Defect solitons supported by parity-time symmetric defects in superlattices*

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The existence and stability of defect solitons supported by parity-time (PT) symmetric defects in superlattices are investigated. In the semi-infinite gap, in-phase solitons are found to exist stably for positive defects, zero defects, and negative defects. In the first gap, out-of-phase solitons are stable for positive defects or zero defects, whereas in-phase solitons are stable for negative defects. For both the in-phase and out-of-phase solitons with the positive defect and in-phase solitons with negative defect in the first gap, there exists a cutoff point of the propagation constant below which the defect solitons vanish. The value of the cutoff point depends on the depth of defect and the imaginary parts of the PT symmetric defect potentials. The influence of the imaginary part of the PT symmetric defect potentials on soliton stability is revealed.

Keywords: defect solitons, parity-time symmetric defect, superlattices

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1. Introduction

Since Bender *et al.* reported that non-Hermitian Hamiltonians can exhibit entirely real spectra if the system has parity-time (PT) symmetry,[1] physical systems with PT symmetry have been widely studied in recent years, both theoretically and experimentally.[2–22] The notion of PT symmetry was introduced within the framework of optics.[5] This suggestion was based on judicious designs that involve both optical gain/loss regions and the process of index guiding, which makes the complex refractive-index distribution obey the condition $n(x) = n^*(-x)$. In other words, the real part of the refractive index is an even function of position, whereas the imaginary part is odd.[4–6] Previous research mainly focused on the PT symmetric optical lattices,[5,6,8–10,12,14–19] and many intriguing features stemming from PT symmetry have been found, such as power oscillation,[6,12] absorption enhanced transmission,[11] nonlinear switching structure,[14] and unidirectional invisibility.[15] In experiments, Guo *et al.* firstly observed the PT symmetry breaking in complex optical potentials,[11] and Rüter *et al.* have observed the non-reciprocal wave propagation in an active PT symmetric coupled waveguide system.[12] The PT symmetry has been realized in the LRC circuits,[20,21] and dual behavior of PT symmetric scattering has been observed.[22] Recently, Regensburger *et al.* applied PT symmetry to a new generation of multifunctional optical devices and networks experimentally.[13] Those theoretical and experimental results led to the proposal of a new class of PT symmetric synthetic materials with intriguing and unexpected properties that rely on non-reciprocal light propagation.[12,15]

Soliton phenomena in optical lattices and superlattices have been attracting considerable attention.[23–29] Optical superlattices can be considered as regular lattice hosting a periodic chain of defects. Superlattices are fascinating because the structures exhibit collective properties not shared by either constituent, and these properties can be controlled through variation of the structural parameters.[30] Defect modes and defect solitons in PT symmetric optical lattices have been studied.[18,19] Recently, we have studied the defect solitons in PT symmetric superlattices with local and nonlocal nonlinearity.[9,31] In these studies, the PT symmetric potentials are covered with the whole optical lattices, and PT symmetric potentials only located in the defect in superlattices have not been exploited yet. In this paper, we study the defect solitons in superlattices with the PT symmetric defect. The properties of defect solitons are affected by the imaginary parts of the PT symmetric defect potentials. The existence and stability of defect solitons in the first semi-infinite gap for positive, zero, and negative defects are studied.

2. Model and method

We consider the propagation of a light beam in optical lattices with PT symmetric defects in Kerr-type nonlinearity. The evolution of complex amplitude U of the light fields can be described by the following dimensionless nonlinear Schrödinger equation (NLSE):

$$\mathrm{i}\frac{\partial U}{\partial z} + \frac{\partial^2 U}{\partial x^2} + [V(x) + \mathrm{i}W(x)]U + |U|^2 U = 0, \qquad (1)$$

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where x and z are the transverse and longitudinal coordinates, respectively, and $V(x)$ and $W(x)$ are the real and imaginary parts of the PT symmetric potentials, respectively, which are assumed in this paper as

$$V(x) = V_0[\varepsilon\cos^2(x)+(1-\varepsilon)\sin^2(2x)] \times [1+a\exp(-x^8/128)], \quad (2)$$

$$3W(x) = \begin{cases} W_0\sin(2x), & -\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}, \\ 0, & x<-\dfrac{\pi}{2} \text{ or } x>\dfrac{\pi}{2}. \end{cases} \quad (3)$$

