Time-dependent Ginzburg–Landau equation for lattice hydrodynamic model describing pedestrian flow*

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A thermodynamic theory is formulated to describe the phase transition and critical phenomena in pedestrian flow. Based on the extended lattice hydrodynamic pedestrian model taking the interaction of the next–nearest–neighbor persons into account, the time-dependent Ginzburg–Landau (TDGL) equation is derived to describe the pedestrian flow near the critical point through the nonlinear analysis method. The corresponding two solutions, the uniform and the kink solutions, are given. The coexisting curve, spinodal line, and critical point are obtained by the first and second derivatives of the thermodynamic potential.

Keywords: pedestrian flow, lattice hydrodynamic model, time-dependent Ginzburg–Landau equation

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1. Introduction

In recent years, pedestrian flow has attracted considerable attention from physicists. The reason for the pedestrian study is that pedestrian movement is an important component in the analysis and design of transportation facilities, pedestrian walkways, traffic intersections, markets, and other public buildings.

The pedestrian flow is a kind of many-body system of strongly interacting persons. There are several models to study the pedestrian dynamics. Usually, pedestrian modeling is characterized by microscopic and macroscopic models. In the former, pedestrians are treated as individual particles. The evolution of the particles with time is determined by physical and social laws which describe the interaction among the particles as well as their interactions with the physical surrounding. Muramatsu *et al.*[1] studied the counter flow of pedestrians within an underpass by using the lattice gas model of biased-random walkers. Li *et al.*[2,3] investigated the self-organized phenomena generated by pedestrian counter flow in a channel and pedestrian counter flow on a crosswalk. Moreover, Chen *et al.*[4] improved the lattice gas model to describe the weaving pedestrian flow in subway stations. Helbing *et al.*[5] presented the social force in a molecular dynamic model to simulate the movement of people. According to Newton's second law of motion, they got the relationship between the movement of the person and the force, which contains the person's own force to the destination and the force to avoid colliding. Macroscopic models treat the whole crowd as an entity without considering the single individuals. It expresses the law of pedestrian movement through paradigms like fluid mechanics. This approach models the relation of the acceleration term, or mean velocity, to local flow conditions.[6,7]

Pedestrian dynamics has some similarities with traffic flow dynamics. Various models developed for the macroscopic traffic flow can be readily extended to capture the dynamics of pedestrian flow. The lattice hydrodynamic model, which was first proposed by Nagatani[8,9] is one of them. The macroscopic model incorporates the idea of the microscopic optimal velocity model, and is a simplified version of the macroscopic hydrodynamic model. Nagatani[10] extended the one-dimensional lattice hydrodynamic model to the two-dimensional one to investigate the two-way traffic flow. He derived the modified Korteweg-de Vries equation to describe the traffic jamming in the form of kink-antikink density waves from the two-dimensional model. Recently, Xue *et al.*[11] extended the model to the two-dimensional bidirectional pedestrian flow by taking four types of pedestrians into account. Moreover, they considered the effect of path change,[12] and the interaction of the next–nearest–neighbor pedestrians.[13]

The density waves described by the modified Korteweg-de Vries (mKdV) equation and Korteweg-de Vries (KdV) equation in the car following model and lattice hydrodynamic model were investigated frequently. Nagatani[8,9] first derived the time-dependent Ginzburg–Landau (TDGL) equation with two simple lattice hydrodynamic models around the crit-

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ical point by applying the reductive perturbation method, and he derived the thermodynamic potential for the traffic flow and showed that the jamming transition included metastability. However, for other lattice hydrodynamic models, there are rarely any reports on the TDGL equation. It is necessary to construct the traffic flow models by which one can derive the TDGL equation since the thermodynamic theory of jamming transition can be formulated by the TDGL equation. In the light of the traffic flow model, we intend to derive the TDGL equation for describing the pedestrian flow with certain conditions.

In this paper, based on the extended lattice hydrodynamic pedestrian model with consideration of the interaction of the next–nearest–neighbor persons, the linear stability theory is given and the neutral stability condition is obtained. Moreover, we make the nonlinear analysis near the critical point, so the TDGL equation and its corresponding soliton solution are obtained. The simulation results are given.

