Leader–following consensus control for networked multi-teleoperator systems with interval time-varying communication delays*

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We study the leader–following consensus stability and stabilization of networked multi-teleoperator systems with interval time-varying communication delays. With the construction of a suitable Lyapunov–Krasovskii functional and the utilization of the reciprocally convex approach, novel delay-dependent consensus stability and stabilization conditions for the systems are established in terms of linear matrix inequalities, which can easily be solved by various effective optimization algorithms. One illustrative example is given to illustrate the effectiveness of the proposed methods.

Keywords: multi-teleoperator systems, interval time-varying delay, leader–following consensus problem, Lyapunov method

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1. Introduction

The multi-agent model has received much attention due to its wide applications in many fields, such as biology, physics, robotics, and power grids.[1–6] One of the main problems of the model is the concurrence of a group of agents on their states with the leader by interaction. To wit, the problem is a consensus problem with a leader, and is called the leader–following consensus problem or consensus regulation. Recently, the consensus problem has been studied in various fields, such as groups of mobile autonomous agents,[2] networked control systems,[3] and other applications.[4–10] In Refs. [11] and [12], the consensus problems for high-order multi-agent systems were studied.

During the past few years, various approaches to creating sufficient conditions for bilateral teleoperation systems (BTSs) have been investigated in the literature.[13–16] In Ref. [13], by using a two-port time-domain passivity approach, a passive bilateral control scheme for a teleoperator with interval time-varying communication delay was proposed. Based on a novel adaptive coordination architecture, a passive coordination control for bilateral teleoperators was developed to state synchronize the master and the slave in free motion.[14] In Ref. [15], a novel control framework for the bilateral teleoperation of a pair of multi-degree-of-freedom nonlinear robotics systems under constant communication delays was proposed. A new adaptive neural network control architecture was introduced to achieve transparency and stability for a teleoperation system with time-delay in a communication channel.[16] In Ref. [17], based on the wave transformation technique, time-delayed BTSs were studied.

In the real world, many practical systems have time delays due to the finite speed of information processing. It is well known that a time delay often causes undesirable dynamic behaviors such as oscillation, performance degradation, and instability of the system. Thus, it is necessary to study various systems with time delays, see Refs. [18]–[23] for examples. Moreover, in Ref. [24], the sufficient conditions for the stability of the stochastic differential delay equations with nonlinear impulsive effects were studied.

It should be pointed out that analyzing the master–slave synchronization problem of the BTSs with time delays can be regarded as investigating the stability of the systems. Very recently, the networked synchronization of a class of master–slave chaotic systems was investigated.[25] However, a time delay in the communication channel has not been considered in Ref. [25]. Therefore, it is desirable to investigate the problem for multi-teleoperators with communication delays.

In this paper, novel delay-dependent consensus stability and stabilization for networked multi-teleoperator systems (NMTSs) with interval time-varying communication delays are studied. In Fig. 1, a model of NMTSs through a network, such as the internet,[26] is drawn based on the multi-agent scheme and the graph theory. The multi-agent scheme is utilized in the structure of the system such as leader ↔

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master and agent ↔ slave. The interconnection information of the leader and each agent is described by the graph theory. In general, the networked scheme has structurally interval time-varying delays.[27] Since the delay-dependent analysis utilizes the time-delay information, the delay-dependent analysis has received more attention than the delay-independent one.[28] That is, the former is generally less conservative than the latter. By the construction of a suitable Lyapunov–Krasovskii functional and the utilization of the reciprocally convex approach,[29] a leader–following consensus delay-dependent criterion for NMTSs with interval time-varying delays and a corresponding controller design method are derived in terms of linear matrix inequalities (LMIs), which can be formulated as convex optimization algorithms that are amenable to the computer solution.[30] One illustrative example is included to show the effectiveness of the proposed methods.

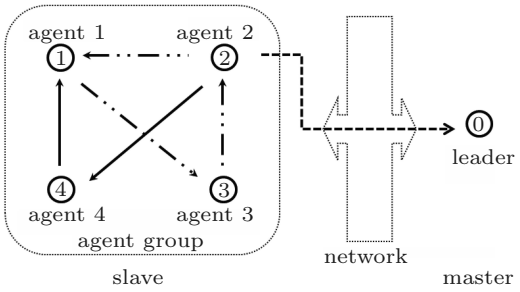


Fig. 1. Example of the networked multi-teleoperator scheme.

The following notations are used. $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. For symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, $\boldsymbol{X}>\boldsymbol{Y}$ ($\boldsymbol{X}\geq\boldsymbol{Y}$) means that the matrix $\boldsymbol{X}-\boldsymbol{Y}$ is positive definite (nonnegative). $\boldsymbol{X}^{\perp}$ denotes a basis for the null space of $\boldsymbol{X}$, and $\boldsymbol{I}_n$, $\boldsymbol{0}_n$, and $\boldsymbol{0}_{m\times n}$ denote the $n\times n$ identity matrix, $n\times n$, and $m\times n$ zero matrices, respectively. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix norm, diag$\{\cdots\}$ denotes the block diagonal matrix, and $\star$ represents the elements below the main diagonal of a symmetric matrix.

