotes a vector norm defined by $\|\boldsymbol{x}\|=\left(\sum_{i=1}^{n}x_i^2\right)^{1/2}$; $\bigwedge \boldsymbol{X}\circ\boldsymbol{Y}=(\bigwedge_{j=1}^{n}x_{1j}y_j,\ldots,\bigwedge_{j=1}^{n}x_{nj}y_j)^{\mathrm{T}}$, $\bigvee \boldsymbol{X}\circ\boldsymbol{Y}=(\bigvee_{j=1}^{n}x_{1j}y_j,\ldots,\bigvee_{j=1}^{n}x_{nj}y_j)^{\mathrm{T}}$; $*$ denotes the symmetric terms in a symmetric matrix.

2. Model formulation and preliminaries

Consider the following chaotic FCNNs (drive system) with discrete and unbounded distributed delays:

$$\begin{cases}\mathrm{d}\boldsymbol{x}(t)=\Big[-\boldsymbol{D}\boldsymbol{x}(t)+\boldsymbol{A}f(\boldsymbol{x}(t))+\boldsymbol{B}f(\boldsymbol{x}(t-\tau_1(t)))+\boldsymbol{I}\\ \qquad\qquad +\bigwedge\boldsymbol{\alpha}\circ\int_{-\infty}^{t}K(t-s)f(\boldsymbol{x}(s))\,\mathrm{d}s\\ \qquad\qquad +\bigvee\boldsymbol{\beta}\circ\int_{-\infty}^{t}K(t-s)f(\boldsymbol{x}(s))\,\mathrm{d}s\Big]\,\mathrm{d}t,\\ \boldsymbol{x}(s)=\boldsymbol{\phi}(s),\quad s\in(-\infty,0],\end{cases}\tag{1}$$

and the corresponding response system with the Wiener process defined on the probability space $(\Omega,\Upsilon,\boldsymbol{P})$, taking the form of

$$\begin{cases}\mathrm{d}\boldsymbol{y}(t)=[-\boldsymbol{D}\boldsymbol{y}(t)+\boldsymbol{A}f(\boldsymbol{y}(t))+\boldsymbol{B}f(\boldsymbol{y}(t-\tau_1(t)))+\boldsymbol{I}\\ \qquad\qquad +\bigwedge\boldsymbol{\alpha}\circ\int_{-\infty}^{t}K(t-s)f(\boldsymbol{y}(s))\,\mathrm{d}s\\ \qquad\qquad +\bigvee\boldsymbol{\beta}\circ\int_{-\infty}^{t}K(t-s)f(\boldsymbol{y}(s))\,\mathrm{d}s+u(t)]\,\mathrm{d}t\\ \qquad\qquad +\boldsymbol{W}\boldsymbol{e}(t)\,\mathrm{d}\boldsymbol{\omega}(t),\\ \boldsymbol{y}(s)=\boldsymbol{\varphi}(s),\quad s\in(-\infty,0],\end{cases}\tag{2}$$

where $\boldsymbol{\omega}(t)$ is a one-dimensional standard Wiener process on the given probability space satisfying $\mathbb{E}\{\mathrm{d}\boldsymbol{\omega}(t)=0\}$ and $\mathbb{E}\{\mathrm{d}\boldsymbol{\omega}^2(t)=\mathrm{d}t\}$; $\boldsymbol{W}$ is the Wiener process matrix that is supposed to be known; $\boldsymbol{\phi}(\cdot)\in C((-\infty,0],\mathbb{R})$ and $\boldsymbol{\varphi}(\cdot)\in C((-\infty,0],\mathbb{R})$ are the initial conditions of the drive system (1) and the response system (2), respectively; $\boldsymbol{\alpha}=(\alpha_{ij})_{n\times n}$ and $\boldsymbol{\beta}=(\beta_{ij})_{n\times n}$ are the elements of fuzzy feedback minimum template and fuzzy feedback maximum template, respectively; $\boldsymbol{A}=(a_{ij})_{n\times n}$ and $\boldsymbol{B}=(b_{ij})_{n\times n}$ are the elements of feedback template; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $\boldsymbol{x}(t)=(x_1(t),x_2(t),\ldots,x_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ and $\boldsymbol{y}(t)=(y_1(t),y_2(t),\ldots,y_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ denote the state vectors in drive system (1) and response system (2), respectively; $\boldsymbol{I}=(I_1,I_2,\ldots,I_n)^{\mathrm{T}}$ denotes the external input of neurons; $\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\ldots,d_n)$ is a positive diagonal matrix; $f(\cdot)$ represents the neuron activation function; $\boldsymbol{u}(t)=(u_1(t),u_2(t),\ldots,u_n(t))^{\mathrm{T}}$ is a control input to be designed; $K(s)\geq 0$ is the feedback kernel and satisfies

$$\int_0^{\infty}K(s)\,\mathrm{d}s=1.\tag{3}$$

In the probability space $(\Omega,\Upsilon,\boldsymbol{P})$, Ω is the sample space, Υ is the algebra of events, and $\boldsymbol{P}$ is the probability measure defined on $\boldsymbol{\Upsilon}$. Let the random form process $\{\eta_t,\ t\in[0,+\infty)\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories with generator $\Pi=(\pi_{ij})$ and transition probability from mode i at time t to mode j at time $t+\delta$ $(i,j\in\mathfrak{M},\mathfrak{M}=\{1,2,\ldots,N\})$,

$$p_{ij}=Pr(\eta_{t+\delta}=j|\eta_t=i)=\begin{cases}\pi_{ij}\delta+o(\delta), & i\neq j,\\ 1+\pi_{ij}\delta+o(\delta), & i=j,\end{cases}$$

with transition probability rates $\pi_{ij} \geq 0$ for $i, j \in \mathfrak{M}$ $(i \neq j)$ and $\pi_{ii} = -\sum_{j=1,\ j\neq i}^{N} \pi_{ij}$, where $\delta > 0$ and $\lim_{\delta\to 0} o(\delta)/\delta = 0$.

In this paper, we consider the following drive-response type chaotic delayed FCNNs with Markovian jumping parameters and Wiener process, which is a modification of systems (1) and (2):

$$\begin{cases} \mathrm{d}\boldsymbol{x}(t) = [-\boldsymbol{D}(\eta_t)\boldsymbol{x}(t) + \boldsymbol{A}(\eta_t)f(\boldsymbol{x}(t)) \\ \qquad + \boldsymbol{B}(\eta_t)f(\boldsymbol{x}(t-\tau_1(t))) + \boldsymbol{I}(\eta_t) \\ \qquad + \bigwedge \boldsymbol{\alpha}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)f(\boldsymbol{x}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)f(\boldsymbol{x}(s))\,\mathrm{d}s]\,\mathrm{d}t, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in (-\infty, 0], \end{cases} \tag{4}$$

and

$$\begin{cases} \mathrm{d}\boldsymbol{y}(t) = [-\boldsymbol{D}(\eta_t)\boldsymbol{y}(t) + \boldsymbol{A}(\eta_t)f(\boldsymbol{y}(t)) \\ \qquad + \boldsymbol{B}(\eta_t)f(\boldsymbol{y}(t-\tau_1(t))) + \boldsymbol{I}(\eta_t) \\ \qquad + \bigwedge \boldsymbol{\alpha}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)f(\boldsymbol{y}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)f(\boldsymbol{y}(s))\,\mathrm{d}s + u(t)]\,\mathrm{d}t \\ \qquad + \boldsymbol{W}(\eta_t)\boldsymbol{e}(t)\,\mathrm{d}\omega(t), \\ y(s) = \boldsymbol{\varphi}(s), \quad s \in (-\infty, 0]. \end{cases} \tag{5}$$

Let $\boldsymbol{e}(t) = \big(e_1(t), e_2(t), \ldots, e_n(t)\big) := \boldsymbol{y}(t) - \boldsymbol{x}(t)$ be the error state. Then, the error dynamical system between (4) and (5) is given by

$$\begin{cases} \mathrm{d}\boldsymbol{e}(t) = [-\boldsymbol{D}(\eta_t)\boldsymbol{e}(t) + \boldsymbol{A}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t)) \\ \qquad + \boldsymbol{B}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) \\ \qquad + \bigwedge \boldsymbol{\alpha}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s + u(t)]\,\mathrm{d}t \\ \qquad + \boldsymbol{W}(\eta_t)\boldsymbol{e}(t)\,\mathrm{d}\omega(t), \\ \boldsymbol{e}(s) = \boldsymbol{\phi}(s) - \boldsymbol{\varphi}(s) = \boldsymbol{\psi}(s), \quad s \in (-\infty, 0], \end{cases} \tag{6}$$

where $\boldsymbol{g}(\boldsymbol{e}(\cdot)) = f(\boldsymbol{y}(\cdot)) - f(\boldsymbol{x}(\cdot))$.