2. Model

We investigate the bidirectional pedestrian flow considering the next–nearest–neighbor pedestrians which was proposed by Wen *et al.*[13] The densities of eastbound, westbound, northbound, and southbound pedestrians at site (x,y) at time t are denoted as $\rho_{x+}(x,y,t)$, $\rho_{x-}(x,y,t)$, $\rho_{y+}(x,y,t)$, and $\rho_{y-}(x,y,t)$, respectively. The flux of eastbound, westbound, northbound, and southbound pedestrians at site (x,y) at time t are denoted as $Q_{x+}(x,y,t)$, $Q_{x-}(x,y,t)$, $Q_{y+}(x,y,t)$, and $Q_{y-}(x,y,t)$, respectively. The fraction of eastbound and westbound pedestrians is c, while the fraction of eastbound pedestrians from eastbound and westbound pedestrians is c_1. The fraction of northbound pedestrians from northbound and southbound pedestrians is c_2. The total average density is ρ_0. The continuity equations of the eastbound, westbound, northbound, and southbound pedestrians are given by

$$\begin{aligned}
&\partial_t\rho_{x+}(j,m,t)+cc_1\rho_0\\
&\times[Q_{x+}(j,m,t)-Q_{x+}(j-1,m,t)]=0, \qquad (1)\\
&\partial_t\rho_{x-}(j,m,t)+c(1-c_1)\rho_0\\
&\times[Q_{x-}(j,m,t)-Q_{x-}(j+1,m,t)]=0, \qquad (2)\\
&\partial_t\rho_{y+}(j,m,t)+(1-c)c_2\rho_0\\
&\times[Q_{y+}(j,m,t)-Q_{y+}(j,m-1,t)]=0, \qquad (3)\\
&\partial_t\rho_{y-}(j,m,t)+(1-c)(1-c_2)\rho_0\\
&\times[Q_{y-}(j,m,t)-Q_{y-}(j,m+1,t)]=0, \qquad (4)
\end{aligned}$$

where $\partial_t=\partial/\partial_t$. In the light of Nagatani's ideas, the flux along each direction is determined by the total optimal current with delay time τ. Here, the pedestrian flow on site (j,m) at time $t+\tau$ is determined by the nearest-neighbor pedestrian flow and the next-nearest-neighbor pedestrian flow at time t. Hence, we have

$$\begin{aligned}
Q_{x+}(j,m,t+\tau) &= cc_1[(1-\gamma)\rho_0V(\rho(j+1,m,t))\\
&\quad+\gamma\rho_0V(\rho(j+2,m,t))], \qquad (5)\\
Q_{x-}(j,m,t+\tau) &= c(1-c_1)[(1-\gamma)\rho_0V(\rho(j-1,m,t))\\
&\quad+\gamma\rho_0V(\rho(j-2,m,t))], \qquad (6)\\
Q_{y+}(j,m,t+\tau) &= (1-c)c_2[(1-\gamma)\rho_0V(\rho(j,m+1,t))\\
&\quad+\gamma\rho_0V(\rho(j,m+2,t))], \qquad (7)\\
Q_{y-}(j,m,t+\tau) &= (1-c)(1-c_2)[(1-\gamma)\rho_0V(\rho(j,m-1,t))\\
&\quad+\gamma\rho_0V(\rho(j,m-2,t))], \qquad (8)
\end{aligned}$$

where τ denotes the delay time, which is the time taken for the pedestrian current to reach the optimal current, and τ is the inverse of the sensitivity a, γ is the weighting value, and $0<\gamma\leq 0.5$. The optimal velocity function $V(\rho(j,m,t))$ is given as[10]

$$\begin{aligned}
V(\rho(j,m,t)) &= \tanh\left[\frac{2}{\rho_0}-\frac{\rho(j,m,t)}{\rho_0^2}-\frac{1}{\rho_c}\right]\\
&\quad+\tanh\left(\frac{1}{\rho_c}\right), \qquad (9)
\end{aligned}$$

where ρ_c is the inverse of the safety distance.