2. Problem statements

The interaction topology of a network of agents is represented using a directed graph (digraph) $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$ and edges $\mathcal{E}=\{(i,j):i,j\in\mathcal{V}\}\subset\mathcal{V}\times\mathcal{V}$. An adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ of the digraph $\mathcal{G}$ is the matrix with nonnegative elements satisfying $a_{ii}=0$ and $a_{ij}\geq 0$. If there is an edge between i and j, then $a_{ij}>0\Leftrightarrow(i,j)\in\mathcal{E}$. The digraph $\mathcal{G}$ is said to be undirected if $(i,j)\in\mathcal{E}\Leftrightarrow(j,i)\in\mathcal{E}$. The set of neighbors of agent i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$. The degree of node i is denoted by $d(i)=\sum_{j\in\mathcal{N}_i}a_{ij}$, and the degree matrix of digraph $\mathcal{G}$ is diagonal and defined as $\mathcal{D}=\text{diag}\{d(1),\ldots,d(N)\}$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. More details can be found in Ref. [31].

Consider the NMTSs with the following dynamics of agent i:

$$\dot{\boldsymbol{z}}_i(t)=\boldsymbol{F}\boldsymbol{z}_i(t)+\boldsymbol{G}\boldsymbol{u}_i(t),\quad i\in\mathcal{V}=\{1,2,\ldots,N\},\tag{1}$$

where $\boldsymbol{z}_i(t)\in\mathbb{R}^n$ and $\boldsymbol{u}_i(t)\in\mathbb{R}^l$ are the state vector and the consensus protocol vector of agent i, respectively, and $\boldsymbol{F}\in\mathbb{R}^{n\times n}$ and $\boldsymbol{G}\in\mathbb{R}^{n\times l}$ are known constant matrices.

A consensus algorithm[25] can be described as

$$\begin{aligned}\boldsymbol{u}_i(t)&=-\boldsymbol{K}\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{z}_i(t)-\boldsymbol{z}_j(t))-\boldsymbol{K}(b_i(\boldsymbol{z}_i(t)-\boldsymbol{z}_0(t))),\\ i\in\mathcal{V}&=\{1,2,\ldots,N\},\end{aligned}\tag{2}$$

where $\boldsymbol{K}\in\mathbb{R}^{l\times n}$ is a protocol gain matrix, $\boldsymbol{z}_0(t)\in\mathbb{R}^n$ is the state vector of the leader, and a_{ij} and b_i are the interconnection weights

$$\begin{cases}a_{ij}>0, & \text{if } i \text{ is connected to } j,\\ a_{ij}=0, & \text{otherwise},\\ b_i>0, & \text{if } i \text{ is connected to the leader},\\ b_i=0, & \text{otherwise}.\end{cases}$$

Based on Eq. (2), a consensus algorithm with time-varying communication delay is proposed

$$\begin{aligned}&\boldsymbol{u}_i(t)\\ &=-\boldsymbol{K}\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{z}_i(t-h(t))-\boldsymbol{z}_j(t-h(t)))-\boldsymbol{K}b_i(\boldsymbol{z}_i(t)-\boldsymbol{z}_0(t)),\\ &i\in\mathcal{V}=\{1,2,\ldots,N\}.\end{aligned}\tag{3}$$

Here, $h(t)$ is an interval time-varying continuous function satisfying

$$0\leq h_{\mathrm{m}}\leq h(t)\leq h_{\mathrm{M}},\quad \dot{h}(t)\leq h_{\mathrm{D}},$$

where h_{m}, h_{M}, and h_{D} are known constant scalars.

Let us define $\boldsymbol{x}_i(t)=\boldsymbol{z}_i(t)-\boldsymbol{z}_0(t)$, which is the state error between agent i and the leader. Since the following equality with the dynamics of the leader $\dot{\boldsymbol{z}}_0(t)=\boldsymbol{F}\boldsymbol{z}_0(t)$ holds:

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t)&=\dot{\boldsymbol{z}}_i(t)-\dot{\boldsymbol{z}}_0(t)\\ &=\boldsymbol{F}\boldsymbol{z}_i(t)+\boldsymbol{G}\boldsymbol{u}_i(t)-\boldsymbol{F}\boldsymbol{z}_0(t)\\ &=\boldsymbol{F}(\boldsymbol{z}_i(t)-\boldsymbol{z}_0(t))-\boldsymbol{G}\boldsymbol{K}b_i(\boldsymbol{z}_i(t)-\boldsymbol{z}_0(t))\\ &\quad-\boldsymbol{G}\Big\{\boldsymbol{K}\sum_{j\in\mathcal{N}_i}a_{ij}\big[\boldsymbol{z}_i(t-h(t))-\boldsymbol{z}_j(t-h(t))\\ &\quad+\boldsymbol{z}_0(t-h(t))-\boldsymbol{z}_0(t-h(t))\big]\Big\},\end{aligned}$$

from Eqs. (1) and (3), the NMTSs with the error dynamics for agent i can be written as

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t)&=(\boldsymbol{F}-\boldsymbol{G}\boldsymbol{K}b_i)\boldsymbol{x}_i(t)\\ &\quad-\boldsymbol{G}\boldsymbol{K}\sum_{j\in\mathcal{N}_i}a_{ij}(\boldsymbol{x}_i(t-h(t))-\boldsymbol{x}_j(t-h(t))),\\ i\in\mathcal{V}&=\{1,2,\ldots,N\}.\end{aligned}\tag{4}$$

For the convenience of the consensus analysis for system (4), the following Kronecker product and its properties are used.