In many real-world applications, it is difficult to guarantee that the state variables transmitted to controllers are continuous. In addition, in order to make full use of modern computer technique, the sampled-data feedback control is applied to synchronize delayed MJFCNNs with the Wiener process. Obviously, the sampled-data control law can be adopted as follows:

$$\boldsymbol{u}(t) := \boldsymbol{H}(\eta_t)\boldsymbol{e}(t_k), \quad t_k \leq t < t_{k+1}, \tag{7}$$

where $\boldsymbol{H}(\eta_t)$ is the element of sampled-data controller gain matrix to be determined, $\boldsymbol{e}(t_k)$ is the discrete measurement of $\boldsymbol{e}(t)$ at the sampling instant t_k, and t_k satisfies the following conditions:

$$0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty.$$

Moreover, the sampling period under consideration is assumed to be bounded by a known constant τ_2, that is, $\Delta_k = t_{k+1} - t_k \leq \tau_2$ for $k \geq 0$.

Substituting control law (7) into the error system (6) yields

$$\begin{cases} \mathrm{d}\boldsymbol{e}(t) = [-\boldsymbol{D}(\eta_t)\boldsymbol{e}(t) + \boldsymbol{A}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t)) \\ \qquad + \boldsymbol{B}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) \\ \qquad + \bigwedge \boldsymbol{\alpha}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \boldsymbol{H}(\eta_t)\boldsymbol{e}(t_k)]\,\mathrm{d}t + \boldsymbol{W}(\eta_t)\boldsymbol{e}(t)\,\mathrm{d}\omega(t), \\ \boldsymbol{e}(s) = \boldsymbol{\psi}(s), \quad s \in (-\infty, 0]. \end{cases} \tag{8}$$

Clearly, it is difficult to analyze the synchronization of chaotic MJFCNNs with a Wiener process based on error system (8) because of the discrete term, $\boldsymbol{e}(t_k)$. Therefore, the input delay approach[35] is applied by defining

$$\tau_2(t) = t - t_k, \quad t_k \leq t < t_{k+1}. \tag{9}$$

It is easily seen that

$$0 \leq \tau_2(t) < \tau_2. \tag{10}$$

Therefore, the state-feedback controller takes the following form:

$$\boldsymbol{u}(t) = \boldsymbol{H}(\eta_t)\boldsymbol{e}(t - \tau_2(t)), \quad t_k \leq t < t_{k+1}. \tag{11}$$

Consequently, connecting (11) to system (8) yields

$$\begin{cases} \mathrm{d}\boldsymbol{e}(t) = [-\boldsymbol{D}(\eta_t)\boldsymbol{e}(t) + \boldsymbol{A}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t)) \\ \qquad + \boldsymbol{B}(\eta_t)\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) \\ \qquad + \bigwedge \boldsymbol{\alpha}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}(\eta_t) \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \boldsymbol{H}(\eta_t)\boldsymbol{e}(t-\tau_2(t))]\,\mathrm{d}t + \boldsymbol{W}(\eta_t)\boldsymbol{e}(t)\,\mathrm{d}\omega(t), \\ \boldsymbol{e}(s) = \boldsymbol{\psi}(s), \quad s \in (-\infty, 0]. \end{cases} \tag{12}$$

For simplicity, while $\eta_t = i$, the matrices $\boldsymbol{D}(\eta_t)$, $\boldsymbol{A}(\eta_t)$, $\boldsymbol{B}(\eta_t)$, $\boldsymbol{\alpha}(\eta_t)$, $\boldsymbol{\beta}(\eta_t)$, $\boldsymbol{H}(\eta_t)$, and $\boldsymbol{W}(\eta_t)$ are represented by $\boldsymbol{D}_i$, $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{\alpha}_i$, $\boldsymbol{\beta}_i$, $\boldsymbol{H}_i$, and $\boldsymbol{W}_i$, respectively. Hence, we

extract the following system (12) for $\eta_t = i \in \mathfrak{M}$,

$$\begin{cases} \mathrm{d}\boldsymbol{e}(t) = [-\boldsymbol{D}_i\boldsymbol{e}(t) + \boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t)) + \boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) \\ \qquad + \bigwedge \boldsymbol{\alpha}_i \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \bigvee \boldsymbol{\beta}_i \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ \qquad + \boldsymbol{H}_i\boldsymbol{e}(t-\tau_2(t))]\,\mathrm{d}t + \boldsymbol{W}_i\boldsymbol{e}(t)\,\mathrm{d}\boldsymbol{\omega}(t), \\ \boldsymbol{e}(s) = \boldsymbol{\psi}(s), \quad s \in (-\infty, 0]. \end{cases} \tag{13}$$

Throughout the paper, the following assumptions are needed.

i) The neuron activation functions $f_j(\cdot)$ $(j \in \Lambda)$ are continuously bounded and satisfy

$$l_j^- \le \frac{f_j(u) - f_j(v)}{u - v} \le l_j^+, \quad u, v \in \mathbb{R}, \ u \ne v, \ j \in \Lambda,$$

where l_j^- and l_j^+ are some real constants and may be positive, zero, or negative.

ii) The transmission delay $\tau_1(t)$ is a differentiable function, satisfying for $t \ge 0$ that

$$0 \le \tau_1(t) \le \tau_1, \quad \dot{\tau}_1(t) \le \mu,$$

where τ_1 and μ are constants.

iii) The transmission delay $\tau_1(t)$ is not differentiable or the upper bound of the derivative of $\tau_1(t)$ is unknown, and $\tau_1(t)$ satisfies $0 \le \tau_1(t) \le \tau_1$.

Definition 2.1 The trivial solution of the error dynamical delayed MJFCNNs is said to be stochastic, asymptotically stable if for all initial states ψ_0 and modes η_0,

$$\lim_{t\to\infty} \mathbb{E}\{\|\boldsymbol{e}(t, \psi_0, \eta_0)\|^2\} = 0.$$

We state the following lemmas which will be used in the sequel to prove the main results.

Lemma 2.1[36] Suppose $\boldsymbol{e} = (e_1, e_2, \ldots, e_n)^{\mathrm{T}}$ is a state of system (13), then we have

$$\left|\bigwedge \boldsymbol{\alpha} \circ \boldsymbol{g}(\boldsymbol{e})\right| \le |\boldsymbol{\alpha}||\boldsymbol{g}(\boldsymbol{e})|,$$

$$\left|\bigvee \boldsymbol{\beta} \circ \boldsymbol{g}(\boldsymbol{e})\right| \le |\boldsymbol{\beta}||\boldsymbol{g}(\boldsymbol{e})|.$$

Lemma 2.2[37] The LMI

$$\begin{bmatrix} \boldsymbol{Q}(x) & \boldsymbol{S}(x) \\ \boldsymbol{S}^{\mathrm{T}}(x) & \boldsymbol{R}(x) \end{bmatrix} > 0,$$

where $\boldsymbol{Q}(x) = \boldsymbol{Q}^{\mathrm{T}}(x)$ and $\boldsymbol{R}(x) = \boldsymbol{R}^{\mathrm{T}}(x)$, is equivalent to

$$\boldsymbol{R}(x) > 0, \quad \boldsymbol{Q}(x) - \boldsymbol{S}(x)\boldsymbol{R}^{-1}(x)\boldsymbol{S}^{\mathrm{T}}(x) > 0.$$

Lemma 2.3[38] For any $x, y \in \mathbb{R}^n$, $\epsilon > 0$ and positive definite matrix $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$, the following matrix inequality holds:

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \le \epsilon\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} + \epsilon^{-1}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{y}.$$

Lemma 2.4[39] For any $\boldsymbol{x} \in \mathbb{R}^n$, any constant matrix $\boldsymbol{A} = [a_{ij}]_{n\times n}$ with $a_{ij} \ge 0$, the following matrix inequality holds:

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{x} \le n\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{A}_{\mathrm{s}}\boldsymbol{x},$$

where $\boldsymbol{A}_{\mathrm{s}} = \mathrm{diag}\left(\sum_{i=1}^{n} a_{i1}, \sum_{i=1}^{n} a_{i2}, \ldots, \sum_{i=1}^{n} a_{in}\right)$.

