Heat transfer in boundary layer stagnation-point flow towards a shrinking sheet with non-uniform heat flux*

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In this paper, the effect of non-uniform heat flux on heat transfer in boundary layer stagnation-point flow over a shrinking sheet is studied. The variable boundary heat fluxes are considered of two types: direct power-law variation with the distance along the sheet and inverse power-law variation with the distance. The governing partial differential equations (PDEs) are transformed into non linear self-similar ordinary differential equations (ODEs) by similarity transformations, and then those are solved using very efficient shooting method. The direct variation and inverse variation of heat flux along the sheet have completely different effects on the temperature distribution. Moreover, the heat transfer characteristics in the presence of non-uniform heat flux for several values of physical parameters are also found to be interesting.

Keywords: non-uniform heat flux, heat transfer, stagnation-point flow, shrinking sheet

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1. Introduction

In fluid dynamics, the stagnation-point flow and the flow due to a stretching sheet are equally important in theoretical and application points of view. The steady two-dimensional stagnation-point flow towards a stretching sheet was first investigated by Chaim[1] considering the strain-rate of the stagnation-point flow and the stretching rate of the sheet to be equal, and he found no boundary layer structure near the sheet. Whereas, Mahapatra and Gupta[2] studied the same problem considering the strain-rate and the stretching rate to be different, and they obtained boundary layer adjacent to sheet, which completely depends on the ratio between the strain-rate of stagnation-point flow and the stretching rate of sheet. Furthermore, the behavior of stagnation-point flow over stretching sheet under different physical aspects was discussed by Mahapatra and Gupta,[3] Nazar *et al.*,[4] Ishak *et al.*,[5] Layek *et al.*,[6] Ding and Zhang,[7] Nadeem *et al.*,[8] and Bhattacharyya *et al.*[9,10]

On the other hand, a new type of flow, the flow due to shrinking sheet, which was recently developed, is quite different from the usual stretching sheet flow.[11–16] The boundary layer flow caused due to a shrinking sheet was first observed by Wang.[17] Actually, the shrinking sheet flow shows different physical characteristics from those of the stretching sheet flow. This unusual type of flow is very much relevant to many manufacturing industries, such as, paper production industry, polymer processing industry, and glass-fiber production industry. The physical structure of shrinking flow is very complicated. The vorticity generated due to the shrinking of the sheet is not confined within the boundary layer and so the steady flow over a shrinking sheet is not possible unless some opposite force is used to prevent the vorticity diffusion and to maintain boundary layer structure. In this regards, Miklavčič and Wang[18] found that the steady shrinking sheet flow is dependent on the wall mass suction at the porous sheet. Further this shrinking sheet flow was extended by Hayat *et al.*,[19–23] Fang *et al.*,[24–27] and Bhattacharyya,[28,29] Bhattacharyya and Pop,[30] and Bhattacharyya *et al.*[31,32] It should be noted that if, in addition to shrinking force, the stagnation-point flow is imposed then the generated vorticity remains inside the boundary layer for a certain level of shrinking without any mass suction. This physical phenomenon established by Wang[33] through studying the stagnation-point flow towards a shrinking sheet. Recently, Wang's problem was extended by some researchers. Ishak *et al.*[34] studied the steady two-dimensional boundary layer stagnation-point flow of micropolar fluid passing through a shrinking sheet. Bhattacharyya and Layek[35] discussed the influences of suction/blowing on the boundary layer stagnation-point flow and heat transfer over a shrinking sheet with thermal radiation effects. Bhattacharyya *et al.*[36] described the steady stagnation-point flow and heat transfer towards a shrinking sheet with slip at the boundary. The MHD stagnation-point flow over a shrinking sheet was investigated by Mahapatra *et al.*[37] and Lok *et al.*[38] Yacob *et al.*[39] investigated the melting heat transfer in steady boundary layer stagnation-point flow towards a horizontal linearly stretching/shrinking sheet in a micropolar fluid. The mass transfer characteristic in boundary layer stagnation-point flow towards a shrinking sheet was explained by Bhattacharyya,[40] and Bachok *et al.*[41]