Lemma 1[32] Let $\otimes$ denote the Kronecker product. Then, the following properties of the Kronecker product are easily established:

(i) $(\alpha \boldsymbol{A}) \otimes \boldsymbol{B} = \boldsymbol{A} \otimes (\alpha \boldsymbol{B})$,

(ii) $(\boldsymbol{A}+\boldsymbol{B}) \otimes \boldsymbol{C} = \boldsymbol{A} \otimes \boldsymbol{C} + \boldsymbol{B} \otimes \boldsymbol{C}$,

(iii) $(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = (\boldsymbol{AC}) \otimes (\boldsymbol{BD})$.

Let us define

$$\boldsymbol{x}(t) = [\boldsymbol{x}_1^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

Then, system (4) can be rewritten in the matrix form as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{F} - \mathcal{B} \otimes \boldsymbol{GK})\boldsymbol{x}(t) - (\mathcal{L} \otimes \boldsymbol{GK})\boldsymbol{x}(t-h(t)), \quad (5)$$

where

$$\begin{aligned} \mathcal{A} &= [a_{ij}]_{N\times N}, \\ \mathcal{D} &= \mathrm{diag}\Big\{ \sum_{j\in\mathcal{N}_1} a_{1j}, \ldots, \sum_{j\in\mathcal{N}_N} a_{Nj} \Big\}, \\ \mathcal{L} &= \mathcal{D} - \mathcal{A}, \\ \mathcal{B} &= \mathrm{diag}\{b_1, \ldots, b_N\}. \end{aligned}$$

The aim of this paper is to design a delay-dependent consensus control (in other words, to design a protocol gain $\boldsymbol{K}$ which guarantees the asymptotic stability) of the NMTSs (5). This means that protocol $\boldsymbol{u}_i(t)$ solves the consensus problem if and only if the states of the agents satisfy

$$\lim_{t\to\infty} \|\boldsymbol{z}_i(t) - \boldsymbol{z}_0(t)\| = \lim_{t\to\infty} \|\boldsymbol{x}_i(t)\| = 0, \quad \forall i \in \mathcal{V}.$$

In order to do so, we introduce the following lemmas.

Lemma 2 (Jensen's inequality)[33] For any constant matrix $\boldsymbol{M} \in \mathbb{R}^{n\times n}$, $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, scalar γ, vector function $\boldsymbol{x} : [0,\gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$-\gamma \int_0^\gamma \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{x}(s)\,\mathrm{d}s \le -\Big(\int_0^\gamma \boldsymbol{x}(s)\,\mathrm{d}s\Big)^{\mathrm{T}} \boldsymbol{M} \Big(\int_0^\gamma \boldsymbol{x}(s)\,\mathrm{d}s\Big).$$

Lemma 3 (Finsler's lemma)[34] Let $\boldsymbol{\zeta} \in \mathbb{R}^n$, $\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, and $\boldsymbol{\Upsilon} \in \mathbb{R}^{m\times n}$ with rank $(\Upsilon) < n$. The following statements are equivalent:

(i) $\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{\zeta} < 0, \forall \boldsymbol{\Upsilon}\boldsymbol{\zeta} = 0, \boldsymbol{\zeta} \ne 0$,

(ii) ${\boldsymbol{\Upsilon}^{\perp}}^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{\Upsilon}^{\perp} < 0$,

(iii) $\exists \mathcal{F} \in \mathbb{R}^{n\times m} : \boldsymbol{\Phi} + \mathcal{F}\boldsymbol{\Upsilon} + \boldsymbol{\Upsilon}^{\mathrm{T}}\mathcal{F}^{\mathrm{T}} < 0$.

3. Main results

In this section, we propose new stability criteria for system (5). For the simplicity of matrix representation, $\boldsymbol{e}_i \in \mathbb{R}^{5Nn\times Nn}$ are defined as block entry matrices; e.g., $\boldsymbol{e}_2 = [\boldsymbol{0}_{Nn}, \boldsymbol{I}_{Nn}, \boldsymbol{0}_{Nn}, \boldsymbol{0}_{Nn}, \boldsymbol{0}_{Nn}]^{\mathrm{T}}$. Several matrices are defined as