Here, a and ε represent the strength of the defect and the modulation parameter of superlattice, respectively. When $0.1\leqslant\varepsilon\leq 0.7$, equation (2) describes the superlattice shape.[23] Without loss of generality, we take $\varepsilon_1 = 0.5$ throughout the paper. The parameters V_0 and W_0 represent the modulation depth of the real and imaginary parts of the PT symmetric potentials, respectively.

The lattices with PT symmetric defects governed by Eq. (2) have a Bloch band structure when $a = 0$. The Bloch band structure obtained by the plane-wave expansion method for $V_0 = 4$ is shown in Fig. 1(a). One can see that the region of the semi-infinite gap is $b > 2.11$, and the first gap is $0.64 < b < 1.51$, where b is the propagation constant. Figures 1(b)–1(d) show the intensity distributions of the PT superlattice potentials with the strength of the defect $a = 0.3$, $a = 0$, and $a = -0.3$, respectively. The value of $a = 0$ corresponds to the uniform superlattice.

We search for stationary solutions of Eq. (1) in the form of $U = f(x)\exp(\mathrm{i}bz)$, where $f(x)$ is a complex function satisfying

$$bf = \frac{\partial^2 f}{\partial x^2} + [V(x)+\mathrm{i}W(x)]f + |f|^2 f. \quad (4)$$

The solutions of defect solitons are obtained numerically from Eq. (4) and shown in the next section. To elucidate the stability of defect solitons, we search for perturbed solutions of Eq. (1) in the form of $U(x,z) = [f(x)+u(x,z)+\mathrm{i}v(x,z)]\exp(\mathrm{i}bz)$, where the real part ($u(x,z)$) and the imaginary part ($v(x,z)$) of the perturbation can increase with a complex rate δ upon propagation. Linearization of Eq. (1) around the stationary solution $f(x)$ leads to the following eigenvalue problem:

$$\delta v = \frac{\partial^2 u}{\partial x^2} - bu + (Vu - Wv) + 2|f|^2 u + \{[Re(f(x))]^2 - [\mathrm{Im}(f(x))]^2\}u + 2\mathrm{Re}(f(x))\mathrm{Im}(f(x))v, \quad (5)$$

$$\delta u = -\frac{\partial^2 v}{\partial x^2} + bv - (Wu + Vv) - 2|f|^2 v + \{[\mathrm{Re}(f(x))]^2 - [\mathrm{Im}(f(x))]^2\}v - 2\mathrm{Re}(f(x))\mathrm{Im}(f(x))u. \quad (6)$$

The above eigenvalue problem is solved numerically to find the maximum value of $\mathrm{Re}(\delta)$. If $\mathrm{Re}(\delta) > 0$, solitons are unstable. Otherwise, they are stable.

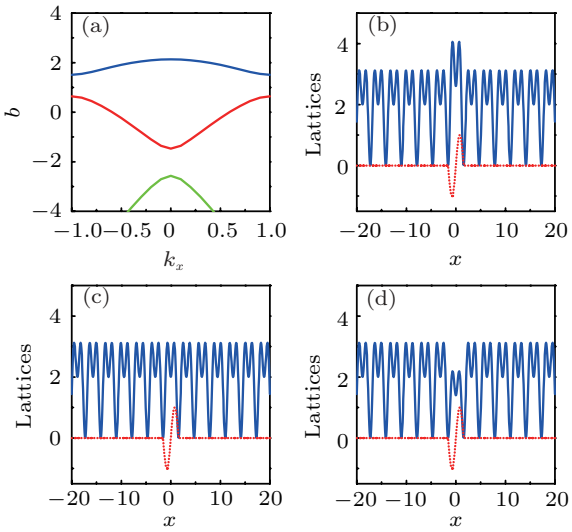


Fig. 1. (color online) (a) Band structure of the superlattice with $V_0 = 4$ and $\varepsilon = 0.5$. Lattice intensity profiles with $V_0 = 4$, $W_0 = 1$, and (b) $a = 0.3$, (c) $a = 0$, and (d) $a = -0.3$. Blue solid lines and red dotted lines represent the real and imaginary parts, respectively.

