Frequency dependence of the magnetoelectric effect in a magnetostrictive-piezoelectric heterostructure

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(Received 22 September 2012; revised manuscript received 29 January 2013)

The frequency dependence of the magnetoelectric effect in a magnetostrictive-piezoelectric heterostructure is theoretically studied by solving combined magnetic, elastic, and electric equations with boundary conditions. Both the mechanical coupling coefficient and the losses of the magnetostrictive and piezoelectric phases are taken into account. The numerical result indicates that the magnetoelectric coefficient and the resonance frequency are determined by the mechanical coupling coefficient, losses, and geometric parameters. Moreover, at the electromechanical resonance frequency, the module of the magnetoelectric coefficient is mostly contributed by the imaginary part. The relationship between the real and the imaginary parts of the magnetoelectric coefficient fit well to the Cole–Cole circle. The magnetostrictive-piezoelectric heterostructure has a great potential application as miniature and no-secondary coil solid-state transformers.

Keywords: magnetoelectric effect, magnetostrictive-piezoelectric heterostructure, electromechanical resonance

PACS: 75.80.+q, 75.85.+t **DOI:** 10.1088/1674-1056/22/7/077504

1. Introduction

The magnetoelectric (ME) effect in composites consisting of magnetostrictive (MS) and piezoelectric (PE) phases has attracted great interest. The composites can be used as transducers for magnetic field energy and electric field energy, and have potential applications in high sensitive magnetic field sensors, voltage transformers, and microwave devices.[1–7] The ME effect in composite structures is a product property, which can be explained as follows. When a magnetic field is applied to the composite, the MS phase changes its shape due to the magnetostrictive/piezomagnetic effect, the shape change then induces strains in the composite, the strains spread out and act on the PE phase, resulting in an electric polarization because of the piezoelectric effect. Thus, the ME effect is associated with the appearance of mechanical stress. The magnitude of the ME voltage coefficient increases by 1 to 2 orders when the frequency of the alternating-current (ac) magnetic field coincides with that of the electromechanical resonance (EMR) of the structure due to the resonance enhancement of strains in the PE layers.[8] Several theories for the frequency dependence of the ME effect have been proposed to predict the ME coefficient in the previous works.[9–16] Recent experiments show that the interface connection between the two phases is not perfect, and the ME coefficient is a complex quantity.[17] However, in the previous models, the mechanical coupling between the two phases is assumed to be perfect, the mechanical stress loss is not taken into account, and the compliance coefficients ${}^{\mathrm{m}}s_{ij}$, ${}^{\mathrm{p}}s_{ij}$, piezomagnetic coefficient ${}^{\mathrm{m}}q_{ij}$, permittivity ${}^{\mathrm{p}}\varepsilon_{ij}$, and piezoelectric coefficient ${}^{\mathrm{p}}d_{ij}$ of the constitute phases are all treated as real quantities. Here, superscripts p and m denote the PE and MS phases, respectively. It is important to theoretically investigate the interface mechanical coupling parameter k and the complex parameters of the constituted phases to characterize the realistic ME coefficient. The imaginary parts of the complex parameters denote the energy loss that transfers to heat or the phase delay of the induced electric field compare with the external magnetic field. So the complex parameters can be described by loss angle tangent $\tan\delta$ as[18]

$$
\begin{aligned}
&{}^{\mathrm{m}}s_{ij} = {}^{\mathrm{m}}s'_{ij}(1-\mathrm{j}^{\mathrm{m}}\tan\delta_s),\ {}^{\mathrm{m}}q_{ij} = {}^{\mathrm{m}}q'_{ij}(1-\mathrm{j}^{\mathrm{m}}\tan\delta_q),\\
&{}^{\mathrm{p}}s_{ij} = {}^{\mathrm{m}}s'_{ij}(1-\mathrm{j}^{\mathrm{p}}\tan\delta_s),\ {}^{\mathrm{p}}d_{ij} = {}^{\mathrm{m}}d'_{ij}(1-\mathrm{j}^{\mathrm{p}}\tan\delta_d),\\
&{}^{\mathrm{p}}\varepsilon_{ij} = {}^{\mathrm{m}}\varepsilon'_{ij}(1-\mathrm{j}^{\mathrm{p}}\tan\delta_\varepsilon).
\end{aligned}
$$

If the imperfect mechanical coupling and losses are taken into account, the theoretical modeling of the frequency-dependent ME coefficient can be more accurate.