Lemma 2.5[40] Given any real matrix $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$ of appropriate dimension, and a vector function $\boldsymbol{\omega}(\cdot) : [a, b] \to \mathbb{R}^n$, such that the integrations concerned are well defined, then

$$\left[\int_a^b \boldsymbol{\omega}(s)\,\mathrm{d}s\right]^{\mathrm{T}} \boldsymbol{M} \left[\int_a^b \boldsymbol{\omega}(s)\,\mathrm{d}s\right] \le (b-a)\int_a^b \boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{\omega}(s)\,\mathrm{d}s.$$

Lemma 2.6 The infinitesimal generator of the Markov process $\{t \ge 0, \boldsymbol{e}(t), \eta_t\}$ is given by

$$\begin{aligned} &\mathscr{L}\boldsymbol{V}(t, \boldsymbol{e}(t), i) \\ &= \frac{\partial \boldsymbol{V}(t, \boldsymbol{e}(t), i)}{\partial t} + \frac{\partial \boldsymbol{V}(t, \boldsymbol{e}(t), i)}{\partial \boldsymbol{e}} \\ &\quad \times \Bigg[-\boldsymbol{D}_i\boldsymbol{e}(t) + \boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t)) + \boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) \\ &\quad + \bigwedge \boldsymbol{\alpha}_i \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s \\ &\quad + \bigvee \boldsymbol{\beta}_i \circ \int_{-\infty}^{t} K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\,\mathrm{d}s + \boldsymbol{H}_i\boldsymbol{e}(t-\tau_2(t))] \\ &\quad + \frac{1}{2}\mathrm{trace}\left[\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{W}_i^{\mathrm{T}} \frac{\partial^2 \boldsymbol{V}(t, \boldsymbol{e}(t), i)}{\partial \boldsymbol{e}\partial \boldsymbol{e}} \boldsymbol{W}_i\boldsymbol{e}(t)\right] \\ &\quad + \sum_{j=1}^{N} \pi_{ij}\boldsymbol{V}(t, \boldsymbol{e}(t), j), \quad i \in \mathfrak{M}. \end{aligned}$$

Itô's formula plays an important role in the analysis of stochastic systems. The main purpose of this paper is to design a control gain matrix $\boldsymbol{H}_i$ $(i \in \mathfrak{M})$ such that the error system (13) is stochastic, asymptotically stable in the mean square.

3. Main results

Theorem 3.1 Assume that Assumptions i) and ii) hold. If there exist $n \times n$ positive diagonal matrices $\boldsymbol{P}_i$ $(i \in \mathfrak{M})$, $\boldsymbol{R}$, $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\boldsymbol{U}_3$, some $n \times n$ positive definite symmetric matrices $\boldsymbol{Q}_i$ $(i \in \mathfrak{M})$, J_1, J_2, J_3, Z_1, Z_2, Z_3, some positive scalars γ_1, γ_2, and $2n \times 2n$ matrices

$$\begin{pmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \star & \boldsymbol{T}_{22} \end{pmatrix} > 0, \quad \begin{pmatrix} \boldsymbol{V}_{11} & \boldsymbol{V}_{12} \\ \star & \boldsymbol{V}_{22} \end{pmatrix} > 0,$$

such that the following LMI holds:

$$\bar{\boldsymbol{\Theta}}_i = \begin{bmatrix} \boldsymbol{\Theta}_i & \boldsymbol{\Gamma}_1^{\mathrm{T}} & \boldsymbol{\Gamma}_2^{\mathrm{T}} \\ \star & -\gamma_1 n^{-1}\boldsymbol{I} & 0 \\ \star & \star & -\gamma_2 n^{-1}\boldsymbol{I} \end{bmatrix} < 0, \tag{14}$$

where

$$\boldsymbol{\Theta}_i = \begin{bmatrix} \boldsymbol{\Theta}_{11,i} & \boldsymbol{\Theta}_{12,i} & \boldsymbol{Z}_1/\tau_1 & \boldsymbol{\Theta}_{14,i} & \boldsymbol{Z}_3/\tau_2 & -\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}_i^{\mathrm{T}} & \boldsymbol{\Theta}_{17,i} & \boldsymbol{P}_i\boldsymbol{B}_i & 0 & 0 \\ \star & \boldsymbol{\Theta}_{22,i} & 0 & 0 & 0 & 0 & 0 & \boldsymbol{U}_2\boldsymbol{\Sigma}_2 & 0 & 0 \\ \star & \star & \boldsymbol{\Theta}_{33,i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \boldsymbol{\Theta}_{44,i} & 0 & \boldsymbol{Q}_i^{\mathrm{T}} & 0 & 0 & \boldsymbol{U}_3\boldsymbol{\Sigma}_2 & 0 \\ \star & \star & \star & \star & \boldsymbol{\Theta}_{55,i} & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \boldsymbol{\Theta}_{66,i} & \boldsymbol{P}_i\boldsymbol{A}_i & \boldsymbol{P}_i\boldsymbol{B}_i & 0 & 0 \\ \star & \star & \star & \star & \star & \star & R-\boldsymbol{U}_1 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\boldsymbol{U}_2 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -\boldsymbol{U}_3 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \gamma_1\boldsymbol{I}+\gamma_2\boldsymbol{I}-\boldsymbol{R} \end{bmatrix},$$