$$\begin{aligned}
\boldsymbol{\zeta}(t) &= [\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{m}}), \boldsymbol{x}^{\mathrm{T}}(t-h(t)), \boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{M}}), \dot{\boldsymbol{x}}^{\mathrm{T}}(t)]^{\mathrm{T}}, \\
\boldsymbol{\Xi}_1 &= \boldsymbol{e}_1(\boldsymbol{I}_N \otimes \boldsymbol{P})\boldsymbol{e}_5^{\mathrm{T}} + \boldsymbol{e}_5(\boldsymbol{I}_N \otimes \boldsymbol{P})\boldsymbol{e}_1^{\mathrm{T}}, \\
\boldsymbol{\Xi}_2 &= \boldsymbol{e}_1(\boldsymbol{I}_N \otimes (\boldsymbol{Q}_1+\boldsymbol{Q}_2))\boldsymbol{e}_1^{\mathrm{T}} - \boldsymbol{e}_2(\boldsymbol{I}_N \otimes (\boldsymbol{Q}_1-\boldsymbol{Q}_3))\boldsymbol{e}_2^{\mathrm{T}} \\
&\quad -(1-h_{\mathrm{D}})\boldsymbol{e}_3(\boldsymbol{I}_N \otimes \boldsymbol{Q}_2)\boldsymbol{e}_3^{\mathrm{T}} - \boldsymbol{e}_4(\boldsymbol{I}_N \otimes \boldsymbol{Q}_3)\boldsymbol{e}_4^{\mathrm{T}}, \\
\boldsymbol{\Xi}_3 &= \boldsymbol{e}_5(\boldsymbol{I}_N \otimes (h_{\mathrm{m}}^2\boldsymbol{R}_1 + (h_{\mathrm{M}}-h_{\mathrm{m}})^2\boldsymbol{R}_2))\boldsymbol{e}_5^{\mathrm{T}} \\
&\quad -(\boldsymbol{e}_1-\boldsymbol{e}_2)(\boldsymbol{I}_N \otimes \boldsymbol{R}_1)(\boldsymbol{e}_1-\boldsymbol{e}_2)^{\mathrm{T}} \\
&\quad -(\boldsymbol{e}_2-\boldsymbol{e}_3)(\boldsymbol{I}_N \otimes \boldsymbol{R}_2)(\boldsymbol{e}_2-\boldsymbol{e}_3)^{\mathrm{T}} \\
&\quad -(\boldsymbol{e}_3-\boldsymbol{e}_4)(\boldsymbol{I}_N \otimes \boldsymbol{R}_2)(\boldsymbol{e}_3-\boldsymbol{e}_4)^{\mathrm{T}} \\
&\quad -(\boldsymbol{e}_2-\boldsymbol{e}_3)(\boldsymbol{I}_N \otimes \boldsymbol{S})(\boldsymbol{e}_3-\boldsymbol{e}_4)^{\mathrm{T}} \\
&\quad -(\boldsymbol{e}_3-\boldsymbol{e}_4)(\boldsymbol{I}_N \otimes \boldsymbol{S})^{\mathrm{T}}(\boldsymbol{e}_2-\boldsymbol{e}_3)^{\mathrm{T}}, \\
\boldsymbol{\Xi}_4 &= \boldsymbol{e}_1(\boldsymbol{I}_N \otimes (h_{\mathrm{m}}\boldsymbol{R}_3 + (h_{\mathrm{M}}-h_{\mathrm{m}})\boldsymbol{R}_5 + \boldsymbol{Z}_1))\boldsymbol{e}_1^{\mathrm{T}} \\
&\quad -\boldsymbol{e}_2(\boldsymbol{I}_N \otimes (\boldsymbol{Z}_1-\boldsymbol{Z}_2))\boldsymbol{e}_2^{\mathrm{T}} - \boldsymbol{e}_3(\boldsymbol{I}_N \otimes (\boldsymbol{Z}_2-\boldsymbol{Z}_3))\boldsymbol{e}_3^{\mathrm{T}} \\
&\quad -\boldsymbol{e}_4(\boldsymbol{I}_N \otimes \boldsymbol{Z}_3)\boldsymbol{e}_4^{\mathrm{T}} + \boldsymbol{e}_5(\boldsymbol{I}_N \otimes (h_{\mathrm{m}}\boldsymbol{R}_4 + (h_{\mathrm{M}}-h_{\mathrm{m}})\boldsymbol{R}_6))\boldsymbol{e}_5^{\mathrm{T}}, \\
\boldsymbol{\Phi} &= \boldsymbol{\Xi}_1 + \boldsymbol{\Xi}_2 + \boldsymbol{\Xi}_3 + \boldsymbol{\Xi}_4, \\
\boldsymbol{\Upsilon} &= [(\boldsymbol{I}_N \otimes \boldsymbol{F} - \mathcal{B} \otimes \boldsymbol{GK}), \boldsymbol{0}_{Nn}, -(\mathcal{L} \otimes \boldsymbol{GK}), \boldsymbol{0}_{Nn}, -\boldsymbol{I}_{Nn}]. \quad (6)
\end{aligned}$$

Now, the following theorem is given for the stability of the NMTSs (5).

Theorem 1 For given scalars $0 < h_{\mathrm{m}} \le h_{\mathrm{M}}$, h_{D} and gain $\boldsymbol{K}$, the agents in system (5) converge to the state of the leader asymptotically, if there exist positive definite matrices $\boldsymbol{P} \in \mathbb{R}^{n\times n}$, $\boldsymbol{Q}_i \in \mathbb{R}^{n\times n}$ $(i = 1,2,3)$, $\boldsymbol{R}_i \in \mathbb{R}^{n\times n}$ $(i = 1,\ldots,6)$, any symmetric matrices $\boldsymbol{Z}_i \in \mathbb{R}^{n\times n}$ $(i = 1,2,3)$, and any matrix $\boldsymbol{S} \in \mathbb{R}^{n\times n}$ satisfying the following LMIs:

$$[\boldsymbol{\Upsilon}^{\perp}]^{\mathrm{T}}\boldsymbol{\Phi}[\boldsymbol{\Upsilon}^{\perp}] < \boldsymbol{0}_{4Nn}, \quad (7)$$

$$\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{S} \\ \star & \boldsymbol{R}_2 \end{bmatrix} \ge \boldsymbol{0}_{2n}, \quad (8)$$

$$\begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{Z}_1 \\ \star & \boldsymbol{R}_4 \end{bmatrix} > \boldsymbol{0}_{2n}, \ \begin{bmatrix} \boldsymbol{R}_5 & \boldsymbol{Z}_2 \\ \star & \boldsymbol{R}_6 \end{bmatrix} > \boldsymbol{0}_{2n}, \ \begin{bmatrix} \boldsymbol{R}_5 & \boldsymbol{Z}_3 \\ \star & \boldsymbol{R}_6 \end{bmatrix} > \boldsymbol{0}_{2n}, \quad (9)$$

where $\boldsymbol{\Phi}$ and $\boldsymbol{\Upsilon}$ are defined in Eq. (6).

Proof Let us consider the following Lyapunov–Krasovskii functional candidate:

$$\boldsymbol{V} = \boldsymbol{V}_1 + \boldsymbol{V}_2 + \boldsymbol{V}_3 + \boldsymbol{V}_4, \quad (10)$$

where

$$\begin{aligned}
\boldsymbol{V}_1 &= \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N \otimes \boldsymbol{P})\boldsymbol{x}(t), \\
\boldsymbol{V}_2 &= \int_{t-h_{\mathrm{m}}}^{t} \boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N \otimes \boldsymbol{Q}_1)\boldsymbol{x}(s)\,\mathrm{d}s \\
&\quad + \int_{t-h(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N \otimes \boldsymbol{Q}_2)\boldsymbol{x}(s)\,\mathrm{d}s \\
&\quad + \int_{t-h_{\mathrm{M}}}^{t-h_{\mathrm{m}}} \boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N \otimes \boldsymbol{Q}_3)\boldsymbol{x}(s)\,\mathrm{d}s, \\
\boldsymbol{V}_3 &= h_{\mathrm{m}} \int_{t-h_{\mathrm{m}}}^{t} \int_{s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_1)\dot{\boldsymbol{x}}(u)\,\mathrm{d}u\,\mathrm{d}s \\
&\quad + (h_{\mathrm{M}}-h_{\mathrm{m}}) \int_{t-h_{\mathrm{M}}}^{t-h_{\mathrm{m}}} \int_{s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_2)\dot{\boldsymbol{x}}(u)\,\mathrm{d}u\,\mathrm{d}s, \\
\boldsymbol{V}_4 &= \int_{t-h_{\mathrm{m}}}^{t} \int_{s}^{t} \Big(\boldsymbol{x}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_3)\boldsymbol{x}(u) \\
&\quad + \dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_4)\dot{\boldsymbol{x}}(u) \Big)\,\mathrm{d}u\,\mathrm{d}s \\
&\quad + \int_{t-h_{\mathrm{M}}}^{t-h_{\mathrm{m}}} \int_{s}^{t} \Big(\boldsymbol{x}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_5)\boldsymbol{x}(u) \\
&\quad + \dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N \otimes \boldsymbol{R}_6)\dot{\boldsymbol{x}}(u) \Big)\,\mathrm{d}u\,\mathrm{d}s.
\end{aligned}$$

$\dot{V}_1$ and $\dot{V}_2$ are calculated as

$$\begin{aligned}\dot{V}_1 &= 2\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{P})\dot{\boldsymbol{x}}(t)\\ &= \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Xi}_1\boldsymbol{\zeta}(t),\\ \dot{V}_2 &\le \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(\boldsymbol{Q}_1+\boldsymbol{Q}_2))\boldsymbol{x}(t)\\ &\quad -\boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{m}})(\boldsymbol{I}_N\otimes(\boldsymbol{Q}_1-\boldsymbol{Q}_3))\boldsymbol{x}(t-h_{\mathrm{m}})\\ &\quad -(1-h_{\mathrm{D}})\boldsymbol{x}^{\mathrm{T}}(t-h(t))(\boldsymbol{I}_N\otimes\boldsymbol{Q}_2)\boldsymbol{x}(t-h(t))\\ &\quad -\boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{M}})(\boldsymbol{I}_N\otimes\boldsymbol{Q}_3)\boldsymbol{x}(t-h_{\mathrm{M}})\\ &= \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Xi}_2\boldsymbol{\zeta}(t),\end{aligned}\tag{11}$$

where $\boldsymbol{\Xi}_1$, $\boldsymbol{\Xi}_2$, and $\boldsymbol{\zeta}(t)$ are defined in Eq. (6).