3. Defect solitons

In the superlattices with PT symmetric defects, we find two types of defect solitons. The first type is in-phase solitons, in which case the real part of optical field is symmetric and the imaginary part is antisymmetric. The other type is out-of-phase solitons, in which case the real part is antisymmetric and the imaginary part is symmetric. We find that in the first gap, out-of-phase solitons can exist stably for the positive and zero defects, whereas in-phase solitons can exist stably for negative defects. In the semi-infinite gap, we find in-phase solitons for positive, zero, and negative defects.

For the positive defect, we assume $a = 0.3$ and show the results in Figs. 2–4. Figure 2(a) shows that the power (defined as $P = \int_{-\infty}^{+\infty}|f(x)|^2\mathrm{d}x$) for both in-phase and out-of-phase solitons increases almost linearly with increasing propagation constant. There exists a cutoff point of the propagation constant for both the in-phase and out-of-phase solitons, below which the defect solitons vanish. The propagation constants of the cutoff point for in-phase solitons decrease with increasing W_0 and decreasing a, whereas the values of the cutoff point for out-of-phase solitons increase with increasing W_0 and a, as shown in Figs. 2(b) and 2(c). When $W_0 > 1.8$ with $a = 0.3$ or $a < 0.2$ with $W_0 = 1$, the cutoff point cannot exist in the semi-infinite gap. In the first gap, the cutoff point cannot exist for $W_0 > 1.25$ with $a = 0.3$ or $a < 0.1$ with $W_0 = 1$. We find that in-phase solitons are stable in the whole regime where solitons exist in the semi-infinite gap when $W_0 = 1$. Out-of-phase solitons are found to exist stably for the low propagation constants in the first gap, as shown in Fig. 2(d).

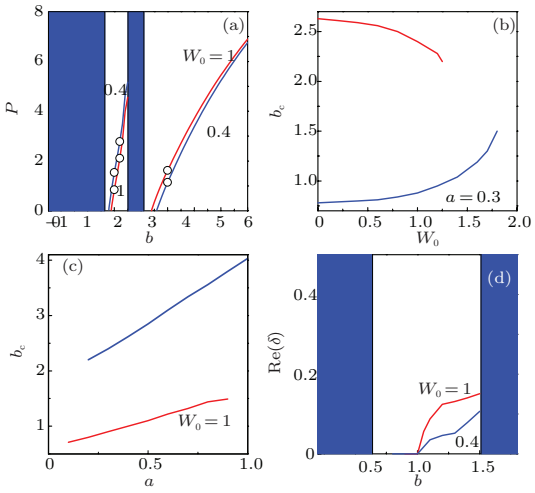


Fig. 2. (color online) (a) Soliton power versus propagation constants for different W_0. (b) Cutoff point b_c versus W_0. (c) Cutoff point b_c versus a for $W_0 = 1$. (c) Unstable growth rate Re(δ) versus propagation constants for out-of-phase solitons in the first gap with different W_0. In all panels except for panel (c), $V_0 = 4$ and $a = 0.3$.