In this work, the interface imperfect mechanical coupling and the complex parameters are investigated for modeling the complex ME coefficient in a magnetostrictive-piezoelectric heterostructure by solving combined magnetic, elastic, and electric equations with boundary conditions. The ME coefficient for the in-plane mode of the heterostructure composite is obtained as a function of the ac magnetic field frequency, interface coupling k, and loss angle tangent $\tan\delta$. The numerical results show that the ME coefficient is largely dependent on the mechanical coupling coefficient and losses.

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2. Theory

A heterostructure like the well-known Rosen type transformer consisting magnetostrictive and piezoelectric phases as shown in Fig. 1 is studied. The piezoelectric phase with electrodes pasted on the left and the right planes is poled along the length direction (direction x illustrated in Fig. 1). The magnetostrictive and the piezoelectric phases are connected with insulated adhesive. The direct-current (dc) and ac bias magnetic fields are applied along the x direction.

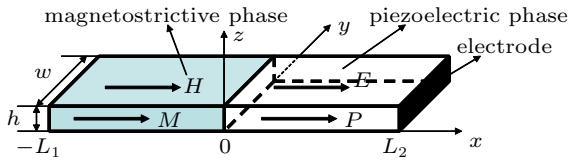


Fig. 1. Configuration of a long-type magnetostrictive-piezoelectric heterostructure.

In our model, we assume that the lengths of the magnetostrictive and the piezoelectric phases are L_1 and L_2, respectively, which are much larger than thickness h and width w. Since the heterostructure is thin and narrow, strain components S_2, S_3 and stress components T_2, T_3 are supposed to be zero. Thus, the basic equations under this condition are

$$^{\mathrm{p}}S_1 = {}^{\mathrm{p}}s_{33}{}^{\mathrm{p}}T_1 + {}^{\mathrm{p}}d_{33}{}^{\mathrm{p}}E_1, \tag{1}$$

$$^{\mathrm{p}}D_1 = {}^{\mathrm{p}}d_{33}{}^{\mathrm{p}}T_1 + {}^{\mathrm{p}}\varepsilon_{33}{}^{\mathrm{p}}E_1, \tag{2}$$

$$^{\mathrm{m}}S_1 = {}^{\mathrm{m}}s_{11}{}^{\mathrm{m}}T_1 + {}^{\mathrm{m}}q_{11}{}^{\mathrm{m}}H_1. \tag{3}$$

The equation of motion for this heterostructure is

$$\rho\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial T_1}{\partial x}, \tag{4}$$

where u_x is the displacement along the wave traveling direction x, and ρ is the density.

The stress tensor S_1 is connected with the displacement vector u_x by

$$S_1 = \frac{\partial u_x}{\partial x}. \tag{5}$$

Displacement ${}^{\mathrm{m}}u_x$ of the oscillation of the magnetostrictive phase induced by the ac magnetic field with frequency ω is transfer into the piezoelectric phase through the interface mechanical coupling. The interface mechanical coupling coefficient is k, and $k = 1$ means the perfect coupling at the interface, while $k = 0$ denotes a fully damped interface. As the mechanical coupling is not perfect, the displacements of the two phases are related by

$$^{\mathrm{p}}u_x(0) = k^{\mathrm{m}}u_x(0). \tag{6}$$

The surfaces of the heterostructure are assumed to be free, therefore, the left surface of the magnetostrictive phase and the right surface of the piezoelectric phase are stress-free, and the surface stresses are zero. There are three boundary conditions for this heterostructure

$$^{\mathrm{m}}T_1(0) = {}^{\mathrm{p}}T_1(0), \tag{7}$$

$$^{\mathrm{m}}T_1(-L_1) = 0, \tag{8}$$

$$^{P}T_1(L_2) = 0. \tag{9}$$

The induced electric field in the piezoelectric phase can be determined using Eq. (2) with the open circuit condition

$$\int_0^w \mathrm{d}y \int_0^{L_2} {}^{\mathrm{p}}D_1\,\mathrm{d}x = 0. \tag{10}$$