By Lemma 2, an estimation of $\dot{V}_3$ can be obtained as

$$\begin{aligned}\dot{V}_3 &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}^2\boldsymbol{R}_1+(h_{\mathrm{M}}-h_{\mathrm{m}})^2\boldsymbol{R}_2))\dot{\boldsymbol{x}}(t)\\ &\quad -h_{\mathrm{m}}\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_1)\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &\quad -(h_{\mathrm{M}}-h_{\mathrm{m}})\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_2)\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &\quad -(h_{\mathrm{M}}-h_{\mathrm{m}})\int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_2)\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &\le \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}^2\boldsymbol{R}_1+(h_{\mathrm{M}}-h_{\mathrm{m}})^2\boldsymbol{R}_2))\dot{\boldsymbol{x}}(t)\\ &\quad -\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{R}_1)\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)\\ &\quad -\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\\ &\quad \times\begin{bmatrix}\frac{1}{1-\alpha(t)}(\boldsymbol{I}_N\otimes\boldsymbol{R}_2) & \boldsymbol{0}_{Nn}\\ \boldsymbol{0}_{Nn} & \frac{1}{\alpha(t)}(\boldsymbol{I}_N\otimes\boldsymbol{R}_2)\end{bmatrix}\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{bmatrix}\\ &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}^2\boldsymbol{R}_1+(h_{\mathrm{M}}-h_{\mathrm{m}})^2\boldsymbol{R}_2))\dot{\boldsymbol{x}}(t)\\ &\quad -\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{R}_1)\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)\\ &\quad -\sum_{i=1}^{N}\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\frac{1}{1-\alpha(t)}\boldsymbol{R}_2 & \boldsymbol{0}_n\\ \boldsymbol{0}_n & \frac{1}{\alpha(t)}\boldsymbol{R}_2\end{bmatrix}\\ &\quad \times\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix},\end{aligned}\tag{12}$$

where $\alpha(t)=(h_{\mathrm{M}}-h(t))(h_{\mathrm{M}}-h_{\mathrm{m}})^{-1}$, which satisfies $0<\alpha(t)<1$ when $h_{\mathrm{m}}<h(t)<h_{\mathrm{M}}$. Then, using the reciprocally convex approach,[29] the following inequality for any matrix $\boldsymbol{S}$ holds:

$$\begin{bmatrix}-\sqrt{\frac{\alpha(t)}{1-\alpha(t)}}\boldsymbol{I}_n & \boldsymbol{0}_n\\ \boldsymbol{0}_n & \sqrt{\frac{1-\alpha(t)}{\alpha(t)}}\boldsymbol{I}_n\end{bmatrix}\begin{bmatrix}\boldsymbol{R}_2 & \boldsymbol{S}\\ \star & \boldsymbol{R}_2\end{bmatrix}\times\begin{bmatrix}-\sqrt{\frac{\alpha(t)}{1-\alpha(t)}}\boldsymbol{I}_n & \boldsymbol{0}_n\\ \boldsymbol{0}_n & \sqrt{\frac{1-\alpha(t)}{\alpha(t)}}\boldsymbol{I}_n\end{bmatrix}>\boldsymbol{0}_{2n}.\tag{13}$$

Also, when $h(t)=h_{\mathrm{m}}$ or $h(t)=h_{\mathrm{M}}$, we get

$$\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s=\int_{t-h_{\mathrm{m}}}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s=\boldsymbol{0}_n,$$

or

$$\int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s=\int_{t-h_{\mathrm{M}}}^{t-h_{\mathrm{M}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s=\boldsymbol{0}_n,\tag{14}$$

respectively.

Thus, from Eqs. (13) and (14), the following inequality still holds:

$$\begin{aligned}&-\sum_{i=1}^{N}\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\frac{1}{1-\alpha(t)}\boldsymbol{R}_2 & \boldsymbol{0}_n\\ \boldsymbol{0}_n & \frac{1}{\alpha(t)}\boldsymbol{R}_2\end{bmatrix}\\ &\times\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix}\\ &\le-\sum_{i=1}^{N}\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{R}_2 & \boldsymbol{S}\\ \star & \boldsymbol{R}_2\end{bmatrix}\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}_i(s)\mathrm{d}s\end{bmatrix}.\end{aligned}\tag{15}$$

Here, if inequality (8) holds, then an upper bound of $\dot{V}_3$ can be rebounded as

$$\begin{aligned}\dot{V}_3 &\le \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}^2\boldsymbol{R}_1+(h_{\mathrm{M}}-h_{\mathrm{m}})^2\boldsymbol{R}_2))\dot{\boldsymbol{x}}(t)\\ &\quad -\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)^{\mathrm{T}}(\boldsymbol{I}_N\otimes\boldsymbol{R}_1)\left(\int_{t-h_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right)\\ &\quad -\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}(\boldsymbol{I}_N\otimes\boldsymbol{R}_2) & (\boldsymbol{I}_N\otimes\boldsymbol{S})\\ \star & (\boldsymbol{I}_N\otimes\boldsymbol{R}_2)\end{bmatrix}\\ &\quad \times\begin{bmatrix}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ \int_{t-h_{\mathrm{M}}}^{t-h(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{bmatrix}\\ &= \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Xi}_3\boldsymbol{\zeta}(t),\end{aligned}\tag{16}$$

where $\boldsymbol{\Xi}_3$ and $\boldsymbol{\zeta}(t)$ are defined in Eq. (6).

