Deformed soliton, breather, and rogue wave solutions of an inhomogeneous nonlinear Schrödinger equation*

Tao Yong-Sheng(陶勇胜)[a], He Jing-Song(贺劲松)[a,†], and K. Porsezian[b]

[a] Department of Mathematics, Ningbo University, Ningbo 315211, China
[b] Department of Physics, Pondicherry University, Puducherry 605014, India

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We use the 1-fold Darboux transformation (DT) of an inhomogeneous nonlinear Schrödinger equation (INLSE) to construct the deformed-soliton, breather, and rogue wave solutions explicitly. Furthermore, the obtained first-order deformed rogue wave solution, which is derived from the deformed breather solution through the Taylor expansion, is different from the known rogue wave solution of the nonlinear Schrödinger equation (NLSE). The effect of inhomogeneity is fully reflected in the variable height of the deformed soliton and the curved background of the deformed breather and rogue wave. By suitably adjusting the physical parameter, we show that a desired shape of the rogue wave can be generated. In particular, the newly constructed rogue wave can be reduced to the corresponding rogue wave of the nonlinear Schrödinger equation under a suitable parametric condition.

Keywords: inhomogeneous nonlinear Schrödinger equation, Lax pair, Darboux transformation, soliton

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1. Introduction

In recent years, rogue waves (freak waves or killer waves) which cause many marine disasters[1,2] have attracted considerable attention of scientists.[3] Though it was first observed and encountered in the area of oceanography,[4–6] in recent years, several theoretical and experimental attempts have been made to observe this peculiar and interesting phenomenon in many other branches of science, such as matter rogue waves[7,8] in Bose–Einstein condensates (BECs), rogue waves in surface and space plasma,[9,10] financial rogue waves describing the possible physical mechanisms in financial markets and related fields,[11] rogue waves in superfluid,[12] and especially those in optics observed when high power optical radiation propagates through photonic crystal fibers.[13,14] The development of a spectral theory of modulation instability has already explained why white light supercontinuum spectra in experiments almost always start from a phase of a triangular spectrum. Such spectra appear universally as a result of the exponential decay of the nonlinearly generated frequency modes of many parametrically-driven systems.[15] This has been confirmed by recent observations of noise-driven supercontinuum generation in optical fibers and can be well-described by the analytical expressions for the spectra of Akhmediev breather solutions at the point of extreme pulse compression.[16] So the study of the breather and rogue wave solutions of nonlinear physical systems is quite fascinating and widely carried out in many optical systems with modern high power lasers and fibres. Most of those studies have reported idealistic systems without any controlling mechanisms. For many of the experimental situations, one has to achieve suitable pulse parameters by using suitable management and control of the system parameters. From our detailed investigation, we have realized that the nonuniform physical systems are more suitable candidates for achieving the required controlling mechanisms.

The most significant feature of rogue waves is that rogue waves are localized in both directions of the plane and decay to disappear quickly. A recent experiment in Ref. [17] revealed that the inhomogeneity of a medium plays an important role in the generation of optical rogue waves, and the inhomogeneity is usually described by the variable-coefficient nonlinear evolution equations. Moreover, the physical medium with inhomogeneous time–space dependence or defects[18] always has some irregular changes, which are related to the realistic understanding of the nonlinear phenomena, including nonautonomous soliton,[19–24] similariton,[25–29] and designable integrability.[30,31] Therefore, we are interested in analyzing the impact of inhomogeneities on the rogue waves and how these localization properties in both directions of the (x,t) plane are affected by an inhomogeneous medium. Motivated by the above discussion, in this paper we consider an inhomogeneous nonlinear Schrödinger equation (INLSE) of the form[26,32–35]

$$\mathrm{i}q_t + q_{xx} + 2|q|^2 q + \alpha^2 x^2 q + \mathrm{i}\alpha q = 0, \tag{1}$$

where α is a real number. If $\alpha = 0$, INLSE (1) reduces to the standard NLSE. The INLSE is one of the completely

