# Sphere-cone-polynomial special window with good aberration characteristic*

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Optical windows with external surfaces shaped to satisfy operational environment needs are known as special windows. A novel special window, a sphere-cone-polynomial (SCP) window, is proposed. The formulas of this window shape are given. An SCP $MgF_2$ window with a fineness ratio of 1.33 is designed as an example. The field-of-regard (FOR) angle is $\pm 75^\circ$. From the window system simulation results obtained with the calculated fluid dynamics (CFD) and optical design software, we find that compared to the conventional window forms, the SCP shape can not only introduce relatively less drag in the airflow, but also have the minimal effect on imaging. So the SCP window optical system can achieve a high image quality across a super wide FOR without adding extra aberration correctors. The tolerance analysis results show that the optical performance can be maintained with a reasonable fabricating tolerance to manufacturing errors.

**Keywords:** special optical window, sphere-cone-polynomial surface, aberration characteristic

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## 1. Introduction

A special window/dome optical system refers to the one whose first optical surface conforms to both operational and imaging requirements. At present, special window optical systems, with their excellent aerodynamic performance, wide field-of-regard (FOR), and flexibility in determining where the sensors should be housed in the window, are drawing increasing attention, and will be widely adopted in various types of aircrafts.[1]

The shape of a special window might be ellipse, torus, ogive, or other aspheric formulas that produce a higher fineness ratio.[2–5] Considering both drag and imaging influence factors, the ellipse is the most common form of external surface in the special window, though this shape still induces severe non-central symmetry aberrations that change significantly with the FOR. A number of design techniques have been developed to overcome this problem; however, they greatly increase the length, weight, and cost of the whole system.[6–10] There is a need for a novel window that produces minimal wavefront distortion and keeps the configuration of the whole window optical system simple and easy to fabricate.

A window comprised of a curved piece of transparent material having a sphere-cone-polynomial (SCP) shape has the above advantages. In this paper, the concept and formulas of the SCP shape are proposed. Then, an SCP window example is given, and its drag coefficient is studied and compared to that of two types of traditional windows with identical diameters in the calculated fluid dynamics (CFD) simulation. After that, the aberration characteristics and the sensitivity to fabrication errors are analyzed through optical design tools. The analysis results confirm that the SCP window can improve the aerodynamic performance whilst rarely degrading the image quality so that the system hardly needs an extra correction method.

## 2. Construction of the SCP surface

Both front and back surfaces of the SCP window are SCP surfaces. The SCP surface is a complex aspherical surface, whose sag is defined in the polar coordinates by

$$\begin{aligned} Z_{\text{sag}} &= Z_{\text{pol}}(\rho,\theta)+Z_{\text{base}}(\rho) \\ &= \sum_{j=1}^{n} c_{j+1}p_j(\rho,\theta) \\ &\quad + \begin{cases} \dfrac{c\rho^2}{1+\sqrt{1-c^2\rho^2}}, & (\rho \le h), \\ \dfrac{c\rho^2}{1+\sqrt{1-(1+k)c^2\rho^2}} - d, & (\rho > h), \end{cases} \end{aligned} \tag{1}$$

where $c$ denotes the curvature, $\rho$ and $\theta$ are the radial coordinates, $k$ is the conic constant, $d$ represents an axial translation amount for keeping the spherical and conical segments of the surface tangent to each other, and $c_j$ and $p_j$ are the polynomial coefficient and the polynomial term, respectively.

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### 2.1. Base sphere-cone of the SCP surface

The SCP surface is defined by a sphere-cone plus a polynomial overlay. The schematic diagram of the sphere-cone is shown in Fig. 1. The rear section of the surface is the rear portion of an ellipsoid in this paper. The front portion of the ellipsoid, whose length (distance from the vertex to the center of the base) is $t$, is replaced by a spherical crown. The ellipsoidal section is tangent to the spherical section at their intersection line. So it is easy to know that the stitched surface has a unique tangent plane at each point of the surface, and when a point takes a continuous movement on the surface, the corresponding tangent plane rotates continuously. Consequently, we conclude that the stitched surface is smooth.[11]