Lastly, $\dot{V}_4$ is calculated as

$$\begin{aligned}\dot{V}_4 &= \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}\boldsymbol{R}_3+(h_{\mathrm{M}}-h_{\mathrm{m}})\boldsymbol{R}_5))\boldsymbol{x}(t)\\ &\quad +\dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes(h_{\mathrm{m}}\boldsymbol{R}_4+(h_{\mathrm{M}}-h_{\mathrm{m}})\boldsymbol{R}_6))\dot{\boldsymbol{x}}(t)\\ &\quad -\int_{t-h_{\mathrm{m}}}^{t}\Big(\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_3)\boldsymbol{x}(s)\\ &\quad +\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_4)\dot{\boldsymbol{x}}(s)\Big)\mathrm{d}s\\ &\quad -\int_{t-h_{\mathrm{M}}}^{t-h_{\mathrm{m}}}\Big(\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_5)\boldsymbol{x}(s)\\ &\quad +\dot{\boldsymbol{x}}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{R}_6)\dot{\boldsymbol{x}}(s)\Big)\mathrm{d}s.\end{aligned}\tag{17}$$

Inspired by Ref. [35], the following three zero equalities with any symmetric matrices $\boldsymbol{Z}_i$ $(i=1,2,3)$ are considered:

$$\begin{aligned}0 &= \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{m}})(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\boldsymbol{x}(t-h_{\mathrm{m}})\\ &\quad -2\int_{t-h_{\mathrm{m}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\dot{\boldsymbol{x}}(s)\mathrm{d}s,\\ 0 &= \boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{m}})(\boldsymbol{I}_N\otimes\boldsymbol{Z}_2)\boldsymbol{x}(t-h_{\mathrm{m}}))\\ &\quad -\boldsymbol{x}^{\mathrm{T}}(t-h(t))(\boldsymbol{I}_N\otimes\boldsymbol{Z}_2)\boldsymbol{x}(t-h(t))\\ &\quad -2\int_{t-h(t)}^{t-h_{\mathrm{m}}}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_2)\dot{\boldsymbol{x}}(s)\mathrm{d}s,\\ 0 &= \boldsymbol{x}^{\mathrm{T}}(t-h(t))(\boldsymbol{I}_N\otimes\boldsymbol{Z}_3)\boldsymbol{x}(t-h(t))\\ &\quad -\boldsymbol{x}^{\mathrm{T}}(t-h_{\mathrm{M}})(\boldsymbol{I}_N\otimes\boldsymbol{Z}_3)\boldsymbol{x}(t-h_{\mathrm{M}})\\ &\quad -2\int_{t-h_{\mathrm{M}}}^{t-h(t)}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_3)\dot{\boldsymbol{x}}(s)\mathrm{d}s.\end{aligned}\tag{18}$$

By adding Eq. (18) to Eq. (17), we get

$$\begin{aligned}\dot{V}_4 &= \zeta^{\mathrm{T}}(t)\Xi_4\zeta(t)\\ &\quad -\int_{t-h_{\mathrm{m}}}^{t}\chi^{\mathrm{T}}(s)\begin{bmatrix}(I_N\otimes R_3) & (I_N\otimes Z_1)\\ \star & (I_N\otimes R_4)\end{bmatrix}\chi(s)\mathrm{d}s\\ &\quad -\int_{t-h(t)}^{t-h_{\mathrm{m}}}\chi^{\mathrm{T}}(s)\begin{bmatrix}(I_N\otimes R_5) & (I_N\otimes Z_2)\\ \star & (I_N\otimes R_6)\end{bmatrix}\chi(s)\mathrm{d}s\\ &\quad -\int_{t-h_{\mathrm{M}}}^{t-h(t)}\chi^{\mathrm{T}}(s)\begin{bmatrix}(I_N\otimes R_5) & (I_N\otimes Z_3)\\ \star & (I_N\otimes R_6)\end{bmatrix}\chi(s)\mathrm{d}s\\ &= \zeta^{\mathrm{T}}(t)\Xi_4\zeta(t)\\ &\quad -\sum_{i=1}^{N}\int_{t-h_{\mathrm{m}}}^{t}\chi_i^{\mathrm{T}}(s)\begin{bmatrix}R_3 & Z_1\\ \star & R_4\end{bmatrix}\chi_i(s)\mathrm{d}s\\ &\quad -\sum_{i=1}^{N}\int_{t-h(t)}^{t-h_{\mathrm{m}}}\chi_i^{\mathrm{T}}(s)\begin{bmatrix}R_5 & Z_2\\ \star & R_6\end{bmatrix}\chi_i(s)\mathrm{d}s\\ &\quad -\sum_{i=1}^{N}\int_{t-h_{\mathrm{M}}}^{t-h(t)}\chi_i^{\mathrm{T}}(s)\begin{bmatrix}R_5 & Z_3\\ \star & R_6\end{bmatrix}\chi_i(s)\mathrm{d}s, \end{aligned}\tag{19}$$

where Ξ_4 and $\zeta(t)$ are defined in Eq. (6), and

$$\chi(t)=[x^{\mathrm{T}}(t),\dot{x}^{\mathrm{T}}(t)]^{\mathrm{T}},\quad \chi_i(t)=[x_i^{\mathrm{T}}(t),\dot{x}_i^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