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demonstrated the effects of homogeneous–heterogeneous reactions on the steady boundary layer stagnation-point flow on a stretching/shrinking surface. Rosali *et al.*[42] studied the stagnation-point flow and heat transfer past shrinking sheet in porous medium. Fan *et al.*[43] investigated the unsteady stagnation flow and heat transfer over a shrinking sheet and obtained a solution using homotopy analysis method, and Bhattacharyya[44] discussed the unsteady boundary layer stagnation-point flow towards a shrinking sheet using numerical method. The stagnation-point flow and heat transfer towards a shrinking sheet in a nanofluid were discussed by Nazar *et al.*[45] Very recently, the steady stagnation-point flow over an exponentially shrinking sheet was studied by Bhattacharyya and Vajravelu.[46]

The heat transfer on a stretching/shrinking sheet becomes more interesting when the heat flux condition at boundary is taken into consideration. In heat transfer engineering, the heat flux phenomenon plays a vital role in controlling the rate of heat transfer. The effect of uniform heat flux on the temperature field in the flow due to a stretching sheet was investigated by Dutta *et al.*[47] After that many researcher considered uniform or non-uniform heat flux at the boundary[48–54] for stretching sheet flow. Ishak *et al.*[55] discussed the MHD flow towards a permeable surface with prescribed wall heat flux. In the present paper, the heat transfer in boundary layer stagnation-point flow towards shrinking sheet with non-uniform heat flux at the boundary is analyzed. Using similarity transformations, the governing PDEs are transformed into self-similar ODEs which are then solved numerically by the shooting method. The numerically obtained results for temperature field are depicted in some figures and discussed in detail.

2. Analysis of the problem

Consider the heat transfer in steady two-dimensional boundary layer stagnation-point flow of viscous incompressible fluid towards a shrinking sheet in the presence of non-uniform heat flux. The governing equations of motion and the energy equation may be written in usual notation as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_{\rm s}\frac{{\rm d}U_{\rm s}}{{\rm d}x}+\upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

and

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are velocity components in the x direction and y direction respectively, $\upsilon(=\mu/\rho)$ is the kinematic viscosity of fluid, ρ is the density of fluid, μ is the coefficient of fluid viscosity, $U_{\rm s}$ is the straining velocity, T is the temperature, κ is the thermal conductivity, c_p is the specific heat, x is the distance along the sheet, and y is the distance perpendicular to the sheet.

The appropriate boundary conditions for the velocity components and temperature are given by

$$u=U_{\rm w}(x)=cx,\ v=0 \text{ at } y=0;$$

$$u\to U_{\rm s}(x)=ax \text{ as } y\to\infty, \tag{4}$$

$$\frac{\partial T}{\partial y}=-\frac{q_{\rm w}}{\kappa}=-\frac{\lambda x^m}{\kappa} \text{ at } y=0;\ T\to T_\infty \text{ as } y\to\infty, \tag{5}$$

where $U_{\rm w}$ is the shrinking velocity of the sheet, $c(<0)$ and $a(>0)$ are the shrinking constant and straining constant respectively, $q_{\rm w}=\lambda x^m$ is the non-uniform heat flux, T_∞ the free stream temperature assumed to be constant, λ a given constant, and m the heat flux parameter.

3. Method of solution

The following similarity transformations are introduced:

$$\psi=\sqrt{a\upsilon}xf(\eta),\ T-T_\infty=\frac{q_{\rm w}}{\kappa}\left(\frac{\upsilon}{a}\right)^{1/2}\theta(\eta),$$

$$\text{and } \eta=y\sqrt{\frac{a}{\upsilon}}, \tag{6}$$

where ψ is the stream function defined in the usual notation as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, η is the similarity variable, f is a dimensionless stream function, and θ is a dimensionless temperature.

In view of Eq. (6), equation (1) is satisfied identically, and substituting Eq. (6) into Eqs. (2) and (3) they reduce to the following self-similar ODEs:

$$f'''+ff''-f'^2+1=0, \tag{7}$$

$$\theta''+Pr[f\theta'-mf'\theta]=0, \tag{8}$$

where primes denote differentiation with respect to η and $Pr=c_p\mu/\kappa$ is the Prandtl number, and f' is the dimensionless velocity.