$$\begin{aligned}
\boldsymbol{\Theta}_{11,i} &= -2\boldsymbol{P}_i\boldsymbol{D}_i + \boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j + \boldsymbol{J}_1 + \boldsymbol{J}_2 + \boldsymbol{J}_3 \\
&\quad - \frac{1}{\tau_1}\boldsymbol{Z}_1 - \frac{(1-\mu)}{\tau_1}\boldsymbol{Z}_2 - \frac{1}{\tau_2}\boldsymbol{Z}_3 - \boldsymbol{U}_1\boldsymbol{\Sigma}_1, \\
\boldsymbol{\Theta}_{12,i} &= \frac{(1-\mu)}{\tau_1}\boldsymbol{Z}_2 + \boldsymbol{T}_{12}^{\mathrm{T}}, \\
\boldsymbol{\Theta}_{14,i} &= \boldsymbol{Q}_i + \boldsymbol{V}_{12}^{\mathrm{T}}, \quad \boldsymbol{\Theta}_{17,i} = \boldsymbol{P}_i\boldsymbol{A}_i + \boldsymbol{U}_1\boldsymbol{\Sigma}_2, \\
\boldsymbol{\Theta}_{22,i} &= -(1-\mu)\boldsymbol{J}_2 - \frac{(1-\mu)}{\tau_1}\boldsymbol{Z}_2 - \boldsymbol{U}_2\boldsymbol{\Sigma}_1 + \tau_1\boldsymbol{T}_{11} - 2\boldsymbol{T}_{12}^{\mathrm{T}}, \\
\boldsymbol{\Theta}_{33,i} &= -\boldsymbol{J}_1 - \frac{1}{\tau_1}\boldsymbol{Z}_1, \quad \boldsymbol{\Theta}_{44,i} = \tau_2\boldsymbol{V}_{11} - 2\boldsymbol{V}_{12}^{\mathrm{T}} - \boldsymbol{U}_3\boldsymbol{\Sigma}_1, \\
\boldsymbol{\Theta}_{55,i} &= -\boldsymbol{J}_3 - \frac{1}{\tau_2}\boldsymbol{Z}_3, \\
\boldsymbol{\Theta}_{66,i} &= -2\boldsymbol{P}_i + \tau_1\boldsymbol{Z}_1 + \tau_1\boldsymbol{Z}_2 + \tau_2\boldsymbol{Z}_3 + \tau_1\boldsymbol{T}_{22} + \tau_2\boldsymbol{V}_{22}, \\
|\boldsymbol{\alpha}_i|_{\mathrm{s}} &= \mathrm{diag}\left(\sum_{j=1}^{n}|\boldsymbol{\alpha}_{j1,i}|, \sum_{j=1}^{n}|\boldsymbol{\alpha}_{j2,i}|, \ldots, \sum_{j=1}^{n}|\boldsymbol{\alpha}_{jn,i}|\right), \\
|\boldsymbol{\beta}_i|_{\mathrm{s}} &= \mathrm{diag}\left(\sum_{j=1}^{n}|\boldsymbol{\beta}_{j1,i}|, \sum_{j=1}^{n}|\boldsymbol{\beta}_{j2,i}|, \ldots, \sum_{j=1}^{n}|\boldsymbol{\beta}_{jn,i}|\right), \\
S_i &= |\boldsymbol{\alpha}_i|_{\mathrm{s}} + |\boldsymbol{\beta}_i|_{\mathrm{s}}, \quad i \in \mathfrak{M}, \\
\boldsymbol{\Sigma}_1 &= \mathrm{diag}(l_1^- l_1^+, \ldots, l_n^- l_n^+), \\
\boldsymbol{\Sigma}_2 &= \mathrm{diag}\left(\frac{l_1^- + l_1^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right), \\
\boldsymbol{\Gamma}_1^{\mathrm{T}} &= \begin{bmatrix} \boldsymbol{P}_i S_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_2^{\mathrm{T}} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \boldsymbol{P}_i S_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}.
\end{aligned}$$

Moreover, the controller gain matrix $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\boldsymbol{Q}_i$ $(i \in \mathfrak{M})$. Then, the error dynamical system (13) is globally stochastic asymptotically stable in the mean square.

Proof Consider the LKF for error-state system (13) as

$$\boldsymbol{V}(t, \boldsymbol{e}(t), i) = \sum_{k=1}^{6}\boldsymbol{V}_k(t, \boldsymbol{e}(t), i), \quad i \in \mathfrak{M}, \tag{15}$$

where

$$\begin{aligned}
\boldsymbol{V}_1(t, \boldsymbol{e}(t), i) &= \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{e}(t), \\
\boldsymbol{V}_2(t, \boldsymbol{e}(t), i) &= \int_{t-\tau_1}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{J}_1\boldsymbol{e}(s)\mathrm{d}s + \int_{t-\tau_1(t)}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{J}_2\boldsymbol{e}(s)\mathrm{d}s \\
&\quad + \int_{t-\tau_2}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{J}_3\boldsymbol{e}(s)\mathrm{d}s, \\
\boldsymbol{V}_3(t, \boldsymbol{e}(t), i) &= \boldsymbol{R}\int_0^{\infty}K(\theta)\int_{t-\theta}^{t}\boldsymbol{g}^2(\boldsymbol{e}(s))\mathrm{d}s\mathrm{d}\theta, \\
\boldsymbol{V}_4(t, \boldsymbol{e}(t), i) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + \int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + \int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta, \\
\boldsymbol{V}_5(t, \boldsymbol{e}(t), i) &= \int_0^{t}\int_{\theta-\tau_1(\theta)}^{\theta}\begin{bmatrix} \boldsymbol{e}(\theta-\tau_1(\theta)) \\ \dot{\boldsymbol{e}}(s) \end{bmatrix}^{\mathrm{T}} \\
&\quad \times \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ * & \boldsymbol{T}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{e}(\theta-\tau_1(\theta)) \\ \dot{\boldsymbol{e}}(s) \end{bmatrix}\mathrm{d}s\mathrm{d}\theta \\
&\quad + \int_0^{t}\int_{\theta-\tau_2(\theta)}^{\theta}\begin{bmatrix} \boldsymbol{e}(\theta-\tau_2(\theta)) \\ \dot{\boldsymbol{e}}(s) \end{bmatrix}^{\mathrm{T}} \\
&\quad \times \begin{bmatrix} \boldsymbol{V}_{11} & \boldsymbol{V}_{12} \\ * & \boldsymbol{V}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{e}(\theta-\tau_2(\theta)) \\ \dot{\boldsymbol{e}}(s) \end{bmatrix}\mathrm{d}s\mathrm{d}\theta, \\
\boldsymbol{V}_6(t, \boldsymbol{e}(t), i) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + \int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{V}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned}$$

From the weak infinitesimal operator $\mathscr{L}$ of the stochastic process $\{t \geq 0, \boldsymbol{e}(t), \eta_t\}$ given in Lemma 2.6, we have

$$\begin{aligned}
\mathscr{L}\boldsymbol{V}_1(t, \boldsymbol{e}(t), i) &= 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i[-\boldsymbol{D}_i\boldsymbol{e}(t) + \boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t)) \\
&\quad + \boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) + \boldsymbol{H}_i\boldsymbol{e}(t-\tau_2(t)) \\
&\quad + \bigwedge\boldsymbol{\alpha}_i \circ \int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s \\
&\quad + \bigvee\boldsymbol{\beta}_i \circ \int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s] \\
&\quad + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i\boldsymbol{e}(t) + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_j\boldsymbol{e}(t).
\end{aligned}$$

Hence, using Lemmas 2.1, 2.3, and 2.4, we obtain

$$\begin{aligned}
&\mathscr{L}\boldsymbol{V}_1(t, \boldsymbol{e}(t), i) \\
&\leq -2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{D}_i\boldsymbol{e}(t) + 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t)) \\
&\quad + 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t))) + 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}_i\boldsymbol{e}(t-\tau_2(t)) \\
&\quad + \gamma_1^{-1}n\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\,(|\boldsymbol{\alpha}_i|_{\mathrm{s}} + |\boldsymbol{\beta}_i|_{\mathrm{s}})(|\boldsymbol{\alpha}_i|_{\mathrm{s}} + |\boldsymbol{\beta}_i|_{\mathrm{s}})^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{e}(t)
\end{aligned}$$