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†Corresponding author. E-mail: hejingsong@nbu.edu.cn; jshe@ustc.edu.cn

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integrable equations, the Lax pair of which is given by the Ablowitz, Kaup, Newell, and Segur (AKNS) method.[34] The explicit form of the breather solution of this equation has been obtained[35] by using the Darboux transformation (DT) of the associated non-isospectral problem. However, the details of the construction of the DT for Eq. (1) has not been given.[35]

To the best of our knowledge, there are at least two approaches to obtain breather and rogue wave solutions of the inhomogeneous soliton equations, that is, transformation mapping inhomogeneous soliton equations to the constant-coefficient soliton equation[30] and the Darboux transformation[36,37] of the non-isospectral problems. In general, the DT is an effective approach to generate rogue waves for constant-coefficient equations, such as the nonlinear Schrodinger equation,[38] the Hirota equation,[39,40] the derivative NLSE (DNLSE),[41] and the NLS-MB equation.[42] The rogue waves of the inhomogeneous equations have been constructed by transformations from known solutions of the corresponding soliton equations.[43–47]

Based on the above results, we first derive the determinant representation[36,48] of the 1-fold DT of INLSE (1) to construct the deformed soliton solution and the breather solution from zero and non-zero seed solutions, respectively. Furthermore, we obtain a deformed rogue wave solution from the deformed breather solution by using the Taylor expansion. These breather and rogue wave solutions are both suitably controlled by a physical parameter associated with the system, which is actually quite different from the reported rogue waves in the literature.

2. Darboux transformation

Let us start from two coupled inhomogeneous nonlinear Schrödinger equations of the form

$$\mathrm{i}q_t + q_{xx} - 2q^2 r + \alpha^2 x^2 q + \mathrm{i}\alpha q = 0, \tag{2}$$

$$\mathrm{i}r_t - r_{xx} + 2r^2 q - \alpha^2 x^2 r + \mathrm{i}\alpha r = 0; \tag{3}$$

these equations can reduce to INLSE (1) if we choose $r = -q^*$. In order to construct the 1-fold DT, we rewrite the Lax pair of Eq. (1) as follows:

$$\varphi_x = \boldsymbol{M}\varphi = (-\mathrm{i}\sigma_3\lambda + \boldsymbol{U})\varphi, \tag{4}$$

$$\varphi_t = \boldsymbol{N}\varphi = (\boldsymbol{V}_1\lambda^2 + \boldsymbol{V}_2\lambda + \boldsymbol{V}_3)\varphi, \tag{5}$$

with

$$\varphi = (\varphi_1, \varphi_2)^{\mathrm{T}}, \quad \boldsymbol{M} = \begin{pmatrix} -\mathrm{i}\lambda & u \\ v & \mathrm{i}\lambda \end{pmatrix},$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} 0 & u \\ v & 0 \end{pmatrix},$$

$$\boldsymbol{N} = \lambda^2 \begin{pmatrix} -2\mathrm{i} & 0 \\ 0 & 2\mathrm{i} \end{pmatrix} + \lambda \begin{pmatrix} 2\mathrm{i}\alpha x & 2u \\ 2v & -2\mathrm{i}\alpha x \end{pmatrix}$$

$$+ \begin{pmatrix} -\mathrm{i}uv & \mathrm{i}u_x - 2\alpha x u \\ -\mathrm{i}v_x - 2\alpha x v & \mathrm{i}uv \end{pmatrix},$$

where $\lambda = \lambda_0 \mathrm{e}^{(-2\alpha t)}$ is a non-isospectral parameter, λ_0 is a complex constant, and $u = q\mathrm{e}^{\frac{-\mathrm{i}\alpha x^2}{2}}, v = r\mathrm{e}^{\frac{\mathrm{i}\alpha x^2}{2}}$. It can be verified that the compatibility condition $\boldsymbol{M}_t - \boldsymbol{N}_x + [\boldsymbol{M}, \boldsymbol{N}] = 0$ gives rise to Eqs. (4) and (5), where the bracket represents the usual matrix commutator.