Using Eqs. (1)–(3) and the general solution for the equation of motion, substituting these expressions into the boundary condition, and using the definition of the ME coefficient $\alpha = E/H$, we can obtain the ME coefficient for this mode as

$$\alpha_{11} = \frac{{}^{\mathrm{p}}E_1}{{}^{\mathrm{m}}H_1} = -\frac{{}^{\mathrm{m}}q_{11}{}^{\mathrm{p}}d_{33}}{{}^{\mathrm{m}}s_{11}{}^{\mathrm{p}}\varepsilon_{33}}\frac{1}{\Delta_{\mathrm{a}}}\frac{k[1-\cos(\kappa_2)][1-\cos(\kappa_1)]}{\kappa_2\cos(\kappa_2)\cos(\kappa_1)[k\tan(\kappa_2)+\gamma\tan(\kappa_1)]}, \tag{11}$$

where ω is the circular frequency of the ac magnetic field,

$$^{\mathrm{m}}k = \omega({}^{\mathrm{m}}\rho{}^{\mathrm{m}}s_{11})^{1/2}, \quad {}^{\mathrm{p}}k = \omega({}^{\mathrm{p}}\rho{}^{\mathrm{p}}s_{33})^{1/2},$$

$$\gamma = ({}^{\mathrm{m}}k{}^{\mathrm{p}}s_{33})/({}^{\mathrm{p}}k{}^{\mathrm{m}}s_{11}) = [({}^{\mathrm{m}}\rho{}^{\mathrm{p}}s_{33})/({}^{\mathrm{p}}\rho{}^{\mathrm{m}}s_{11})]^{1/2},$$

$$\kappa_1 = {}^{\mathrm{m}}kL_1, \quad \kappa_2 = {}^{\mathrm{p}}kL_2,$$

$$\Delta_{\mathrm{a}} = 1 - K_{33}^2\left\{1 - \frac{\tan(\kappa_2)}{\kappa_2} + \frac{k[\cos(\kappa_2)-1]^2}{\cos^2(\kappa_2)}\frac{1}{\kappa_2[\gamma\tan(\kappa_1)+k\tan(\kappa_2)]}\right\}.$$

Here, $K_{33}^2 = {}^{\mathrm{p}}d_{33}^2/({}^{\mathrm{p}}s_{33}{}^{\mathrm{p}}\varepsilon_{33})$ is the square of the electromechanical coupling coefficient under planar oscillations.

3. Results and discussion

We numerically study the specific case of Terfenol-D/PZT heterostructure using the theoretical model. The real parts of the parameters for calculation are as follows:

$${}^{\mathrm{m}}s'_{11} = 45.4\times10^{-12}\ \mathrm{m^2/N},\ {}^{\mathrm{m}}q'_{11} = 1970\times10^{-12}\ \mathrm{m/A},$$

$${}^{\mathrm{m}}\rho = 9.2\times10^{3}\ \mathrm{kg/m^3},\ {}^{\mathrm{p}}s'_{11} = 16.5\times10^{-12}\ \mathrm{m^2/N},$$

$${}^{\mathrm{p}}s'_{33} = 20.7\times10^{-12}\ \mathrm{m^2/N},\ {}^{\mathrm{p}}d'_{31} = -274\times10^{-12}\ \mathrm{C/N},$$

$${}^{\mathrm{p}}d'_{33} = 593\times10^{-12}\ \mathrm{C/N},\ {}^{\mathrm{p}}\rho = 7.5\times10^{3}\ \mathrm{kg/m^3},$$

$${}^{\mathrm{p}}\varepsilon'_{33}/\varepsilon_0 = 1470,\ \varepsilon_0 = 8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2},$$

$$L_1 = L_2 = 10\ \mathrm{mm}.$$

In order to simplify the calculation, we suppose that the electric and magnetic losses have the same loss $\tan\delta$. The frequency-dependent ME coefficient is calculated from 1 kHz to 150 kHz. The module of predicted ME coefficient $|\alpha_{11}|$ as a function of the frequency is shown in Fig. 2 with $\tan\delta = 0.02$ and series k values. Two giant sharp resonance peaks, which correspond to the half-wavelength (fundamental) and full-wavelength (second longitudinal) shape resonances in the heterostructure, respectively, are observed for various k values. The $|\alpha_{11}|$ and the resonance frequencies have a strong dependence on the k value. The maximum $|\alpha_{11}|$ decreases as the mechanical coupling weakens, while the resonance frequency increases as the coupling weakens. To test the theoretical mode,

we apply it to the specific case of Terfenol-D/PMN-PT structure, which has been studied in Ref. [19]. The material parameters of PMN-PT for calculation are given as follows:[20]