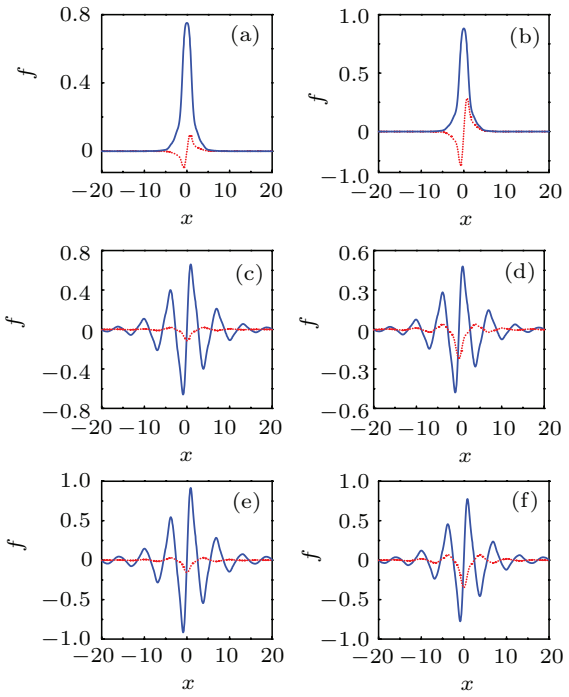


Fig. 3. (color online) Complex fields (blue solid line: real part; red dotted line: imaginary part) for in-phase solitons in the semi-infinite gap with (a) $b = 3$, $W_0 = 0.4$, (b) $b = 3$, $W_0 = 1$, and for out-of-phase solitons in the first gap with (c) $b = 1$, $W_0 = 0.4$, (d) $b = 1$, $W_0 = 1$, (e) $b = 1.5$, $W_0 = 0.4$, (f) $b = 1.5$, $W_0 = 1$. In all panels, $V_0 = 4$ and $a = 0.3$.

Figures 5–7 show the results for zero defects ($a = 0$). The properties of defect solitons with zero defects are similar to those with positive defects. From Figs. 5(a) and 5(b), we can see that the cutoff points approach the gap edges. For $W_0 = 0.4$, in-phase solitons are stable in the whole semi-infinite gap, whereas out-of-phase solitons are stable in the first gap for the low propagation constant. When $W_0 = 1$, both in-phase solitons and out-of-phase solitons are unstable in the semi-infinite gap and first gap, respectively, as shown in Fig. 5(b). Figures 6(a)–6(f) show the complex profiles of solitons for zero defects, which correspond to the circle symbols in Fig. 5(a). We can see that the field distributions are similar to those with positive defects. However, in-phase solitons in the semi-infinite gap have more peaks than those with positive defects. Figure 7 shows that in-phase solitons are stable in the whole semi-infinite gap, whereas out-of-phase solitons are stable at low propagation constants for $W_0 = 0.4$. For $W_0 = 1$, both in-phase solitons and out-of-phase solitons are unstable.

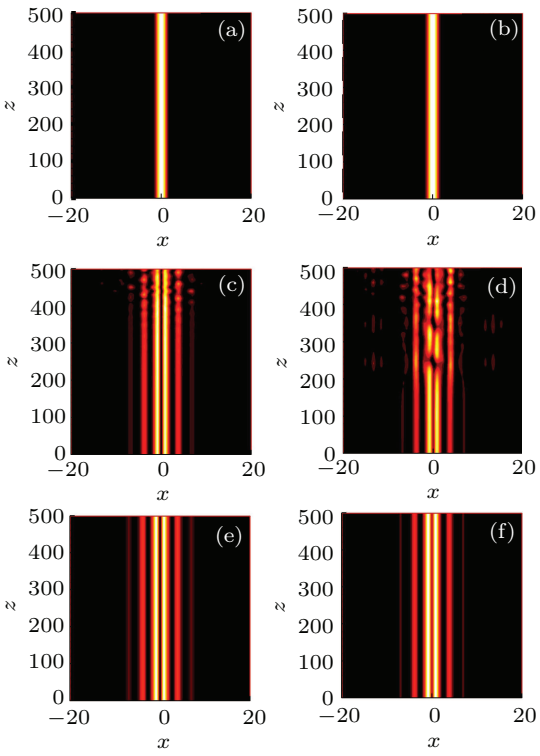


Fig. 4. (color online) (a)–(f) Evolutions of defect solitons corresponding to Figs. 3(a)–3(f), respectively.