Inserting Eqs. (5)–(8) into Eqs. (1)–(4), the total density $\rho(j,m,t)=\rho_{x+}(j,m,t)+\rho_{x-}(j,m,t)+\rho_{y+}(j,m,t)+\rho_{y-}(j,m,t)$, we obtain

$$\begin{aligned}
&\rho(j,m,t+2\tau)-\rho(j,m,t+\tau)+\tau\rho_0^2(cc_1)^2\\
&\times[V(\rho(j+1,m,t))-V(\rho(j,m,t))]\\
&+\gamma\tau\rho_0^2(cc_1)^2[V(\rho(j+2,m,t))\\
&-2V(\rho(j+1,m,t))+V(\rho(j,m,t))]\\
&+\tau\rho_0^2[c(1-c_1)]^2[V(\rho(j-1,m,t))-V(\rho(j,m,t))]\\
&+\gamma\tau\rho_0^2[c(1-c_1)]^2[V(\rho(j-2,m,t))\\
&-2V(\rho(j-1,m,t))+V(\rho(j,m,t))]\\
&+\tau\rho_0^2[(1-c)c_2]^2[V(\rho(j,m+1,t))-V(\rho(j,m,t))]\\
&+\gamma\tau\rho_0^2[(1-c)c_2]^2[V(\rho(j,m+2,t))\\
&-2V(\rho(j,m+1,t))+V(\rho(j,m,t))]\\
&+\tau\rho_0^2[(1-c)(1-c_2)]^2[V(\rho(j,m-1,t))-V(\rho(j,m,t))]\\
&+\gamma\tau\rho_0^2[(1-c)(1-c_2)]^2[V(\rho(j,m-2,t))\\
&-2V(\rho(j,m-1,t))+V(\rho(j,m,t))]=0. \qquad (10)
\end{aligned}$$

As $c=0$, $c_1=1$, and $\gamma=0$, or $c=0$, $c_2=1$, and $\gamma=0$, equation (10) reduces to the density equation of the one-dimensional lattice hydrodynamic equation.[9] As $c_1=1, c_2=1$, and $\gamma=0$, it becomes the density equation of the two-dimensional lattice hydrodynamic equation with unidirectional flow.[10] As $\gamma=0$, it reduces to the density equation of the two-dimensional bidirectional pedestrian lattice hydrodynamic equation.[11]

3. Linear stability analysis

We use the linear stability method to describe the pedestrian flow described by Eq. (10). We consider the stability of the uniform pedestrian flow with constant density ρ_0 and constant $V(\rho_0)$ along any direction. Obviously, the steady-state solution for Eq. (10) is given by

$$\begin{aligned} \rho(j,m,t) &= \rho_0, \\ v_{x+}(j,m,t) &= v_{x-}(j,m,t) = v_{y+}(j,m,t) \\ &= v_{y-}(j,m,t) = V(\rho_0). \end{aligned} \tag{11}$$

Supposing $y(j,m,t)$ to be a small deviation from the steady-state solution

$$\rho(j,m,t) = \rho_0 + y(j,m,t). \tag{12}$$

Substituting Eqs. (11)–(12) into Eq. (10), then we obtain the linearized equation

$$\begin{aligned} & y(j,m,t+2\tau) - y(j,m,t+\tau) \\ & + \tau\rho_0^2 (cc_1)^2 V'(\rho_0)[y(j+1,m,t) - y(j,m,t)] \\ & + \gamma\tau\rho_0^2 (cc_1)^2 V'(\rho_0)[y(j+2,m,t) \\ & - 2y(j+1,m,t) + y(j,m,t)] \\ & + \tau\rho_0^2 [c(1-c_1)]^2 V'(\rho_0)[y(j-1,m,t) - y(j,m,t)] \\ & + \gamma\tau\rho_0^2 [c(1-c_1)]^2 V'(\rho_0)[y(j-2,m,t) - 2y(j-1,m,t) \\ & + y(j,m,t)] + \tau\rho_0^2 [(1-c)c_2]^2 V'(\rho_0) \\ & \times [y(j,m+1,t) - y(j,m,t)] + \gamma\tau\rho_0^2 [(1-c)c_2]^2 V'(\rho_0) \\ & \times [y(j,m+2,t) - 2y(j,m+1,t) + y(j,m,t)] \\ & + \tau\rho_0^2 [(1-c)(1-c_2)]^2 V'(\rho_0)[y(j,m-1,t)) - y(j,m,t)] \\ & + \gamma\tau\rho_0^2 [(1-c)(1-c_2)]^2 V'(\rho_0)[y(j,m-2,t) \\ & - 2y(j,m-1,t) + y(j,m,t)] = 0, \end{aligned} \tag{13}$$

where $V'(\rho_0) = \mathrm{d}V(\rho)/\mathrm{d}\rho|_{\rho=\rho_0}$ is the derivative of the optimal velocity $V(\rho_0)$ at $\rho = \rho_0$.