The boundary conditions (4) and (5) also reduce to

$$f(\eta)=0,\ f'(\eta)=B \text{ at } \eta=0;\ f'(\eta)\to1 \text{ as } \eta\to\infty, \tag{9}$$

$$\theta'(\eta)=-1 \text{ at } \eta=0;\ \theta(\eta)\to0 \text{ as } \eta\to\infty, \tag{10}$$

where $B=c/a$ is the velocity ratio parameter.

The highly nonlinear coupled self-similar equations (7) and (8) together with the boundary conditions (9) and (10) form a two-point boundary value problem (BVP) and are solved using the shooting method,[56–58] by converting it into an initial value problem (IVP). In this method, it is necessary to choose a suitable finite value of $\eta\to\infty$, say η_∞. The following first-order system is set

$$f'=p,\ p'=q,\ q'=p^2-fq-1, \tag{11}$$

$$\theta'=h,\ h'=-Pr[fh-mp\theta], \tag{12}$$

with the boundary conditions

$$f(0)=0,\ p(0)=B,\ h(0)=-1. \tag{13}$$

To solve Eqs. (11) and (12) with boundary conditions (13) as an IVP the values for $q(0)$, i.e. $f''(0)$, and $\theta(0)$ are required but no such values are given. The initial guess values for $f''(0)$ and $\theta(0)$ are chosen and the fourth order Runge–Kutta method is used to obtain a solution. Then the calculated values of $f'(\eta)$ and $\theta(\eta)$ at $\eta_\infty(=15)$ are compared with those under the given boundary conditions $f'(\eta_\infty)=0$ and $\theta(\eta_\infty)=0$ and the values of $f''(0)$ and $\theta(0)$ are adjusted using "secant method" to give a better approximation for the solution. The step-size is taken as $\Delta\eta=0.01$. The process is repeated until we obtain the results corrected up to the desired accuracy of 10^{-5} level.

4. Results and discussion

To clearly understand the effect of non-uniform heat flux on the temperature distribution, the numerically obtained results are plotted in some figures (Figs. 1–12). Wang[33] investigated the dual nature of solutions for stagnation-point flow and heat transfer over a shrinking sheet with prescribed surface temperature (PST), and similarly, here the similarity solution is also of dual nature for $-1.24657 \le B \le -1$ and the solution is unique solution for $B > -1$. Since the highlight of this investigation is the effect of the non-uniform heat flux condition at the wall and since the flow is a forced convective flow, the focus is given only on the variations in temperature profiles.

The influences of heat flux parameter m on the temperature distribution are presented in Figs. 1–4. In the variation of m, both positive and negative values are considered. For positive values of m the heat flux varies directly with the power of distance along the surface and for which both temperature profiles (in dual solutions case) show an interesting characteristic. Each temperature profile (Fig. 1) initially decreases and then for large η it increases and ultimately goes to zero. This nature is prominent in the second solution case. Also, it is important to note that the values of the dimensionless temperature at the surface in these cases ($m < 1.5$) are negative. Moreover, in temperature gradient profile (Fig. 2) the overshoot is observed. These all occur due to the direct variation of heat flux along the sheet. Due to this direct variation, the heat flux increases along the surface and the wall temperature significantly reduces. On the other hand, for negative values of m the heat flux varies inversely with the power of distance along the sheet, and the effect of this variation is quite different from the above case. For negative m, in each case the wall temperature is positive and the profiles (Fig. 3) go to zero with slightly wavy nature. This wavy nature is also observed in each of the temperature gradient profiles (Fig. 4). For the inverse variation the heat flux decreases along the surface and consequently, the wall temperature remains at a higher level. So, the non-uniform heat flux strongly affects the temperature distribution, depending on the nature of the variation.