$$^{\mathrm{p}}s'_{33} = 13.3\times10^{-12}\ \mathrm{m^2/N}, \quad ^{\mathrm{p}}d'_{33} = 190\times10^{-12}\ \mathrm{C/N},$$
$$^{\mathrm{p}}\rho = 8.06\times10^{3}\ \mathrm{kg/m^3}, \quad ^{\mathrm{p}}\varepsilon'_{33}/\varepsilon_0 = 640,$$

and the geometry size of the structure is $L_1 = L_2 = 14$ mm. The calculated resonance frequencies under the condition $k = 0.8$ are 37 kHz and 79 kHz, which agree with the experimental results 36 kHz and 72 kHz.[19] The calculated maxima $|\alpha_{11}|$ under the condition $\tan\delta = 0.08$ are 11.5 V/Oe and 6.4 V/Oe, the experimental results are 11.6 V/Oe and 7.6 V/Oe. The difference between calculation and experimental results may be caused by eddy current loss, dynamic Young's modulus of elasticity, and the dynamic piezomagnetic coefficient of the magnetostrictive material and the bonding layer.

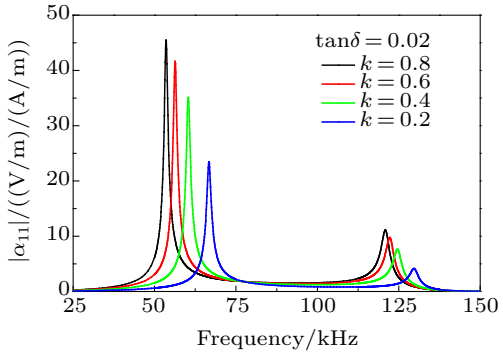


Fig. 2. (color online) Predicted module of ME coefficient as a function of frequency with $\tan\delta = 0.02$ and series k values.

Figure 3 shows the frequency dependence of the module of the predicted ME coefficient of the structure with $k = 0.8$ and series $\tan\delta$ values. With increasing the value of $\tan\delta$, the power losses increase, thus the module of the predicted ME coefficient decreases sharply. The resonant frequency is determined by the zero denominator of Eq. (11) when we do not take the internal losses into account, and the predicted ME coefficient is infinite at the resonance frequency. When any one of the internal losses is considered in the model, the resonant frequency is determined by the zero real part of the denominator of Eq. (11). The $\tan\delta$ only changes the imaginary part of the denominator. So, with increasing $\tan\delta$, the resonance frequency remains unchanged. Therefore, the maximum value of the ME coefficient of the resonance frequency is dependent on the internal loss $\tan\delta$ and the interface mechanical coupling coefficient k; and the resonance frequency is largely dependent on the interface mechanical coupling coefficient.

According to Eq. (11), the resonance frequency is determined not only by the material parameters but also by the geometry, such as the lengths of the two phases. Figure 4(a) shows the dependence of the first resonance frequency on the geometric parameters L_1 and L_2. It is clear that as L_1 or L_2 increases, the resonance frequency decreases. As shown in Fig. 4(b), the maximum $|\alpha_{11}|$ at the resonance frequency is also dependent on the geometric parameters. As the length ratio L_2/L_1 increases, the maximum $|\alpha_{11}|$ increases first and then decreases.

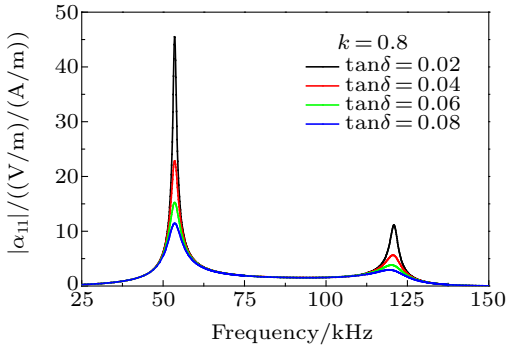


Fig. 3. (color online) Predicted module of magnetoelectric coefficient as a function of frequency with $k = 0.8$ and series $\tan\delta$ values.