$$
\begin{aligned}
&+\gamma_1\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right)^{\mathrm{T}}\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right)\\
&+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i\boldsymbol{e}(t)+\sum_{j=1}^{N}\pi_{ij}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_j\boldsymbol{e}(t)\\
&+2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i[-\dot{\boldsymbol{e}}(t)+\dot{\boldsymbol{e}}(t)]\\
\le\ &-2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{D}_i\boldsymbol{e}(t)+2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t))\\
&+2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t)))+2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}_i\boldsymbol{e}(t-\tau_2(t))\\
&+\gamma_1^{-1}n\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\left(|\boldsymbol{\alpha}_i|_{\mathrm{s}}+|\boldsymbol{\beta}_i|_{\mathrm{s}}\right)\left(|\boldsymbol{\alpha}_i|_{\mathrm{s}}+|\boldsymbol{\beta}_i|_{\mathrm{s}}\right)^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{e}(t)\\
&+\gamma_1\left(\int_{-\infty}^{t}K(t-s)g\big(\boldsymbol{e}(s)\big)\mathrm{d}s\right)^{\mathrm{T}}\left(\int_{-\infty}^{t}K(t-s)g\big(\boldsymbol{e}(s)\big)\mathrm{d}s\right)\\
&+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i\boldsymbol{e}(t)+\sum_{j=1}^{N}\pi_{ij}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_j\boldsymbol{e}(t)\\
&-2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i\dot{\boldsymbol{e}}(t)-2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{D}_i\boldsymbol{e}(t)+2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{A}_i\boldsymbol{g}(\boldsymbol{e}(t))\\
&+2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t)))+2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{Q}_i\boldsymbol{e}(t-\tau_2(t))\\
&+\gamma_2^{-1}n\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}_i(|\boldsymbol{\alpha}_i|_{\mathrm{s}}+|\boldsymbol{\beta}_i|_{\mathrm{s}})(|\boldsymbol{\alpha}_i|_{\mathrm{s}}+|\boldsymbol{\beta}_i|_{\mathrm{s}})^{\mathrm{T}}\boldsymbol{P}_i\dot{\boldsymbol{e}}(t)\\
&+\gamma_2\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right)^{\mathrm{T}}\\
&\times\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right),
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\mathscr{L}\boldsymbol{V}_2(t,\boldsymbol{e}(t),i)\\
\le\ &\boldsymbol{e}^{\mathrm{T}}(t)[\boldsymbol{J}_1+\boldsymbol{J}_2+\boldsymbol{J}_3]\boldsymbol{e}(t)-\boldsymbol{e}^{\mathrm{T}}(t-\tau_1)\boldsymbol{J}_1\boldsymbol{e}(t-\tau_1)\\
&-(1-\mu)\boldsymbol{e}^{\mathrm{T}}(t-\tau_1(t))\boldsymbol{J}_2\boldsymbol{e}(t-\tau_1(t))\\
&-\boldsymbol{e}^{\mathrm{T}}(t-\tau_2)\boldsymbol{J}_3\boldsymbol{e}(t-\tau_2),
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&\mathscr{L}\boldsymbol{V}_3(t,\boldsymbol{e}(t),i)\\
=\ &\boldsymbol{R}\int_0^{\infty}K(\theta)\boldsymbol{g}^2(\boldsymbol{e}(t))\mathrm{d}\theta-\boldsymbol{R}\int_0^{\infty}K(\theta)\boldsymbol{g}^2(\boldsymbol{e}(t-\theta))\mathrm{d}\theta\\
=\ &\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{R}\boldsymbol{g}(\boldsymbol{e}(t))\\
&-\boldsymbol{R}\int_0^{\infty}K(\theta)\mathrm{d}\theta\int_0^{\infty}K(\theta)\boldsymbol{g}^2(\boldsymbol{e}(t-\theta))\mathrm{d}\theta\\
=\ &\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{R}\boldsymbol{g}(\boldsymbol{e}(t))-\boldsymbol{R}\left(\int_0^{\infty}K(\theta)\boldsymbol{g}(\boldsymbol{e}(t-\theta))\mathrm{d}\theta\right)^2\\
=\ &\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{R}\boldsymbol{g}(\boldsymbol{e}(t))-\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right)^{\mathrm{T}}\\
&\times\boldsymbol{R}\left(\int_{-\infty}^{t}K(t-s)\boldsymbol{g}(\boldsymbol{e}(s))\mathrm{d}s\right),
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\mathscr{L}\boldsymbol{V}_4(t,\boldsymbol{e}(t),i)\\
\le\ &\dot{\boldsymbol{e}}^{\mathrm{T}}(t)[\tau_1\boldsymbol{Z}_1+\tau_1\boldsymbol{Z}_2+\tau_2\boldsymbol{Z}_3]\dot{\boldsymbol{e}}(t)-\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
&-(1-\mu)\int_{t-\tau_1(t)}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
&-\int_{t-\tau_2}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s.
\end{aligned}
\tag{19}
$$

From Jensen's inequality (Lemma 2.5), it follows that

$$
\begin{aligned}
&-\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
\le\ &-\frac{1}{\tau_1}[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)]^{\mathrm{T}}\boldsymbol{Z}_1[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1)],
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&-(1-\mu)\int_{t-\tau_1(t)}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
\le\ &-\frac{(1-\mu)}{\tau_1}[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1(t))]^{\mathrm{T}}\\
&\times\boldsymbol{Z}_2[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_1(t))],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&-\int_{t-\tau_2}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_3\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
\le\ &-\frac{1}{\tau_2}[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_2)]^{\mathrm{T}}\boldsymbol{Z}_3[\boldsymbol{e}(t)-\boldsymbol{e}(t-\tau_2)],
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&\mathscr{L}\boldsymbol{V}_5(t,\boldsymbol{e}(t),i)\\
\le\ &\boldsymbol{e}^{\mathrm{T}}(t-\tau_1(t))[\tau_1\boldsymbol{T}_{11}-2\boldsymbol{T}_{12}^{\mathrm{T}}]\boldsymbol{e}(t-\tau_1(t))\\
&+2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{T}_{12}^{\mathrm{T}}\boldsymbol{e}(t-\tau_1(t))+\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
&+\boldsymbol{e}^{\mathrm{T}}(t-\tau_2(t))[\tau_2\boldsymbol{V}_{11}-2\boldsymbol{V}_{12}^{\mathrm{T}}]\boldsymbol{e}(t-\tau_2(t))\\
&+2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{V}_{12}^{\mathrm{T}}\boldsymbol{e}(t-\tau_2(t))+\int_{t-\tau_2}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{V}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&\mathscr{L}\boldsymbol{V}_6(t,\boldsymbol{e}(t),i)\\
=\ &\tau_1\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{T}_{22}\dot{\boldsymbol{e}}(t)-\int_{t-\tau_1}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{T}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s\\
&+\tau_2\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{V}_{22}\dot{\boldsymbol{e}}(t)-\int_{t-\tau_2}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{V}_{22}\dot{\boldsymbol{e}}(s)\mathrm{d}s.
\end{aligned}
\tag{24}
$$

In addition, for any $n\times n$ diagonal matrices $\boldsymbol{U}_1>0$, $\boldsymbol{U}_2>0$, and $\boldsymbol{U}_3>0$, the following inequality holds by the methods proposed in Ref. [41]

$$
\begin{aligned}
&\left\{\begin{bmatrix}\boldsymbol{e}(t)\\ \boldsymbol{g}(\boldsymbol{e}(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\boldsymbol{U}_1\boldsymbol{\Sigma}_1 & \boldsymbol{U}_1\boldsymbol{\Sigma}_2\\ * & -\boldsymbol{U}_1\end{bmatrix}\begin{bmatrix}\boldsymbol{e}(t)\\ \boldsymbol{g}(\boldsymbol{e}(t))\end{bmatrix}\right.\\
&+\begin{bmatrix}\boldsymbol{e}(t-\tau_1(t))\\ \boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t)))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\boldsymbol{U}_2\boldsymbol{\Sigma}_1 & \boldsymbol{U}_2\boldsymbol{\Sigma}_2\\ * & -\boldsymbol{U}_2\end{bmatrix}\\
&\times\begin{bmatrix}\boldsymbol{e}(t-\tau_1(t))\\ \boldsymbol{g}(\boldsymbol{e}(t-\tau_1(t)))\end{bmatrix}\\
&+\begin{bmatrix}\boldsymbol{e}(t-\tau_2(t))\\ \boldsymbol{g}(\boldsymbol{e}(t-\tau_2(t)))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\boldsymbol{U}_3\boldsymbol{\Sigma}_1 & \boldsymbol{U}_3\boldsymbol{\Sigma}_2\\ * & -\boldsymbol{U}_3\end{bmatrix}\\
&\left.\times\begin{bmatrix}\boldsymbol{e}(t-\tau_2(t))\\ \boldsymbol{g}(\boldsymbol{e}(t-\tau_2(t)))\end{bmatrix}\right\}\ge 0.
\end{aligned}
\tag{25}
$$

Hence, from Eqs. (16)–(25), we have

$$
\mathscr{L}\boldsymbol{V}(t,\boldsymbol{e}(t),i)\le\boldsymbol{\xi}^{\mathrm{T}}(t)\left[\boldsymbol{\Theta}_i+\boldsymbol{\Gamma}_1^{\mathrm{T}}\gamma_1^{-1}n\boldsymbol{\Gamma}_1+\boldsymbol{\Gamma}_2^{\mathrm{T}}\gamma_2^{-1}n\boldsymbol{\Gamma}_2\right]\boldsymbol{\xi}(t).
$$