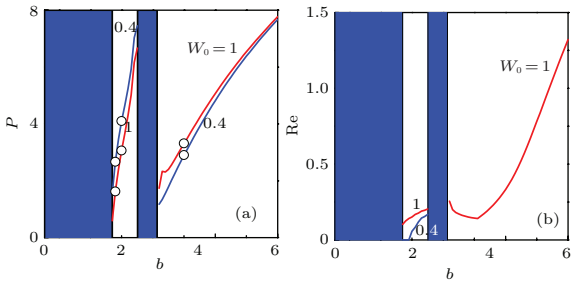


Fig. 5. (color online) (a) Soliton power versus propagation constants for different W_0. (b) Unstable growth rate Re(δ) versus propagation constants for the in-phase solitons in the semi-infinite gap, and for out-of-phase solitons in the first gap with different W_0. For all cases, $V_0 = 4$ and $a = 0$.

For the negative defect, we assume $a = -0.3$. The results are shown in Figs. 8–10. We find that the in-phase solitons in the semi-infinite gap are unstable in their whole existence region for $W_0 = 1$, and out-of-phase solitons in the first gap are unstable in their first gap for both $W_0 = 0.4$ and $W_0 = 1$ (see Fig. 8(c)). In the first gap, differing from positive and zero defects, in-phase solitons are stable in the whole existence region of both $W_0 = 0.4$ and $W_0 = 1$. We can see that there exists a cutoff point of propagation constant in the first gap,

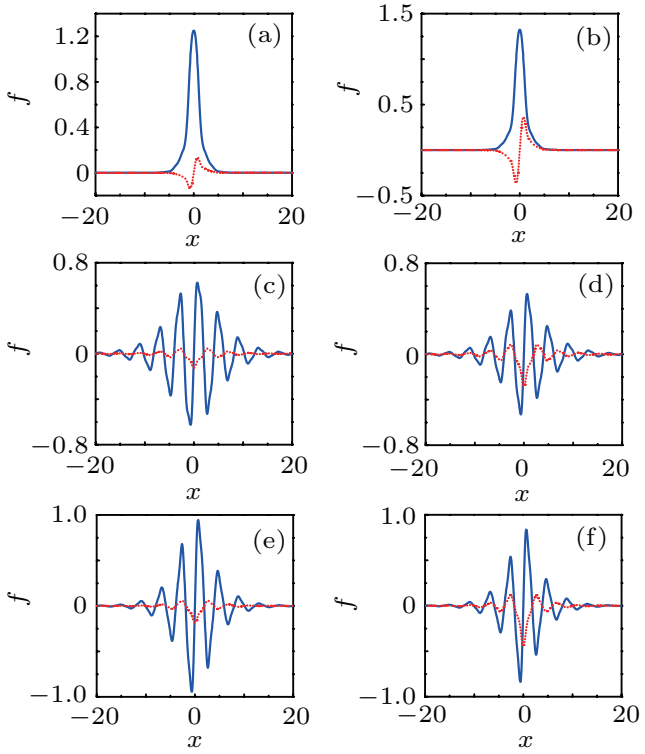


Fig. 6. (color online) Complex fields (blue solid line: real part; red dotted line: imaginary part) for in-phase solitons in the semi-infinite gap with (a) $b = 3$, $W_0 = 0.4$, (b) $b = 3$, $W_0 = 1$, and for out-of-phase solitons in the first gap with (c) $b = 0.8$, $W_0 = 0.4$, (d) $b = 0.8$, $W_0 = 1$, (e) $b = 1$, $W_0 = 0.4$, (f) $b = 1$, $W_0 = 1$, respectively. For all cases, $V_0 = 4$ and $a = 0$.