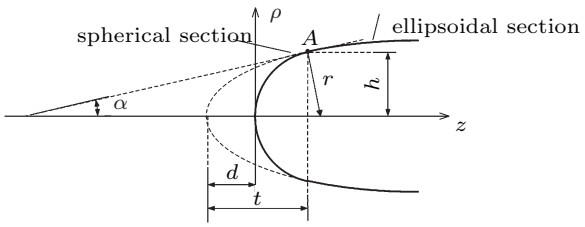



**Fig. 1.** Schematic diagram of a sphere-cone surface.

In Fig. 1, the length of the stitched surface reduces $d$ (with the same definition in Eq. (1)) compared with the original ellipsoid, $r$ is the radius of the spherical section, and $\alpha$ is the angle between the tangent plane of the surface at intersection point $A$ and the $z$ axis. By the laws of geometry, the following equation can be obtained:

$$d = t - r(1 - \sin\alpha). \tag{2}$$

The general equation of the conical surface is

$$z = \frac{c\rho^2}{1 + \sqrt{1 - (1+k)c^2\rho^2}}. \tag{3}$$

The base semi-diameter of the cut portion of the ellipsoid is $h$ (with the same definition in Eq. (1)), so the following equation can be derived:

$$r = h\sqrt{1 + (\tan\alpha)^2}. \tag{4}$$

By substituting $h$ for $\rho$ in Eq. (3), $t$ can be obtained as

$$t = \frac{ch^2}{1 + \sqrt{1 - (1+k)c^2h^2}}. \tag{5}$$

From Eq. (3), the value of $\tan\alpha$ can be obtained as

$$\tan\alpha = \frac{\mathrm{d}\rho}{\mathrm{d}z}\Big|_{\rho=h} = \frac{\sqrt{1 - (1+k)c^2h^2}}{ch}. \tag{6}$$

According to the trigonometric function operation and Eq. (6), the value of $\sin\alpha$ can be solved as

$$\sin\alpha = \sqrt{\frac{\tan\alpha^2}{\tan\alpha^2 + 1}}. \tag{7}$$

Using Eqs. (2)–(7), we can obtain the length $d$ numerically.

### 2.2. Polynomial terms of the SCP surface

It is well known that the Zernike polynomials represent a complete description of the aberrations of any imaging optical system with a circular pupil, and some third-order aberration components are related to certain Zernike terms.[12] Naturally, it is possible to add Zernike polynomial terms to the mathematical expression of the window surface to mitigate specific aberrations induced by a wide FOR, such as spherical aberration, astigmatism, and coma.[12] Therefore, $Z_{\mathrm{pol}}(\rho,\theta)$ in Eq. (1) can be defined as

$$Z_{\mathrm{pol}}(\rho,\theta) = \sum_{j=1}^{66} c_{j+1} z_j(\rho,\theta), \tag{8}$$

where $z_j$ is the $j$-th Zernike polynomial ($j = 1, 2, \ldots, 66$), and $c_{j+1}$ is the coefficient for $z_j$.

## 3. Baseline SCP window

It is necessary to define a particular SCP window for the following detailed discussion. The modeling of this window is accomplished through the selection of the fundamental design parameters, as shown in Table 1.

**Table 1.** SCP window design parameters.

| Fundamental design parameters | Outer surface | Inner surface |
|---|---|---|
| Curvature of the original ellipsoid $c$/mm$^{-1}$ | 0.0800 | 0.07874 |
| Conic constant of the original ellipsoid $k$ | –0.9375 | –0.9238 |
| Diameter of the ellipsoid $D$/mm | 100 | 92 |
| Semi-diameter of the spherical section $h$/mm | 44 | 42 |
| Zernike terms | 0 | 2nd–10th terms |
| Thickness of the window $t$/mm | 4 | |
| Length of the original ellipsoid of outer surface $L$/mm | 200 | |
| Reduced length of the outer surface $d$/mm | 66.5437 | |
| Length of the window $L'$ /mm | 133.4563 | |

If the length of this SCP window is $L'$ and the diameter is $D$, the fineness ratio (length to diameter ratio) of the window will be

$$F = \frac{L'}{D} = 1.33. \tag{9}$$

## 4. Analysis of aerodynamic performance, aberration characteristic and tolerance simulation of the SCP window

To quantitatively explain the advantages of the SCP window, the familiar ellipsoidal window will be chosen for comparison in the analysis of aerodynamic and imaging performances.