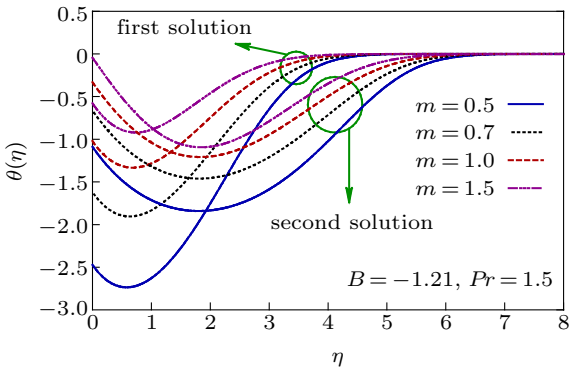


Fig. 1. Dual temperature profiles $\theta(\eta)$ for some values of m (>0).

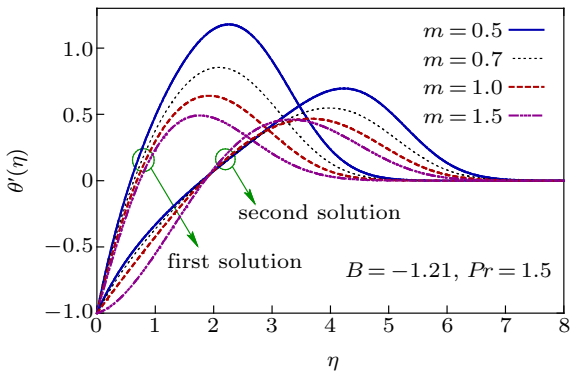


Fig. 2. Dual temperature gradient profiles $\theta'(\eta)$ for some values of m (>0).

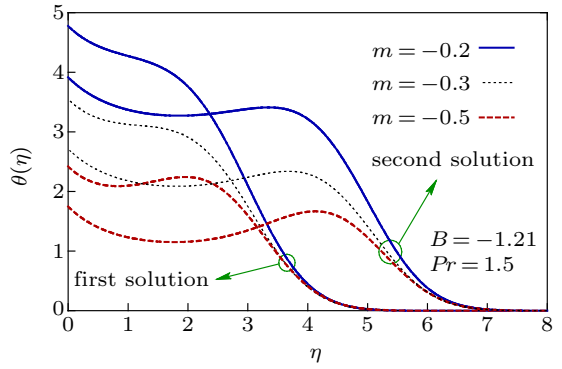


Fig. 3. Dual temperature profiles $\theta(\eta)$ for some values of m (<0).

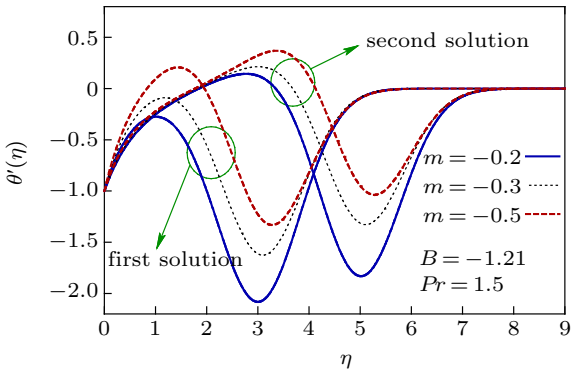


Fig. 4. Dual temperature gradient profiles $\theta'(\eta)$ for some values of m (<0).

Next, the spotlight is transferred to the effects of velocity ratio parameter B on temperature when there is a non-uniform heat flux (with direct/inverse variation). For several values of B, the dual temperature profiles $\theta(\eta)$ and temperature gradient profiles $\theta'(\eta)$ are presented in Figs. 5–8 (for $m = 1 > 0$ and $m = -0.5 < 0$, respectively). It is important to note that though the characteristics of profiles (both, temperature and temperature gradient) are similar to those of positive and negative variations of m respectively, but the parameter B has an noticeable effect on the nature of the profiles, which can be clearly observed from the figures, and it is also noted that the effects are opposite in two solutions. The velocity ratio parameter B is very important in controlling the existence and uniqueness of the similarity solution. For the increase in B, either the shrinking velocity increases or the straining velocity decreases and in both cases the generated vorticity affects steady solution and the boundary layer structure. Moreover, the thermal boundary layer thickness increases with the magnitude of B increasing for first solution and it decreases for second solution.