Subsequently, using Lemma 2.2, we have

$$
\mathscr{L}\boldsymbol{V}(t,\boldsymbol{e}(t),i)\le\boldsymbol{\xi}^{\mathrm{T}}(t)\,\bar{\boldsymbol{\Theta}}_i\,\boldsymbol{\xi}(t),
\tag{26}
$$

where

$$
\begin{aligned}
\boldsymbol{\xi}^{\mathrm{T}}(t)=\Big[&\boldsymbol{e}^{\mathrm{T}}(t),\ \boldsymbol{e}^{\mathrm{T}}(t-\tau_1(t)),\ \boldsymbol{e}^{\mathrm{T}}(t-\tau_1),\\
&\boldsymbol{e}^{\mathrm{T}}(t-\tau_2(t)),\ \boldsymbol{e}^{\mathrm{T}}(t-\tau_2),\ \dot{\boldsymbol{e}}^{\mathrm{T}}(t),\ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t)),\\
&\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t-\tau_1(t))),\ \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(t-\tau_2(t))),\\
&\int_{-\infty}^{t}K(t-s)\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{e}(s))\mathrm{d}s\Big],\\
\bar{\boldsymbol{\Theta}}_i=\ &\boldsymbol{\Theta}_i+\boldsymbol{\Gamma}_1^{\mathrm{T}}\gamma_1^{-1}n\boldsymbol{\Gamma}_1+\boldsymbol{\Gamma}_2^{\mathrm{T}}\gamma_2^{-1}n\boldsymbol{\Gamma}_2.
\end{aligned}
$$

Taking the mathematical expectation on both sides of Eq. (26), considering the LMI $\bar{\boldsymbol{\Theta}}_i<0$, and applying Definition 2.1, one

can derive the following inequality:

$$\mathbb{E}\{\mathscr{L}\boldsymbol{V}(t,\boldsymbol{e}(t),i)\} \leq \mathbb{E}\{\boldsymbol{\xi}^{\mathrm{T}}(t)\,\bar{\boldsymbol{\Theta}}_i\,\boldsymbol{\xi}(t)\} \leq -\lambda_{\min}(-\bar{\boldsymbol{\Theta}}_i)\mathbb{E}\{\|\boldsymbol{e}(t,\psi_0,\eta_0)\|^2\}.$$

From Eq. (14), we conclude that, for each mode i, $\bar{\boldsymbol{\Theta}}_i < 0$. Therefore, if this inequality holds, it follows that $\mathscr{L}\boldsymbol{V}(t,\boldsymbol{e}(t),i) < 0$. That is, the error dynamical MJFCNNs system (13) is globally stochastic asymptotically stable in the mean square and this completes the proof of Theorem 3.1. As a result, the slave chaotic MJFCNNs with discrete and unbounded distributed delays with the Wiener process (5) is globally synchronized with the master MJFCNNs (4).

Corollary 3.1 Assume that Assumptions i) and iii) hold. If there exist $n\times n$ positive diagonal matrices $\boldsymbol{P}_i$ ($i\in\mathfrak{M}$), $\boldsymbol{R}$, $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\boldsymbol{U}_3$, some $n\times n$ positive definite symmetric matrices $\boldsymbol{Q}_i$ ($i\in\mathfrak{M}$), $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, some positive scalars γ_1, γ_2, and $2n\times 2n$ matrices

$$\begin{pmatrix}\boldsymbol{T}_{11} & \boldsymbol{T}_{12}\\ \star & \boldsymbol{T}_{22}\end{pmatrix} > 0, \quad \begin{pmatrix}\boldsymbol{V}_{11} & \boldsymbol{V}_{12}\\ \star & \boldsymbol{V}_{22}\end{pmatrix} > 0,$$

such that the following LMI holds

$$\bar{\boldsymbol{\Psi}}_i = \begin{bmatrix}\boldsymbol{\Psi}_i & \boldsymbol{\Gamma}_1^{\mathrm{T}} & \boldsymbol{\Gamma}_2^{\mathrm{T}}\\ \star & -\gamma_1 n^{-1}\boldsymbol{I} & 0\\ \star & \star & -\gamma_2 n^{-1}\boldsymbol{I}\end{bmatrix} < 0, \tag{27}$$

where

$$\boldsymbol{\Psi}_i = \begin{bmatrix}
\boldsymbol{\Psi}_{11,i} & \boldsymbol{T}_{12}^{\mathrm{T}} & \frac{1}{\tau_1}\boldsymbol{Z}_1 & \boldsymbol{\Psi}_{14,i} & \frac{1}{\tau_2}\boldsymbol{Z}_2 & -\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}_i^{\mathrm{T}} & \boldsymbol{\Psi}_{17,i} & \boldsymbol{P}_i\boldsymbol{B}_i & 0 & 0\\
\star & \boldsymbol{\Psi}_{22,i} & 0 & 0 & 0 & 0 & 0 & \boldsymbol{U}_2\boldsymbol{\Sigma}_2 & 0 & 0\\
\star & \star & \boldsymbol{\Psi}_{33,i} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \boldsymbol{\Psi}_{44,i} & 0 & \boldsymbol{Q}_i^{\mathrm{T}} & 0 & 0 & \boldsymbol{U}_3\boldsymbol{\Sigma}_2 & 0\\
\star & \star & \star & \star & \boldsymbol{\Psi}_{55,i} & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \boldsymbol{\Psi}_{66,i} & \boldsymbol{P}_i\boldsymbol{A}_i & \boldsymbol{P}_i\boldsymbol{B}_i & 0 & 0\\
\star & \star & \star & \star & \star & \star & R-\boldsymbol{U}_1 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -\boldsymbol{U}_2 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & -\boldsymbol{U}_3 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \gamma_1\boldsymbol{I}+\gamma_2\boldsymbol{I}-\boldsymbol{R}
\end{bmatrix},$$

$$\begin{aligned}
\boldsymbol{\Psi}_{11,i} &= -2\boldsymbol{P}_i\boldsymbol{D}_i + \boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j + \boldsymbol{M}_1 + \boldsymbol{M}_2 - \frac{1}{\tau_1}\boldsymbol{Z}_1 - \frac{1}{\tau_2}\boldsymbol{Z}_2 - \boldsymbol{U}_1\boldsymbol{\Sigma}_1,\\
\boldsymbol{\Psi}_{14,i} &= \boldsymbol{Q}_i + \boldsymbol{V}_{12}^{\mathrm{T}}, \quad \boldsymbol{\Psi}_{17,i} = \boldsymbol{P}_i\boldsymbol{A}_i + \boldsymbol{U}_1\boldsymbol{\Sigma}_2,\\
\boldsymbol{\Psi}_{22,i} &= \tau_1\boldsymbol{T}_{11} - 2\boldsymbol{T}_{12}^{\mathrm{T}} - \boldsymbol{U}_2\boldsymbol{\Sigma}_1,\\
\boldsymbol{\Psi}_{33,i} &= -\boldsymbol{M}_1 - \frac{1}{\tau_1}\boldsymbol{Z}_1, \quad \boldsymbol{\Psi}_{44,i} = \tau_2\boldsymbol{V}_{11} - 2\boldsymbol{V}_{12}^{\mathrm{T}} - \boldsymbol{U}_3\boldsymbol{\Sigma}_1,\\
\boldsymbol{\Psi}_{55,i} &= -\boldsymbol{M}_2 - \frac{1}{\tau_2}\boldsymbol{Z}_2,\\
\boldsymbol{\Psi}_{66,i} &= -2\boldsymbol{P}_i + \tau_1\boldsymbol{Z}_1 + \tau_2\boldsymbol{Z}_2 + \tau_1\boldsymbol{T}_{22} + \tau_2\boldsymbol{V}_{22},\\
|\boldsymbol{\alpha}_i|_{\mathrm{s}} &= \mathrm{diag}\left(\sum_{j=1}^{n}|\boldsymbol{\alpha}_{j1,i}|, \sum_{j=1}^{n}|\boldsymbol{\alpha}_{j2,i}|, \ldots, \sum_{j=1}^{n}|\boldsymbol{\alpha}_{jn,i}|\right),\\
|\boldsymbol{\beta}_i|_{\mathrm{s}} &= \mathrm{diag}\left(\sum_{j=1}^{n}|\boldsymbol{\beta}_{j1,i}|, \sum_{j=1}^{n}|\boldsymbol{\beta}_{j2,i}|, \ldots, \sum_{j=1}^{n}|\boldsymbol{\beta}_{jn,i}|\right),\\
\boldsymbol{S}_i &= |\boldsymbol{\alpha}_i|_{\mathrm{s}} + |\boldsymbol{\beta}_i|_{\mathrm{s}}, \quad \boldsymbol{\Sigma}_1 = \mathrm{diag}\left(l_1^- l_1^+, \ldots, l_n^- l_n^+\right),\\
\boldsymbol{\Sigma}_2 &= \mathrm{diag}\left(\frac{l_1^- + l_1^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right),\\
\boldsymbol{\Gamma}_1^{\mathrm{T}} &= \begin{bmatrix}\boldsymbol{P}_i\boldsymbol{S}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},\\
\boldsymbol{\Gamma}_2^{\mathrm{T}} &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & \boldsymbol{P}_i\boldsymbol{S}_i & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}.
\end{aligned}$$