Based on the spectral problems (4) and (5), we would like to introduce the following gauge transformation:

$$\varphi^{[1]} = \boldsymbol{T}\varphi. \tag{6}$$

The gauge transformation converts the old spectral problems (4) and (5) into the same type of new spectral problems, namely,

$$\varphi_x^{[1]} = \boldsymbol{M}^{[1]}\varphi^{[1]}, \quad \varphi_t^{[1]} = \boldsymbol{N}^{[1]}\varphi^{[1]},$$

where $\boldsymbol{M}^{[1]}$ and $\boldsymbol{N}^{[1]}$ have the same forms as those of $\boldsymbol{M}$ and $\boldsymbol{N}$ with u and v replaced by $u^{[1]}$ and $v^{[1]}$. It is easy to find out that $\boldsymbol{M}^{[1]}$ and $\boldsymbol{N}^{[1]}$ are given by

$$\boldsymbol{M}^{[1]} = \boldsymbol{T}_x\boldsymbol{T}^{-1} + \boldsymbol{T}\boldsymbol{M}\boldsymbol{T}^{-1}, \tag{7}$$

$$\boldsymbol{N}^{[1]} = \boldsymbol{T}_t\boldsymbol{T}^{-1} + \boldsymbol{T}\boldsymbol{N}\boldsymbol{T}^{-1}. \tag{8}$$

In general, if the transformation $\boldsymbol{T}$ is a polynomial of parameter λ, we can start with

$$\boldsymbol{T}(\lambda) = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\lambda + \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{9}$$

where a_1, b_1, c_1, d_1, a, b, c, d are all functions of variables x and t. After the direct calculation by substituting Eq. (9) into Eqs. (7) and (8) and collecting similar terms, we can prove that a_1 and d_1 are constants and, for simplicity, we assume both of them equal to 1 and b_1 and c_1 both equal to 0. So the transformation can be expressed as

$$\boldsymbol{T}(\lambda) = \lambda\boldsymbol{I} - \boldsymbol{S}_1, \tag{10}$$

where $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{S}_1$ is a nonsingular matrix.

From Eqs. (7) and (10), comparing the coefficients of λ, we obtain

$$\boldsymbol{U}^{[1]} = \boldsymbol{U} + \mathrm{i}[\boldsymbol{S}_1, \sigma_3], \tag{11}$$

which implies that the relationship between new and old potential functions can be given as

$$u^{[1]} = u - 2\mathrm{i}s_1^{12}, \quad v^{[1]} = v + 2\mathrm{i}s_1^{21},$$

$$\boldsymbol{S}_1 = \begin{pmatrix} s_1^{11} & s_1^{12} \\ s_1^{21} & s_1^{22} \end{pmatrix}. \tag{12}$$

In order to obtain the explicit expression of matrix $\boldsymbol{S}_1$, we define this matrix by the solutions of the Lax pair as follows:

$$\boldsymbol{S}_1 = \varphi\boldsymbol{\Lambda}\varphi^{-1}, \tag{13}$$

with

$$\varphi=(\varphi_1,\varphi_2)=\begin{pmatrix} f_1 & f_2 \\ g_1 & g_2 \end{pmatrix},\quad \boldsymbol{\Lambda}=\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where $\varphi_1=(f_1,g_1)^{\dagger}$ is a solution of the Lax pair associated with the spectral condition $\lambda=\lambda_1$, and $\varphi_2=(f_2,g_2)^{\dagger}$ is a solution of the Lax pair with $\lambda=\lambda_2$. By direct calculation, it is verified that $\boldsymbol{T}(\lambda_k)(\varphi_k)=0$ for $k=1,2$. The $\boldsymbol{T}$ given by Eqs. (10)–(13) also satisfies the condition of Eq. (5). By using these solutions of the Lax pair, new solutions $u^{[1]}$ and $v^{[1]}$ can be written as

$$\begin{aligned} u^{[1]}&=u-2\mathrm{i}\begin{vmatrix} \lambda_1 f_1 & f_1 \\ \lambda_2 f_2 & f_2 \end{vmatrix}\Big/\begin{vmatrix} f_2 & f_1 \\ g_2 & g_1 \end{vmatrix},\\ v^{[1]}&=v+2\mathrm{i}\begin{vmatrix} \lambda_2 g_1 & g_1 \\ \lambda_1 g_2 & g_2 \end{vmatrix}\Big/\begin{vmatrix} f_2 & f_1 \\ g_2 & g_1 \end{vmatrix}. \end{aligned}\tag{14}$$