The dual temperature profiles and temperature gradient profiles for several values of Prandtl number Pr are plotted in Figs. 9–12 for $m = 1, -0.5$. It is seen from the figures that in both solution cases the increase in Pr causes a reduction in thermal boundary layer thickness. The increase in Pr means the reduction in thermal conductivity of the fluid, which reduces the thermal boundary layer thickness. This effect is similar to that in the PST case. Moreover, like velocity ratio parameter B the Prandtl number also affects the temperature profiles (with heat flux) and but in the same manner for both solutions.

Similar to the PST case, here it is also found that in dual solutions, the thermal boundary layer thickness for the first solution is thinner than that of the second solution. Finally, in each figure the far field thermal boundary condition is satisfied asymptotically with a desired accuracy level in both solutions, which ensures the correctness of the applied numerical scheme and the convergence of the solutions. The comparison of the results, for the velocity field, obtained using the numerical scheme described above, with available published results is made by Bhattacharyya,[40] which also confirms the accuracy of the applied numerical method.

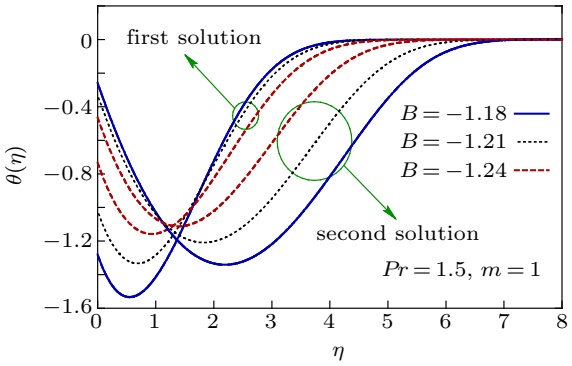


Fig. 5. Dual temperature profiles $\theta(\eta)$ for some values of B with $m > 0$.

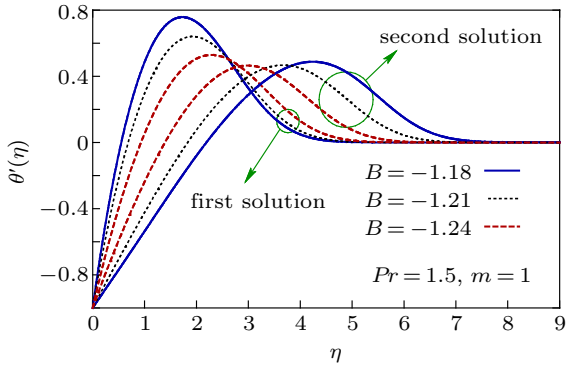


Fig. 6. Dual temperature gradient profiles $\theta'(\eta)$ for some values of B with $m > 0$.

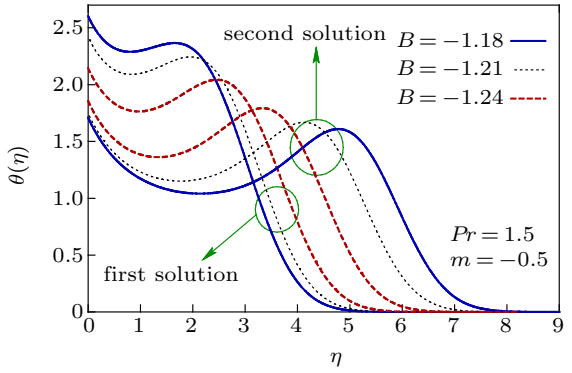


Fig. 7. Dual temperature profiles $\theta(\eta)$ for some values of B with $m < 0$.

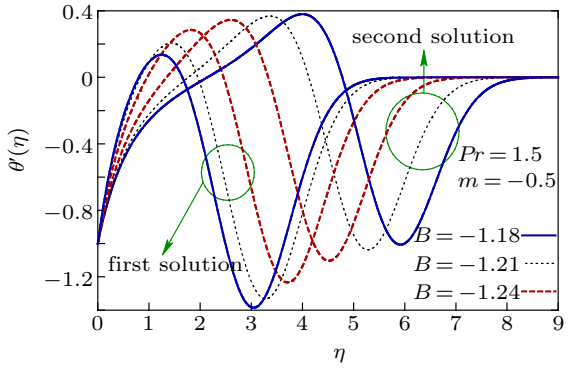


Fig. 8. Dual temperature gradient profiles $\theta'(\eta)$ for some values of B with $m < 0$.