Expanding $y(j,m,t) = \exp[\mathrm{i}k(j+m) + zt]$, $z = z_1(\mathrm{i}k) + z_2(\mathrm{i}k)^2 + \ldots$, and then inserting them into Eq. (13), and keeping the first- and second-order terms of $\mathrm{i}k$, we get the coefficients of z, respectively

$$\begin{aligned} z_1 &= -\rho_0^2 V'\{(cc_1)^2 - [c(1-c_1)]^2 \\ &\quad + [(1-c)c_2]^2 - [(1-c)(1-c_2)]^2\}, \end{aligned} \tag{14}$$

$$\begin{aligned} z_2 &= -\frac{3}{2}\tau(\rho_0^2 V')^2\{(cc_1)^2 - [c(1-c_1)]^2 \\ &\quad + [(1-c)c_2]^2 - [(1-c)(1-c_2)]^2\}^2 \\ &\quad - \left(\frac{1}{2} + \gamma\right)\rho_0^2 V'\{(cc_1)^2 + [c(1-c_1)]^2 \\ &\quad + [(1-c)c_2]^2 + [(1-c)(1-c_2)]^2\}. \end{aligned} \tag{15}$$

If $z_2 > 0$, the uniform flow is always stable. If $z_2 < 0$, the uniform steady flow is unstable for long wavelength modes. Therefore, we have the unstable condition

$$\tau > -\frac{(1+2\gamma)\{(cc_1)^2 + [c(1-c_1)]^2 + [(1-c)c_2]^2 + [(1-c)(1-c_2)]^2\}}{3\rho_0^2 V'\{(cc_1)^2 - [c(1-c_1)]^2 + [(1-c)c_2]^2 - [(1-c)(1-c_2)]^2\}^2}. \tag{16}$$

The neutral stability condition is given by

$$\tau = -\frac{(1+2\gamma)\{(cc_1)^2 + [c(1-c_1)]^2 + [(1-c)c_2]^2 + [(1-c)(1-c_2)]^2\}}{3\rho_0^2 V'\{(cc_1)^2 - [c(1-c_1)]^2 + [(1-c)c_2]^2 - [(1-c)(1-c_2)]^2\}^2}. \tag{17}$$

4. Nonlinear analysis and TDGL equation

We use the long wavelength modes to derive the TDGL equation for describing the pedestrian flow on coarse-grained scale. We analyze the slowly varying behavior at long waves near the critical point. First, we give slow scales for space variables j, m, and time variable t, and define the slow variables X and T

$$X = \varepsilon(j + m + bt), \quad T = \varepsilon^3 t, \quad 0 < \varepsilon \ll 1, \tag{18}$$

where b is a constant to be determined later. Given

$$\rho(j,m,t) = \rho_c + \varepsilon R(X,T). \tag{19}$$

Next, substituting Eqs. (18)–(19) into Eq. (10) and making the Taylor expansion to the fifth-order of ε, we obtain the following nonlinear partial differential equation

$$\begin{aligned} & \varepsilon^2 f_1 \partial_X R + \varepsilon^3 f_2 \partial_X^2 R + \varepsilon^4 (\partial_T R + f_3 \partial_X^3 R + f_4 \partial_X R^3) \\ & + \varepsilon^5 (f_5 \partial_X \partial_T R + f_6 \partial_X^4 R + f_7 \partial_X^2 R^3) = 0. \end{aligned} \tag{20}$$

Here, the coefficients f_i are given in Table 1.

For convenience, in the Table 1 and mentioned later, we denote

$$V' = \frac{\mathrm{d}V(\rho)}{\mathrm{d}\rho}|_{\rho=\rho_c}, \quad V''' = \frac{\mathrm{d}^3 V(\rho)}{\mathrm{d}\rho^3}|_{\rho=\rho_c},$$

$g = (cc_1)^2 - [c(1-c_1)]^2 + [(1-c)c_2]^2 - [(1-c)(1-c_2)]^2$, $f = (cc_1)^2 + [c(1-c_1)]^2 + [(1-c)c_2]^2 + [(1-c)(1-c_2)]^2$. Now, we study the pedestrian flow near the critical point $\tau = (1+\varepsilon^2)\tau_c$, $\tau_c = (1+2\gamma)f/3gb$. By taking $b = -g\rho_c^2 V'$, the second-order terms of ε of Eq. (20) are eliminated. Equa-

tion (20) is rewritten as

$$\varepsilon^4\partial_T R = \varepsilon^4 m_1\partial_X^3 R+\varepsilon^4 m_2\partial_X R^3-\varepsilon^3 m_3\partial_X^2 R -\varepsilon^5 m_4\partial_X^4 R-\varepsilon^5 m_5\partial_X^2 R^3. \quad (21)$$

Here, the coefficients m_i are given in Table 2.