Moreover, the controller gain matrices are given by $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\boldsymbol{Q}_i$ ($i\in\mathfrak{M}$). Then, the error dynamical system (13) is globally stochastic asymptotically stable in the mean square.

Proof Consider the LKF for error-state system (13) as

$$V(t,\boldsymbol{e}(t),i) = \sum_{k=1}^{6}\boldsymbol{V}_k(t,\boldsymbol{e}(t),i), \quad i\in\mathfrak{M}, \tag{28}$$

where

$$\begin{aligned}
\boldsymbol{V}_1(t,\boldsymbol{e}(t),i) &= \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{e}(t),\\
\boldsymbol{V}_2(t,\boldsymbol{e}(t),i) &= \int_{t-\tau_1}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{M}_1\boldsymbol{e}(s)\mathrm{d}s + \int_{t-\tau_2}^{t}\boldsymbol{e}^{\mathrm{T}}(s)\boldsymbol{M}_2\boldsymbol{e}(s)\mathrm{d}s,\\
\boldsymbol{V}_3(t,\boldsymbol{e}(t),i) &= R\int_0^{\infty}K(\theta)\int_{t-\theta}^{t}\boldsymbol{g}^2(\boldsymbol{e}(s))\mathrm{d}s\mathrm{d}\theta,\\
\boldsymbol{V}_4(t,\boldsymbol{e}(t),i) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{e}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{e}}(s)\mathrm{d}s\mathrm{d}\theta,\\
\boldsymbol{V}_5(t,\boldsymbol{e}(t),i) &= \int_0^t\int_{\theta-\tau_1(\theta)}^{\theta}\begin{bmatrix}\boldsymbol{e}(\theta-\tau_1(\theta))\\ \dot{\boldsymbol{e}}(s)\end{bmatrix}^{\mathrm{T}}\times\begin{bmatrix}\boldsymbol{T}_{11} & \boldsymbol{T}_{12}\\ * & \boldsymbol{T}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}(\theta-\tau_1(\theta))\\ \dot{\boldsymbol{e}}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_0^t\int_{\theta-\tau_2(\theta)}^{\theta}\begin{bmatrix}\boldsymbol{e}(\theta-\tau_2(\theta))\\ \dot{\boldsymbol{e}}(s)\end{bmatrix}^{\mathrm{T}}\times\begin{bmatrix}\boldsymbol{V}_{11} & \boldsymbol{V}_{12}\\ * & \boldsymbol{V}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}(\theta-\tau_2(\theta))\\ \dot{\boldsymbol{e}}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

$$V_6(t,e(t),i) = \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)V_{22}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.$$

The proof of Corollary 3.1 immediately follows from Theorem 3.1.

Remark 3.1 In Refs. [32] and [33], authors have solved the synchronization problem for stochastic FCNNs with distributed delays. However, in this paper we found the results on synchronization of stochastic FCNNs with unbounded distributed delays. Note that Assumption i) on activation function and Assumptions ii) and iii) on time-varying delay in this paper are weaker than those generally used in Refs. [32] and [33], namely, the boundedness of the activation function f_j. Furthermore, the restrictions, such as the time-varying delay is required to be differentiable or even its time derivative is assumed to be smaller than one, are no longer required in our analysis. However, for $\mu = 0.9$ at τ_2 =0.3, the maximum allowable upper bound (MAUB) τ_1 is around 10^{15}. Moreover, for μ=1, $1 < \dot{\tau}_1(t) < \infty$; for unknown μ, the MAUB τ_1 is 7.8188.

4. Numerical example

Consider the error-state system (13) with the following parameters:

$$D_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.4 & -0.12 \\ 0.14 & -0.27 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.01 & -0.002 \\ 0.033 & -0.036 \end{bmatrix},$$
$$\alpha_1 = \begin{bmatrix} 1/31 & 1/31 \\ 1/31 & -1/31 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} -1/32 & 1/32 \\ 1/32 & -1/32 \end{bmatrix},$$
$$I_1 = \begin{bmatrix} 0.007\sin(2.5t) \\ 0.007\cos(0.5t) \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 2.1 & 0 \\ 0 & 2.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.25 & -0.2 \\ 0.13 & -0.12 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.023 & -0.029 \\ 0.022 & -0.05 \end{bmatrix},$$
$$\alpha_2 = \begin{bmatrix} 1/37 & -1/37 \\ 1/37 & -1/37 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} -1/33 & 1/33 \\ 1/33 & -1/33 \end{bmatrix},$$
$$I_2 = \begin{bmatrix} 0.73\sin(2.5t) \\ 0.73\cos(0.5t) \end{bmatrix}, \quad \Pi = \begin{bmatrix} -0.07 & 0.07 \\ 0.03 & -0.03 \end{bmatrix},$$

and the Wiener process matrices

$$W_1 = \begin{bmatrix} 0.32 & 0.25 \\ 0.27 & 0.13 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.037 & 0.019 \\ 0.004 & 0.034 \end{bmatrix}.$$

The activation function satisfies Assumption i) with $l^- = -I$ and $l^+ = I$. Thus, we can obtain the following parameters:

$$\Sigma_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The activation function $f(x) = \big(|x+1| - |x-1|\big)/2$, and the time varying delays are chosen as $\tau_1(t) = 7 + 0.8188\sin(t)$, which means $\tau_1 = 7.8188$, derivative of time-varying delay satisfies $\dot{\tau}_1(t) \le \mu = 2$. Moreover, the sampling period is taken as $\tau_2 = 0.3$. Using the Matlab LMI Control Toolbox to solve the LMIs in Theorem 3.1, it can be found that the LMI is feasible. Consequently, the controller gain matrices H_i $(i = 1,2)$ are designed as follows:

$$H_1 = P_1^{-1}Q_1 = \begin{bmatrix} 0.0012 & -0.0008 \\ -0.0007 & 0.0005 \end{bmatrix},$$
$$H_2 = P_2^{-1}Q_2 = \begin{bmatrix} 0.0087 & -0.0071 \\ -0.0060 & 0.0049 \end{bmatrix}. \quad (29)$$

By Theorem 3.1, systems (4) and (5) are stochastic asymptotically synchronized. The simulation results are illustrated in Figs. 1–4, in which the controller designed in Eq. (29) is applied. In the simulations of this example, the time step size h = 0.45, and time segment $T = 100$ for Figs. 2–4 and $T = 500$ for Fig. 1 are chosen, respectively. Moreover, the initial conditions are chosen as follows: $\phi(s) = (-0.001, 0.004)^{\mathrm{T}}$, $\varphi(s) = (-0.007, 0.009)^{\mathrm{T}}$, and $s \in (-\infty, 0]$.