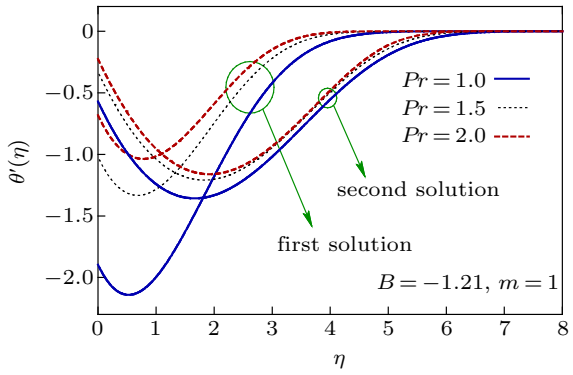


Fig. 9. Dual temperature profiles $\theta(\eta)$ for some values of Pr with $m > 0$.

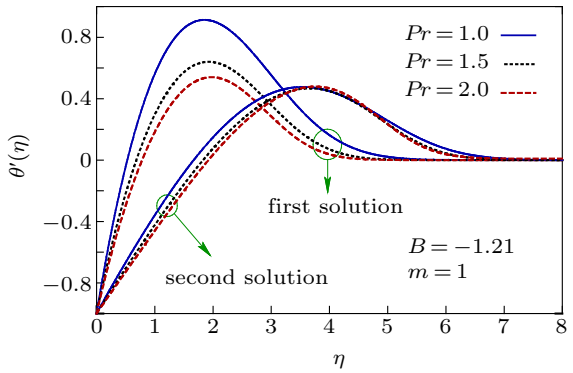


Fig. 10. Dual temperature gradient profiles $\theta'(\eta)$ for some values of Pr with $m > 0$.

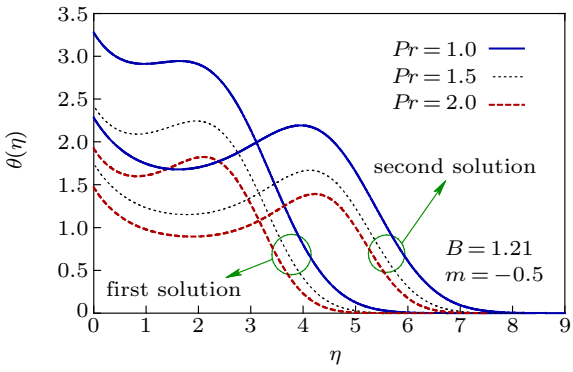


Fig. 11. Dual temperature profiles $\theta(\eta)$ for some values of Pr with $m < 0$.

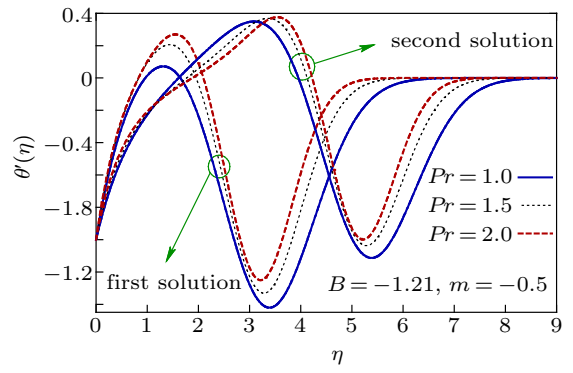


Fig. 12. Dual temperature gradient profiles $\theta'(\eta)$ for some values of Pr with $m < 0$.

5. Conclusions

The heat transfer in the boundary layer stagnation-point flow passing through a shrinking sheet with non-uniform heat flux is investigated. The transformed self-similar ODEs are solved numerically and dual solutions are obtained. For direct variation of heat flux along the surface, the thermal boundary layer thickness reduces and for inverse variation of heat flux, a wavy nature in temperature profiles is found. The analysis also reveals that the velocity ratio parameter and the Prandtl number act as the enhancing or suppressing this unusual nature of temperature due to the presence of non-uniform heat flux. These important results have not been reported previously.

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