The new solutions of the coupled equations (2) and (3) can be expressed as

$$\begin{aligned} q^{[1]}&=q-2\mathrm{i}\begin{vmatrix} \lambda_1 f_1 & f_1 \\ \lambda_2 f_2 & f_2 \end{vmatrix}\Big/\begin{vmatrix} f_2 & f_1 \\ g_2 & g_1 \end{vmatrix}\mathrm{e}^{\frac{\mathrm{i}\alpha x^2}{2}},\\ r^{[1]}&=r+2\mathrm{i}\begin{vmatrix} \lambda_2 g_1 & g_1 \\ \lambda_1 g_2 & g_2 \end{vmatrix}\Big/\begin{vmatrix} f_2 & f_1 \\ g_2 & g_1 \end{vmatrix}\mathrm{e}^{-\frac{\mathrm{i}\alpha x^2}{2}}. \end{aligned}\tag{15}$$

3. Deformed soliton and breather solutions

In this section, we start from different seed solutions to construct deformed soliton and breather solutions by using the DT obtained above.

1) Now assuming the initial seed $q=0$ and $\lambda_1=(\xi+\mathrm{i}\eta)\mathrm{e}^{-2\alpha t}$, we get the following eigenfunctions by directly solving the corresponding Lax pair (4) and (5):

$$f_1=c_1\exp\left(-\mathrm{i}[2x\alpha\int\lambda_1\,\mathrm{d}t+2\int\lambda_1(\lambda_1-x\alpha)\mathrm{d}t+\lambda_1 x]\right),\tag{16}$$

$$f_2=c_2\exp\left(\mathrm{i}[2x\alpha\int\lambda_1\,\mathrm{d}t+2\int\lambda_1(\lambda_1-x\alpha)\mathrm{d}t+\lambda_1 x]\right),\tag{17}$$

where c_1 and c_2 are both real arbitrary numbers. The deformed soliton solution is given (see Fig. 1) by one-fold DT (15) and the reductions $g_1=-f_2^*$, $g_2=f_1^*$, $\lambda_2=\lambda_1^*$ of the form

$$q^{[1]}_{\text{soliton}}=-\frac{4\eta c_1c_2\mathrm{e}^{2\Theta_1}}{c_1^2\mathrm{e}^{2\mathrm{i}\Theta_2}+c_2^2\mathrm{e}^{-2\mathrm{i}\Theta_2}}\mathrm{e}^{\frac{1}{2}\mathrm{i}\alpha x^2-2\alpha t},\tag{18}$$

where

$$\begin{aligned} \Theta_1&=\eta\Gamma-2\mathrm{i}\alpha x\xi\left(\int\mathrm{e}^{-2\alpha t}\mathrm{d}t\right)\\ &\quad-\mathrm{i}x\xi\,\mathrm{e}^{-2\alpha t}-\mathrm{i}\xi\Gamma+\eta\Pi+\mathrm{i}\xi\Pi,\\ \Theta_2&=-\xi\Gamma-2\mathrm{i}\alpha x\eta\left(\int\mathrm{e}^{-2\alpha t}\mathrm{d}t\right)\\ &\quad-\mathrm{i}x\eta\,\mathrm{e}^{-2\alpha t}-\mathrm{i}\eta\Gamma-\xi\Pi+\mathrm{i}\eta\Pi,\\ \Gamma&=\int\mathrm{e}^{-2\alpha t}(\xi\,\mathrm{e}^{-2\alpha t}+\mathrm{i}\eta\,\mathrm{e}^{-2\alpha t}-\alpha x)\mathrm{d}t,\\ \Pi&=\int\mathrm{e}^{-2\alpha t}(-\xi\,\mathrm{e}^{-2\alpha t}+\mathrm{i}\eta\,\mathrm{e}^{-2\alpha t}+\alpha x)\mathrm{d}t. \end{aligned}$$