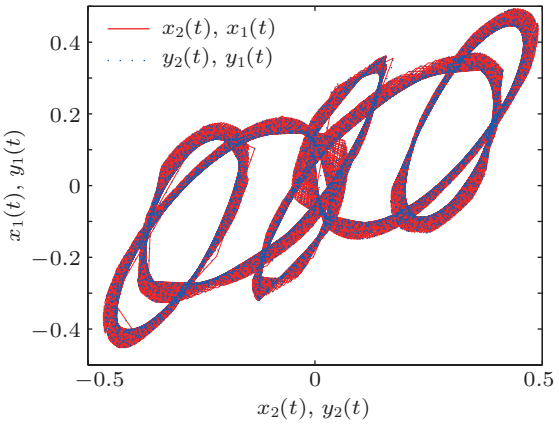


Fig. 1. (color online) Chaotic attractor of drive system (4) and response system (5) with control inputs (29).

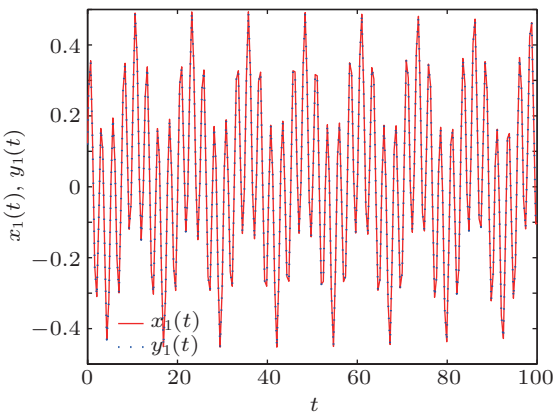


Fig. 2. (color online) State trajectories of drive system (4) with state $x_1(t)$ and response system (5) with state $y_1(t)$ through control inputs (29).

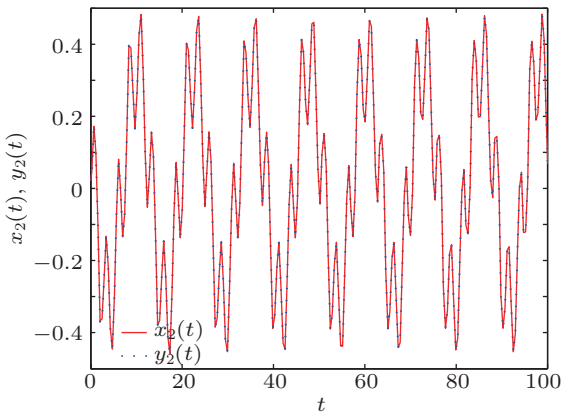


Fig. 3. (color online) State trajectories of drive system (4) with state $x_2(t)$ and response system (5) with state $y_2(t)$ through control inputs (29).

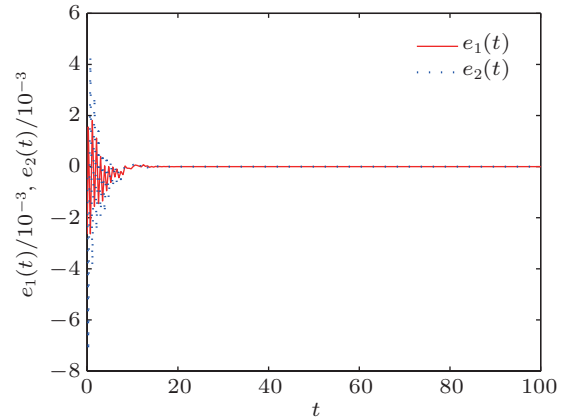


Fig. 4. (color online) Convergence dynamics of errors between drive system (4) and response system (5) with control inputs (29).

Remark 4.1 To easily understand the theoretical results, in this example we used two-neuron networks with delays which display the same behavior as larger networks. Similarly, one can enhance the two-neuron networks to a larger number of networks for practical use.

Remark 4.2 The simulation results can be described as follows. Figure 1 describes the chaotic behavior in the phase space of the drive system (4) and the response system (5) with control inputs (29). Figures 2–4 show that the state trajectories and error trajectories of the drive system (4) and the response system (5) with control inputs (29) to be stochastic asymptotically synchronized. Figures 5–7 provide the state trajectories and the error trajectories between the drive system (4) and the response system (5) without control inputs. One may observe that the drive system (4) and the response system (5) without control inputs (29) cannot be synchronized. Moreover, from Fig. 7, the error trajectories between drive and response systems cannot converge to zero. The numerical simulations clearly verify the effectiveness of the sampled-data control approach to the synchronization of chaotic delayed MJFCNNs with the Wiener process.

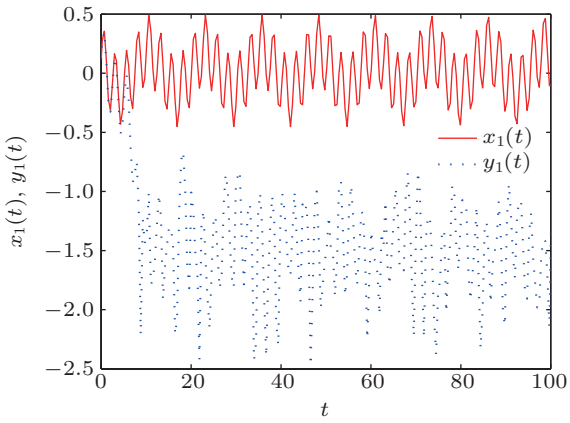


Fig. 5. (color online) State trajectories of drive system (4) with state $x_1(t)$ and response system (5) with state $y_1(t)$ without control inputs (29).

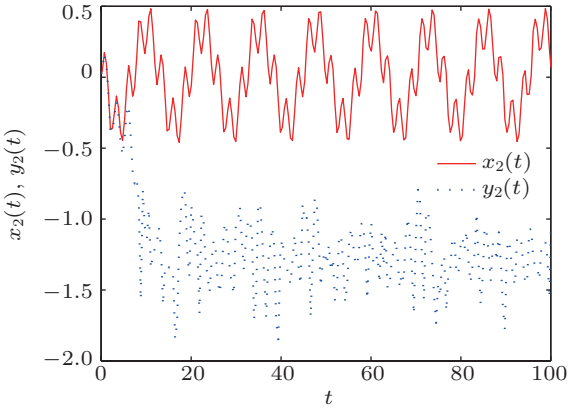


Fig. 6. (color online) State trajectories of drive system (4) with state $x_2(t)$ and response system (5) with state $y_2(t)$ without control inputs (29).

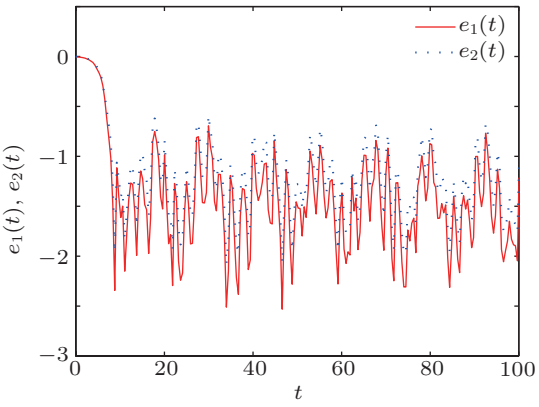


Fig. 7. (color online) Convergence dynamics of errors between drive system (4) and response system (5) without control inputs (29).

5. Conclusion

In this paper, we have dealt with the problem of the synchronization of chaotic MJFCNNs with discrete, unbounded distributed delays and the Wiener process based on sampled-data control. By defining LKF, new LMI-based stability conditions are derived. The synchronization of chaotic delayed MJFCNNs with a Wiener process based on the sampled-data control is new in this literature. In Theorem 3.1, the restrictions, such as the time derivative is assumed to be smaller than

one, are removed. Moreover, in Corollary 3.1, the restrictions, such as the time-varying delay is required to be differentiable or even its time derivative is assumed to be smaller than one, are removed. The effectiveness of the proposed results have been demonstrated through a numerical example well.

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