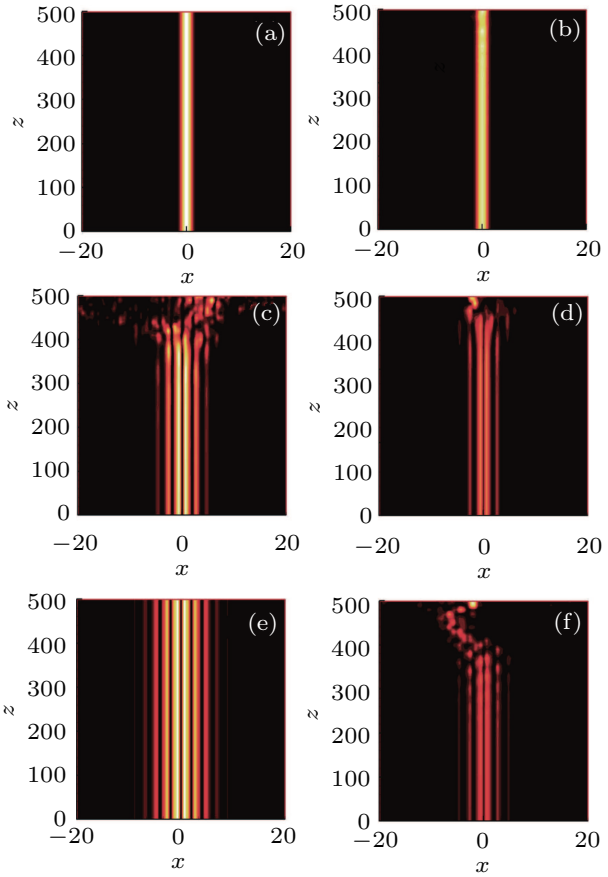


Fig. 7. (color online) (a)–(f) Evolutions of defect solitons corresponding to Figs. 6(a)–6(f), respectively.

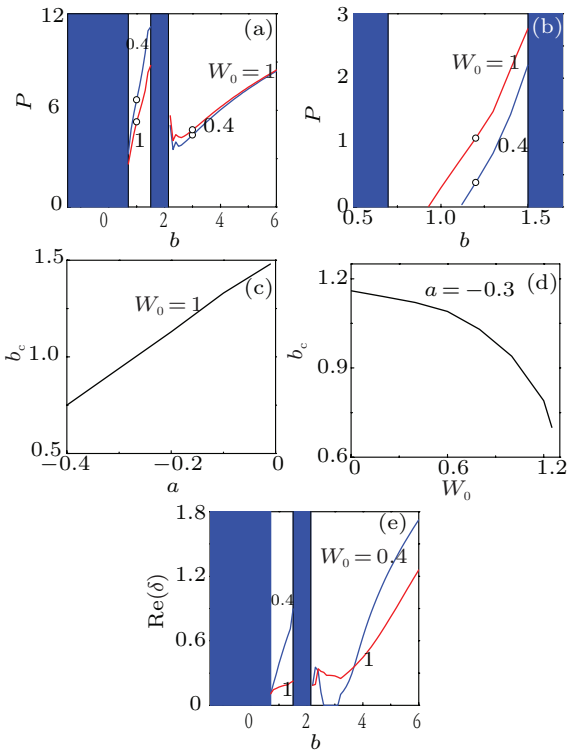


Fig. 8. (color online) (a) Soliton power versus propagation constants for in-phase solitons in the semi-infinite gap and out-of-phase solitons in the first gap with different W_0. (b) Soliton power versus propagation constants for in-phase solitons in the first gap with different W_0. (c) Cutoff point b_c versus W_0 for the in-phase solitons in the first gap. (d) Cutoff point b_c versus a for the in-phase solitons in the first gap with $W_0 = 1$. (e) Unstable growth rate $\mathrm{Re}(\delta)$ versus propagation constants for the in-phase solitons in the semi-infinite gap and out-of-phase solitons in the first gap with different W_0. For all cases, $V_0 = 4$ and $a = -0.3$ except for panel (d).