Table 1. The coefficients f_i of the model.

f_1	$b+g\rho_c^2V'$
f_2	$\frac{3b^2\tau}{2}+\frac{(1+2\gamma)f\rho_c^2V'}{2}$
f_3	$\frac{7b^3\tau^2}{6}+\frac{(1+6\gamma)g\rho_c^2V'}{6}$
f_4	$\frac{g\rho_c^2V'''}{6}$
f_5	$3b\tau$
f_6	$\frac{5b^4\tau^3}{8}+\frac{(1+14\gamma)f\rho_c^2V'}{24}$
f_7	$\frac{(1+2\gamma)f\rho_c^2V'''}{12}$

Table 2. The coefficients m_i of the model.

m_1	$\frac{-7(1+2\gamma)^2f^2+9(1+6\gamma)g^2}{54g}(-g\rho_c^2V')$
m_2	$-\frac{g\rho_c^2V'''}{6}$
m_3	$\frac{(-\rho_c^2V')}{2}(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)$
m_4	$\frac{-23(1+2\gamma)^3f^2+27(16\gamma^2+6\gamma+1)g^2}{216g^2}f(-\rho_c^2V')$
m_5	$-\frac{1+2\gamma}{12}f\rho_c^2V'''$

In order to derive the TDGL equation, we should define

$$\frac{m_4}{m_1}=\frac{m_5}{m_2}=k=\frac{(1+2\gamma)f}{2g}.$$

By transforming variables X and T to variables $x=\varepsilon^{-1}X$ and $t=\varepsilon^{-3}T$, and making $S(x,t)=\varepsilon R(X,T)$, equation (21) is rewritten as follows:

$$\partial_t S = m_1\partial_x^3 S+m_2\partial_x S^3-\frac{(-\rho_c^2V')}{2} \times(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)\partial_x^2 S -km_1\partial_x^4 S-km_2\partial_x^2 S^3. \quad (22)$$

By adding term

$$\frac{(-\rho_c^2V')}{2k}[-3g^2\rho_c^2V'\tau-(1+2\gamma)f]\partial_x S$$

to both the left- and right-hand sides and performing Galilean transformation $t_1=t$ and

$$x_1=x-\frac{(-\rho_c^2V')}{2k}[-3g^2\rho_c^2V'\tau-(1+2\gamma)f]t$$

in Eq. (22), then we obtain

$$\partial_{t_1}S = (\partial_{x_1}-k\partial_{x_1}^2)\{m_1\partial_{x_1}^2S+\frac{(-\rho_c^2V')}{2k} \times[-3g^2\rho_c^2V'\tau-(1+2\gamma)f]S+m_2S^3\}. \quad (23)$$

We define the thermodynamic potential

$$\phi(S)\equiv-\frac{-\rho_c^2V'}{4k}[-3g^2\rho_c^2V'\tau-(1+2\gamma)f]S^2-\frac{m_2}{4}S^4. \quad (24)$$

By taking Eq. (24) into Eq. (23), the TDGL equation is derived

$$\partial_{t_1}S=-(\partial_{x_1}-k\partial_{x_1}^2)\frac{\delta\Phi(S)}{\delta S}$$

with

$$\Phi(S)\equiv\int dx_1[\frac{m_1}{2}(\partial_{x_1}S)^2+\phi(S)], \quad (25)$$

where $\phi(S)$ is given by Eq. (24).

Except for the trivial solution $S=0$, equation (25) has two steady-state solutions: one is the uniform solution

$$S(x_1,t_1)=\pm\left[\frac{3(-\rho_c^2V')(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)}{kg\rho_c^2V'''}\right]^{1/2}, \quad (26)$$

and the other is the kink solution

$$S(x_1,t_1) = \pm\left[\frac{3(-\rho_c^2V')(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)}{kg\rho_c^2V'''}\right]^{1/2} \times\tanh\left\{\left[\frac{135(1+2\gamma)g^2}{54g^2(14\gamma^2+7\gamma+1)-44f^2(1+2\gamma)^3} \times\left(\frac{-3g^2\rho_c^2V'}{(1+2\gamma)f}V'\tau-1\right)\right]^{1/2}(x_1-x_0)\right\}, \quad (27)$$

where x_0 is a constant. Equation (27) represents the coexisting phase.