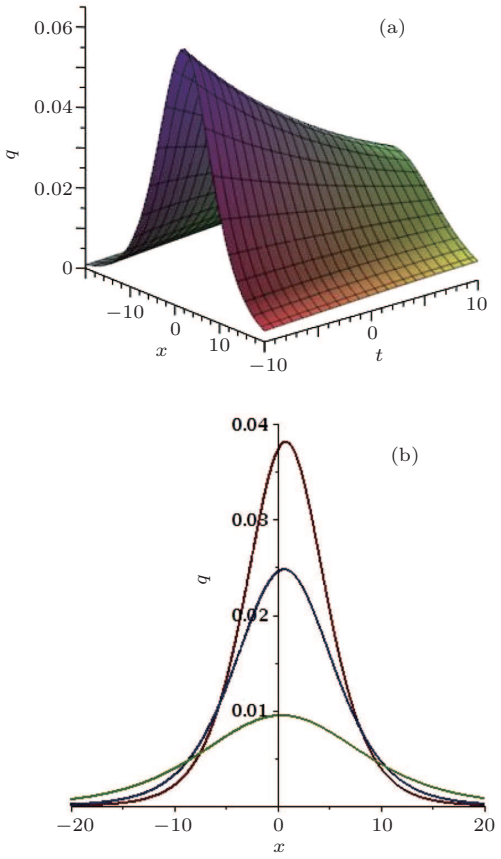


Fig. 1. (color online) (a) Deform solution with $\alpha=0.012$, $\xi=0.01$, $\eta=0.1$, $c_1=1$, $c_2=1$ of INLSE (1) and (b) its profiles at different time: $t=1$ (red, top), $t=10$ (blue, middle), $t=30$ (green, bottom). This figure shows the variable height of the deformed soliton.

2) In general, the breather solution of Eq. (1) can be constructed from a nonzero seed. However, it is difficult to solve the resulting four variable-coefficient equations of the Lax pair if the seed is too general. For this reason, we choose a special kind of initial seed as

$$q=\mathrm{e}^{-2\alpha t+\mathrm{i}\theta},\tag{19}$$

where $\theta=\alpha x^2/2+2\int_0^t\mathrm{e}^{-4\alpha t}\mathrm{d}t$. In order to construct the rogue wave solution, some simplifications are necessary. For example, by assuming the spectral parameter $\lambda=\mathrm{i}v\mathrm{e}^{(-2\alpha t)}$ (v is a real constant), the corresponding solutions of the eigenvalue equations of the Lax pair are given by

$$f_1=(\rho_1\Psi+\rho_2\Phi)\exp\left(-\frac{\mathrm{i}}{4\alpha}\mathrm{e}^{-4\alpha t}\right),\tag{20}$$

$$\begin{aligned} f_2=&-[\rho_1(\mathrm{i}\sqrt{1-v^2}+v)\Psi\\ &-\rho_2(\mathrm{i}\sqrt{1-v^2}-v)\Phi]\exp\left(-\frac{\mathrm{i}}{4\alpha}(2-\mathrm{e}^{-4\alpha t})\right), \end{aligned}\tag{21}$$

where ρ_1 and ρ_2 are constants of integration and

$$\begin{aligned} \Psi&=\exp\left(-\frac{1}{2\alpha}\sqrt{1-v^2}(v\mathrm{e}^{-2\alpha t}+2\mathrm{i}\alpha x)\mathrm{e}^{-2\alpha t}\right),\\ \Phi&=\exp\left(\frac{1}{2\alpha}\sqrt{1-v^2}(v\mathrm{e}^{-2\alpha t}+2\mathrm{i}\alpha x)\mathrm{e}^{-2\alpha t}\right). \end{aligned}$$