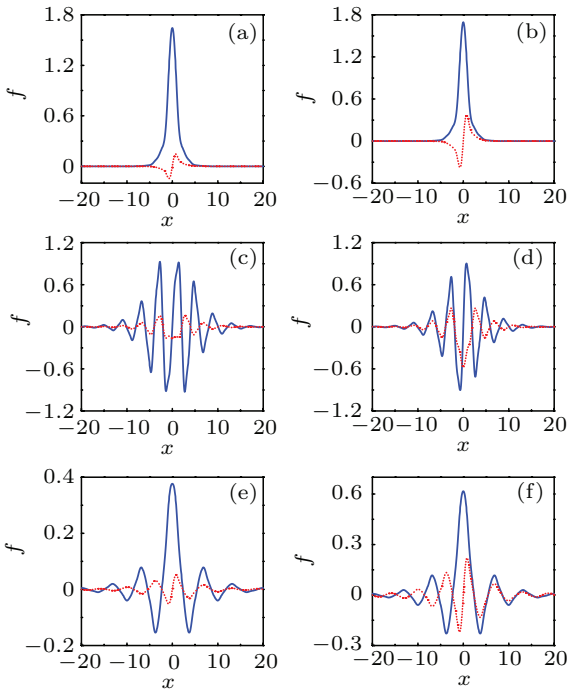


Fig. 9. (color online) Complex fields (blue solid line: real part; red dotted line: imaginary part) for in-phase solitons in the semi-infinite gap with (a) $b = 3$, $W_0 = 0.4$, (b) $b = 3$, $W_0 = 1$, and for out-of-phase solitons in the first gap with (c) $b = 1$, $W_0 = 0.4$, (d) $b = 1$, $W_0 = 1$, and for in-phase solitons in the first gap with (e) $b = 1.2$, $W_0 = 0.4$, (f) $b = 1.2$, $W_0 = 1$, respectively. For all cases, $V_0 = 4$ and $a = -0.3$.

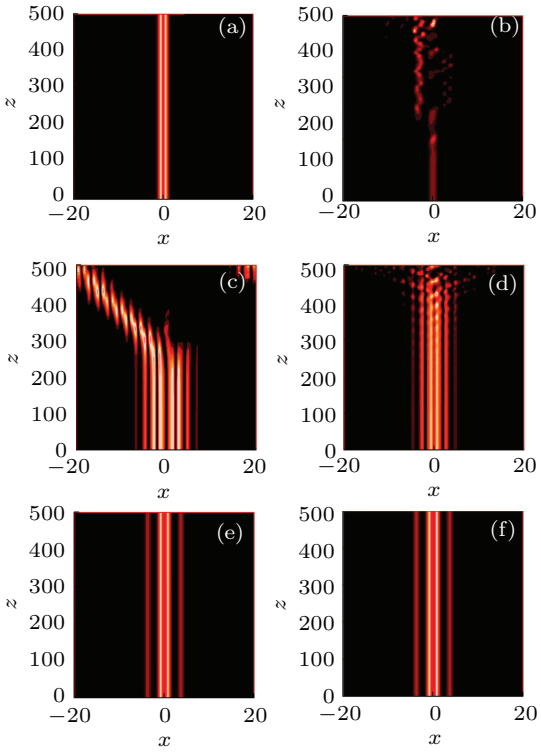


Fig. 10. (color online) (a)–(f) Evolutions of defect solitons corresponding to Figs. 9(a)–9(f), respectively.

above which in-phase solitons exist. This feature is similar to the case of positive defects. Figure 8(c) shows that the cutoff point shifts toward low propagation constants when increasing W_0 and the depth of negative defects $|a|$, and the existing region of in-phase solitons in the first gap increases too.

The examples of in-phase solitons in the semi-infinite gap as well as in-phase or out-of-phase solitons in the first gap are shown in Figs. 9(c)–9(f), and the corresponding propagations are shown in Fig. 10. One can see that in-phase solitons are unstable in the semi-infinite gap for $W_0 = 1$, whereas they are stable in the first gap.

4. Conclusions

In conclusion, we have studied the existence and stability of defect solitons supported by superlattices with parity-time symmetric defects. Unusual properties are found in this system. The solitons can be stable even though the intensities of imaginary parts of fields are close to the real parts. The imaginary parts of the PT symmetric defect potentials and the depth of the defect can drastically affect the existence and stability of defect solitons. For positive defects or zero defects, in-phase solitons can exist stably in the semi-infinite gap, and out-of-phase solitons can exist stably in the first gap. For negative defects, in-phase solitons exist stably in the first gap. Compared to the PT symmetric superlattices, the solitons in the lattices with PT symmetric defects can be stable with larger W_0.

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