We can obtain the coexisting curve, the spinodal line and the critical point according to Eq. (24). The condition of the coexisting curve is given as

$$\frac{\partial\phi}{\partial S}=0 \quad \text{and} \quad \frac{\partial^2\phi}{\partial S^2}>0. \quad (28)$$

Hence, inserting Eq. (24) into Eq. (28), we obtain the coexisting curve in terms of the original parameters

$$\rho_{co}=\rho_c\pm\left[\frac{3(-\rho_c^2V')(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)}{kg\rho_c^2V'''}\right]^{1/2}. \quad (29)$$

The condition of the spinodal line is given as

$$\frac{\partial^2\phi}{\partial S^2}=0. \quad (30)$$

Inserting Eq. (24) into Eq. (30), we have the spinodal line in terms of the original parameters

$$\rho_{sp}=\rho_c\pm\left[\frac{(-\rho_c^2V')(-3g^2\rho_c^2V'\tau-(1+2\gamma)f)}{kg\rho_c^2V'''}\right]^{1/2}. \quad (31)$$

The condition of the critical point is given as

$$\frac{\partial\phi}{\partial S}=0 \quad \text{and} \quad \frac{\partial^2\phi}{\partial S^2}=0. \quad (32)$$

Inserting Eq. (24) into Eq. (32), we obtain the critical point in terms of the original parameters

$$\rho=\rho_c \quad \text{and} \quad \tau_c=\frac{(1+2\gamma)f}{3g^2(-\rho_c^2V')}. \quad (33)$$

Substituting g and f into Eq. (33), we have the same result as Eq. (17).

5. Simulation

In Fig. 1, three pairs of curves based on the theoretical analysis are plotted corresponding to the situation of $\gamma = 0$, 0.1, 0.2, and $c = c_1 = c_2 = 0.2$ and $\rho_0 = \rho_c = 0.2$. The dashed line represents the neutral line given by Eq. (17), the dotted line represents the coexisting line given by Eq. (29), and the solid curve indicates the spinodal curve given by Eq. (31). The apex of each line indicates the existence of a critical point at $\rho = \rho_c = 0.2$ and $a = a_c$. As $a > a_c$, the uniform state irrespective of vehicle density is always linearly stable. The region of stable pedestrian flow is above each neutral stability line and the jam will not appear. Below each line, the pedestrian flow is unstable and the jam emerges. As $\gamma = 0$, the lines are consistent with those in Ref. [11]. From Fig. 1, it can be found that the unstable region shrinks more and more with increasing the parameter γ, which means that considering the next–nearest–neighbor persons has a significant effect on the stability of uniform pedestrian flow.

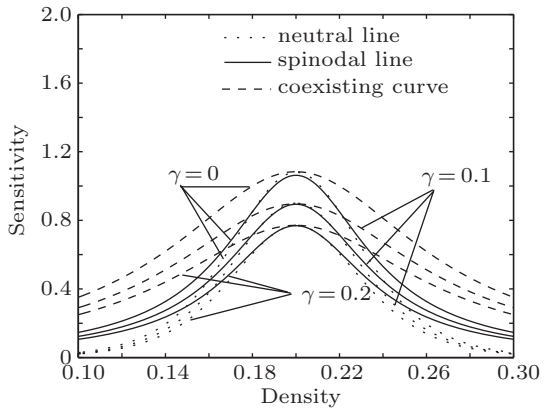


Fig. 1. The phase diagram of the models according to different values of parameter γ.

6. Summary

We have applied the thermodynamic theory to describe the jamming transition in the extended lattice hydrodynamic pedestrian model considering the interaction of the next-nearest-neighbor persons. We have obtained the neutral stability line and the critical point by using the linear stability theory. Furthermore, we have derived the TDGL equation to describe pedestrian behavior near the neutral stability line and the critical point by applying the reductive perturbation method. The corresponding two steady-state solutions except for a trivial solution have been obtained, and the spinodal line and the critical point have been calculated according to the TDGL equation.

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