Finally, after tedious calculations, we get a deformed breather solution (see Fig. 2) of the form

$$q^{[1]}_{\text{breather}} = \frac{(1-2v^2)\cosh(d_1) - 2\mathrm{i}v\sqrt{1-v^2}\sinh(d_1) + v\cos(d_2)}{\cosh(d_1) - v\cos(d_2)} \exp\left(\frac{1}{2\alpha}\mathrm{i}(1+\alpha^2x^2 - \mathrm{e}^{-4\alpha t}) - 2\alpha t\right), \tag{22}$$

where

$$d_1 = \frac{v}{\alpha}(\sqrt{1-v^2})(\mathrm{e}^{-4\alpha t} - 1), \quad d_2 = 2x\sqrt{1-v^2}\,\mathrm{e}^{-2\alpha t}.$$

Comparing our newly obtained breather solutions with those of the Hirota equation[39,40] and the DNLSE,[41] we find that the breather solution of the INLSE has some deformations along the direction of t due to the variable exponential term. In other words, the breather in Eq. (22) has a curved background, which is completely controlled by the system parameter α.

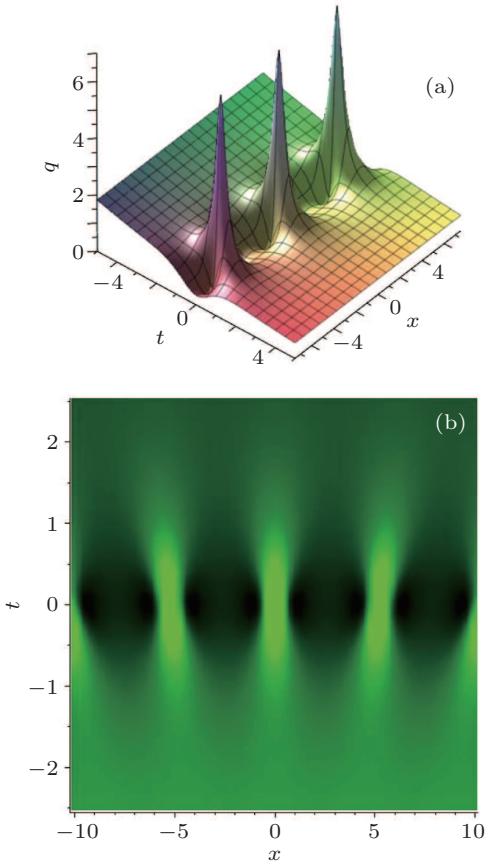


Fig. 2. (color online) (a) Deformed breather solution (22) of INLSE (1) with $\alpha = 0.03$, $v = 0.8$ and (b) its density plot.

4. Deformed rogue wave

In this section, we shall show for the first time that the deformed rogue wave solution of inhomogeneous Eq. (1) can also be obtained by the Taylor expansion of its breather solution.

The Taylor expansion of breather solution (22) with $v = 1$ implies the first order rogue wave of INLSE (1) (see Fig. 3) in the form

$$q^{[1]}_{\text{roguewave}} = \left[\frac{4\alpha^2 - 4\mathrm{i}\alpha(\mathrm{e}^{-4\alpha t} - 1)}{(\mathrm{e}^{-4\alpha t} - 1)^2 + 4x^2\alpha^2(\mathrm{e}^{-2\alpha t})^2 + \alpha^2} - 1\right] \times \exp\left(\frac{1}{2\alpha}\mathrm{i}(1+\alpha^2x^2 - \mathrm{e}^{-4\alpha t}) - 2\alpha t\right). \tag{23}$$

It is not difficult to verify the validity of this solution. This rogue wave is localized in one direction and decays to disappear quickly in both directions of the background. Moreover, the first order rogue wave of INLSE (1) has a curved background, the inhomogeneity or defect leads to some deformations which are commonly observed in most variable-coefficient models,[43,45,46] especially in inhomogeneous media. In addition, it is worth mentioning that the parameter α in rogue wave (23) completely controls the trend, shape, width, and direction of the curved background. (i) The bigger the value of the parameter α is, the steeper the background will be; and the smaller the value of the parameter α is, the flatter the background will be. (ii) If $\alpha > 0$, the background will be curved along the negative direction of the variable t; if $\alpha < 0$, the background will be curved along the positive direction of the variable t.