### 4.1. Drag coefficients

We carry out a contrast simulation experiment to study the aerodynamics of different window shapes. The experiment objects consist of an SCP window whose parameters are given in Table 1, a hemispherical window, and a fineness ratio ($F$) 0.75 ellipsoidal window. The diameter of each window is 100 mm. Figure 2 displays the geometries of these windows. The base of each window is connected to one end face of a 100-mm diameter, 700-mm length cylinder, as the aircraft models in the CFD simulation. We divide the three-dimensional (3D) flow field around the model into unstructured mesh elements, and adopt the solver based density, Spalart–Allmaras viscous model and the pressure far field boundary.[13] The speed of the aircraft is 3 Ma. The simulation with each model is conducted under the above conditions. The drag coefficients of these windows are shown in Table 2. Obviously, compared with the conventional spherical window, there is an aerodynamic benefit in going to the SCP and the ellipsoidal windows. The drag produced by the $F$ 0.75 ellipsoidal window is about 0.8 of that produced by the hemisphere window, and this result coincides with data in the published literature.[2] The drag produced by the $F$ 1.33 SCP window is approximately the same as that of the $F$ 0.75 window.

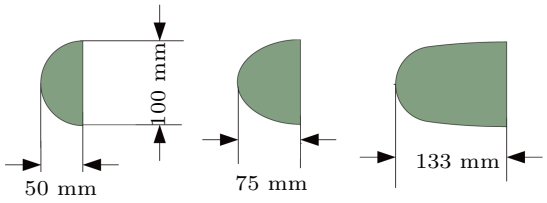



**Fig. 2.** (color online) Window structures used in the contrast simulation.

**Table 2.** Drag coefficients for three window shapes.

| Window shape | Drag coefficient |
|---|---|
| Hemisphere | 0.95378 |
| $F$ 0.75 ellipsoid | 0.75548 |
| $F$ 1.33 SCP | 0.78654 |

### 4.2. Aberration characteristics

The aberration characteristics of the SCP window are analyzed and compared to those of the ellipsoidal window in this section. The $F$ 1.33 SCP window and $F$ 0.75 ellipsoidal window studied in the above CFD simulation are chosen as examples. The $F$ 0.75 window has a thickness of 4 mm, a diameter of 100 mm, and an inner ellipsoidal surface with a fineness ratio of 0.75 and a diameter of 92 mm. For each window, the system wavelength range is 3–5 μm, the material is $MgF_2$, and the imaging system is an $F$/# 1.25 perfect lens gimballed behind the window, which has a half angle range of 75° from boresight to the edge of FOR, as shown in Fig. 3. As is customary, we characterize the aberrations of the windows by decomposing the wavefront at the exit pupil into fringe Zernike polynomials.[14] Figure 4 shows the aberrations versus the look angle. The dominant aberrations caused by the window are 3rd order spherical aberration ($Z_9$), coma ($Z_8$), and astigmatism ($Z_5$).

It can be seen from Fig. 4 that the peak-to-valley of the Zernike aberrations for the $F$ 0.75 window is much larger than that for the $F$ 1.33 window. For the $F$ 1.33 window, the aberration fluctuation is maintained within a relatively small range when the look angle is no more than 60°. This is because the optical system images through the near-spherical section of this window and the axis of rotation of the imaging lens is at the center of the curvature of the base sphere. When the FOR angle is between 60° and 75°, a portion of the light passes through the near-conical section. The two window sections introduce different amounts of optical power, leading to more astigmatism, coma, and defocus in the wavefront of the exit pupil. But the aberrations are still on the range from –0.3 to +0.2 waves.

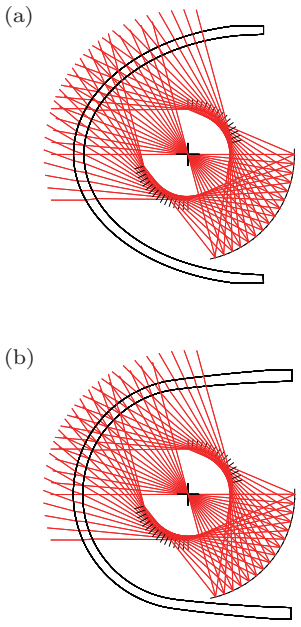



**Fig. 3.** (color online) Layouts of the window optical systems for all zoom positions: (a) $F$ 0.75 ellipsoidal window, (b) $F$ 1.33 SCP window.