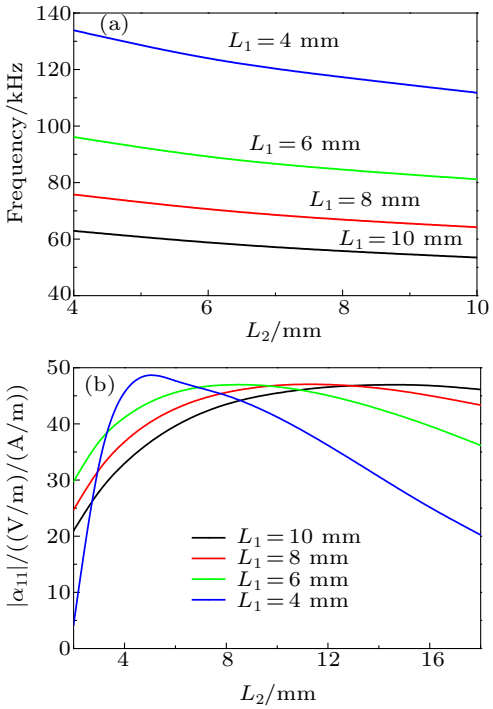


Fig. 4. (color online) Dependences of (a) the first resonance frequency and (b) the first maximum $|\alpha_{11}|$ on geometric parameters L_1 and L_2.

From Eq. (11), a real ME coefficient can also be obtained if not considering the internal losses, however, when any of the losses is taken into account, the ME coefficient becomes complex. As a complex quantity, the ME coefficient can be written as $\alpha_{11} = \alpha'_{11} + \mathrm{j}\alpha''_{11}$.[17] According to the numerical results, at low frequencies far below the resonance frequency, the imaginary part of the ME coefficient is small, and the module is mainly determined by the real part. For example, the predicted ME coefficient at 15 kHz is $(-7.84 + \mathrm{j}0.17)\times10^{-2}$ (V/m)/(A/m) and the module is 7.84×10^{-2} (V/m)/(A/m).

Near the resonance frequency, the real part of the ME coefficient is about zero, and the module of the ME coefficient is mostly contributed by the imaginary part, as shown in Fig. 5. The relation between the real and the imaginary parts fits well to the Cole–Cole circle, as shown in Fig. 6. The frequency is increased in the clockwise direction.

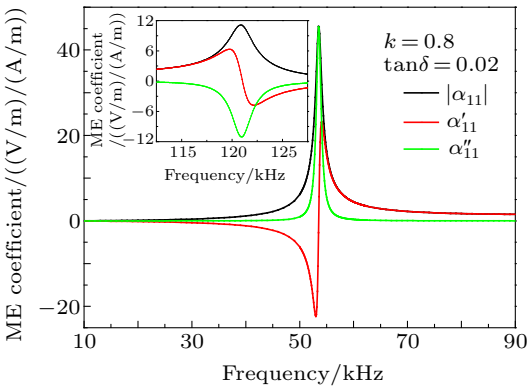


Fig. 5. (color online) Module, real and imaginary parts of the Predicted ME coefficient as a function of frequency from 10 kHz to 90 kHz with $k = 0.8$ and $\tan\delta = 0.02$. Insert shows the module, real and imaginary parts of the predicted ME coefficient as a function of frequency from 112.5 kHz to 137.5 kHz with $k = 0.8$ and $\tan\delta = 0.02$.

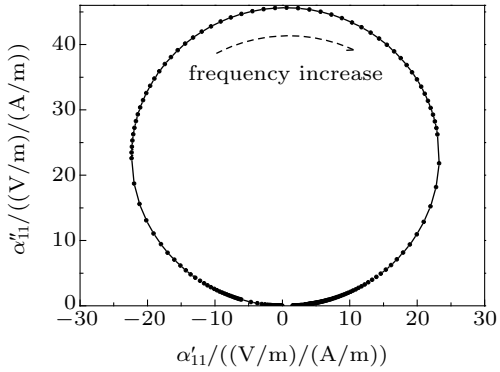


Fig. 6. Cole–Cole circle of the predicted ME coefficient with $k = 0.8$ and $\tan\delta = 0.02$.

4. Conclusion

The ME coefficient of a magnetostrictive-piezoelectric heterostructure as a function of frequency is studied theoretically and calculated numerically. The interface mechanical coupling coefficient between magnetostrictive and piezoelectric phases and the internal losses of the constituent two phases are investigated for the complex and frequency-dependent characteristics of the ME coefficient. The numerical results show that the module of the ME coefficient and the resonance frequency depend on not only the material intrinsic parameters and the interface mechanical coupling coefficient, but also the geometric parameters. Furthermore, around the resonance frequency, the imaginary part of the ME coefficient dominates the module of the ME coefficient. The real part and imaginary parts of the ME coefficient fit well to the Cole–Cole circle. The magnetostrictive-piezoelectric heterostructure has a great potential application as miniature and no-secondary coil solid-state transformer devices. The step-up ratio of these transformers can be changed by the magnetic field frequencies.

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