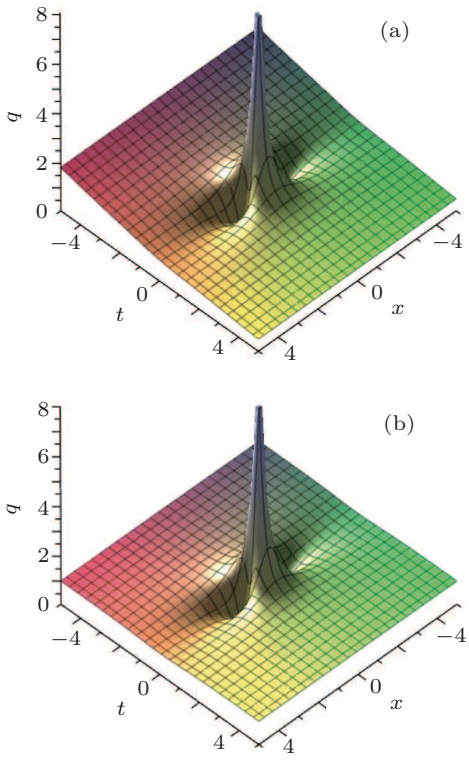


Fig. 3. (color online) (a) Deformed rogue wave solution (23) of INLSE (1) with $\alpha = 0.03$ and (b) rogue wave solution (24) of the standard NLSE.

Specifically, if α goes to 0, the curved background becomes a flat one, and rogue wave solution (23) of INLSE (1) reduces to

$$q^{[11]}_{\text{roguewave}} = \left(\frac{16\mathrm{i}t + 4}{16t^2 + 4x^2 + 1} - 1\right)\mathrm{e}^{2\mathrm{i}t}, \tag{24}$$

which is just the rogue wave solution[3] of the standard NLSE (see Fig. 3). So by taking a limiting condition, the rogue wave of the INLSE reduces to the corresponding one of the NLSE.

5. Discussion and conclusion

The determinant representation of the 1-fold DT of INLSE (1) is given by a detailed deduction. The deformed soliton and breather solutions of this inhomogeneous equation are constructed by using this method from zero and nonzero seed solutions, respectively. Furthermore, the first order rogue wave of INLSE (1) is obtained by the Taylor expansion of the breather solution for the first time. All solutions of INLSE (1) reduce to the corresponding solutions of the nonlinear Schrödinger equation when α goes to zero. In particular, the deformed soliton has a variable height, which is one of the common properties of the nonautonomous soliton,[19] the deformed breather and rogue wave of INLSE (1) also has a curved background completely controlled by the parameter α. Therefore, our result clearly shows that it is possible to control the rogue wave properties by changing α through the external potential (or field). In this way, by adjusting the parameter α, one can achieve suitable and required pulse width and shape. Moreover, by comparing with the rogue wave of the NLSE, we find that the deformation of the rogue wave (23) of the INLSE reflects the inhomogeneity or defect in the optical system or suitable potential in BECs modeled, thus it suggests a way to remotely control the defects in the physical systems. We hope that our results will be realized in some physics experiments in the future, which will be useful to understand the generation mechanism and find possible applications of the rogue wave in realistic systems like optical fiber and BECs.

Similar to the n-fold DT[36,48] of the NLSE, the determinant of the n-fold DT of Eq. (1) can be generated, which is useful to analyse the higher order breathers and rogue waves. The work is in progress and will be published elsewhere.

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