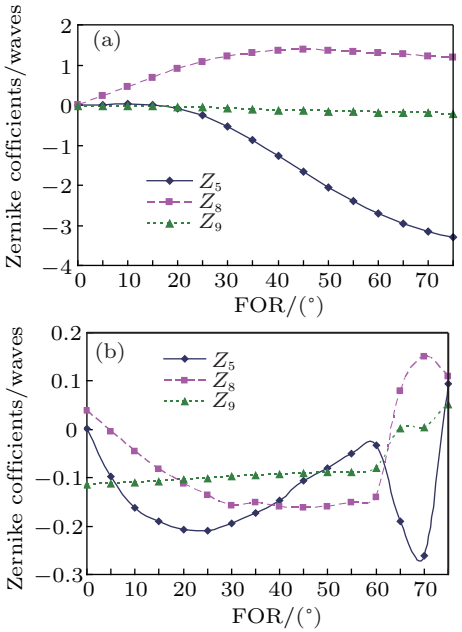



**Fig. 4.** (color online) Zernike aberrations versus look angle: (a) $F$ 0.75 ellipsoidal window, (b) $F$ 1.33 SCP window.

Figure 5 shows the RMS spot size versus look angle of the SCP window system. The spot size is 11–26 μm, which is about 0.86–2.03 times the diffraction limit. The above results show that the image effect over the full FOR of the SCP window is reasonably slight and can be easily eliminated by the real imaging lenses.

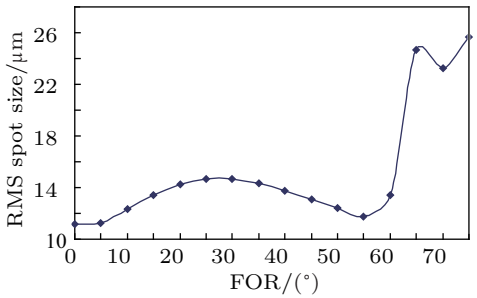



**Fig. 5.** (color online) RMS spot sizes of the SCP optical system.

### 4.3. Manufacturing tolerance analysis

To evaluate the manufacturability for the SCP window, a Monte–Carlo simulation analysis of tolerance budgets of the baseline window is executed to reveal the window optical performance after machining. Table 3 lists the allowable tolerance of each error.[15] The analysis predicts the tolerances on the RMS spot size. The spot size versus look angle of the window with machining errors is plotted in Fig. 6.

**Table 3.** Tolerances for the SCP baseline window.

| Tolerance type | Surface | Value |
|---|---|---|
| Test plate fit | out/in | 1 fringe |
| Thickness | / | 0.01 mm |
| Element wedge | / | 0.025 mm |
| Index error | / | 0.0002 |

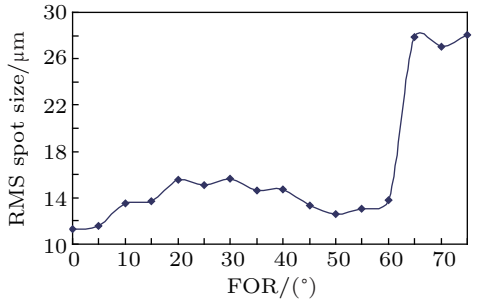



**Fig. 6.** (color online) RMS spot size for the window with machining errors.

In Fig. 6, the spot size of the window with machining errors rises to 11.3–28.1 μm, which is approximately 0.88–2.20 times the diffraction limit. The result illustrates that the high imaging performance of the window will be maintained within the above tolerance limit. The requirement in tolerance can be easily met by the present fabrication technology.[16]

## 5. Conclusion

The expression of the SCP shape is put forward, and a baseline SCP window model is established. Then, with the commercial CFD and optical software, we validate that the SCP window alleviates the air resistance suffered by the aircraft and produces a very small amount of aberrations across super wide FOR angles. The fabrication tolerance of this window can be satisfied by the modern fabrication technology. So the application of the SCP windows will omit the static and dynamic aberration correctors usually adopted in the airborne special window/dome optical system, which will significantly reduce the weight and volume and enhance the reliability of the system.

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