Here, if inequalities (9) hold, then $\dot{V}_4$ has an upper bound as follows:

$$\dot{V}_4\le\zeta^{\mathrm{T}}(t)\Xi_4\zeta(t).\tag{20}$$

Therefore, from Eqs. (11)–(20) and by the application of the S-procedure,[30] $\dot{V}$ has a new upper bound as

$$\dot{V}\le\zeta^{\mathrm{T}}(t)\underbrace{(\Xi_1+\Xi_2+\Xi_3+\Xi_4)}_{\Phi}\zeta(t).\tag{21}$$

System (5) with the augmented vector $\zeta(t)$ can also be rewritten as

$$\Upsilon\zeta(t)=0_{Nn\times1},\tag{22}$$

where Υ is defined in Eq. (6). Then, a delay-dependent stability condition for system (5) is

$$\zeta^{\mathrm{T}}(t)\Phi\zeta(t)<0\quad\text{subject to}\quad\Upsilon\zeta(t)=0_{Nn\times1}.\tag{23}$$

Finally, by utilizing Lemma 3, condition (23) is equivalent to the following inequality:

$$[\Upsilon^{\perp}]^{\mathrm{T}}\Phi[\Upsilon^{\perp}]<0_{4Nn}.\tag{24}$$

From inequality (24), if LMIs (7) are satisfied, then stability condition (23) holds. This completes our proof.

Theorem 1 provides a consensus criterion for system (5) in the framework of LMIs when the consensus protocol gain is known. Based on Theorem 1, we propose a consensus controller design method for system (5), which will be introduced as Theorem 2. To design the consensus protocol gain, we add the following zero equations with any matrices Z_4 and Z_5 to be chosen:

$$0=2(x^{\mathrm{T}}(t)(I_N\otimes Z_4)+\dot{x}^{\mathrm{T}}(t)(I_N\otimes Z_5))\Upsilon\zeta(t).\tag{25}$$

Before deriving this, several matrices are defined for simplicity

$$\begin{aligned}\bar{\Xi}_1 &= e_1(I_N\otimes X)e_5^{\mathrm{T}}+e_5(I_N\otimes X)e_1^{\mathrm{T}},\\ \bar{\Xi}_2 &= e_1(I_N\otimes(\bar{Q}_1+\bar{Q}_2))e_1^{\mathrm{T}}-e_2(I_N\otimes(\bar{Q}_1-\bar{Q}_3))e_2^{\mathrm{T}}\\ &\quad -(1-h_{\mathrm{D}})e_3(I_N\otimes\bar{Q}_2)e_3^{\mathrm{T}}-e_4(I_N\otimes\bar{Q}_3)e_4^{\mathrm{T}},\\ \bar{\Xi}_3 &= e_5(I_N\otimes(h_{\mathrm{m}}^2\bar{R}_1+(h_{\mathrm{M}}-h_{\mathrm{m}})^2\bar{R}_2))e_5^{\mathrm{T}}\\ &\quad -(e_1-e_2)(I_N\otimes\bar{R}_1)(e_1-e_2)^{\mathrm{T}}\\ &\quad -(e_2-e_3)(I_N\otimes\bar{R}_2)(e_2-e_3)^{\mathrm{T}}\\ &\quad -(e_3-e_4)(I_N\otimes\bar{R}_2)(e_3-e_4)^{\mathrm{T}}\\ &\quad -(e_2-e_3)(I_N\otimes\bar{S})(e_3-e_4)^{\mathrm{T}}\\ &\quad -(e_3-e_4)(I_N\otimes\bar{S})^{\mathrm{T}}(e_2-e_3)^{\mathrm{T}},\\ \bar{\Xi}_4 &= e_1(I_N\otimes(h_{\mathrm{m}}\bar{R}_3+(h_{\mathrm{M}}-h_{\mathrm{m}})\bar{R}_5+\bar{Z}_1))e_1^{\mathrm{T}}\\ &\quad -e_2(I_N\otimes(\bar{Z}_1-\bar{Z}_2))e_2^{\mathrm{T}}-e_3(I_N\otimes(\bar{Z}_2-\bar{Z}_3))e_3^{\mathrm{T}}\\ &\quad -e_4(I\otimes\bar{Z}_3)e_4^{\mathrm{T}}+e_5(I_N\otimes(h_{\mathrm{m}}\bar{R}_4+(h_{\mathrm{M}}-h_{\mathrm{m}})\bar{R}_6))e_5^{\mathrm{T}},\\ \bar{\Phi} &= \bar{\Xi}_1+\bar{\Xi}_2+\bar{\Xi}_3+\bar{\Xi}_4,\\ \bar{\Upsilon} &= [(I_N\otimes FX-\mathcal{B}\otimes GY),0_{Nn},-(\mathcal{L}\otimes GY),0_{Nn},\\ &\quad -(I_N\otimes X)],\\ \Pi &= [\gamma_1 I_{Nn},0_{Nn},0_{Nn},0_{Nn},\gamma_2 I_{Nn}]^{\mathrm{T}}.\end{aligned}\tag{26}$$

We will utilize the zero equation (25) in Theorem 2.

Theorem 2 For given scalars $0<h_{\mathrm{m}}\le h_{\mathrm{M}}$, h_{D}, γ_1, and γ_2, the agents in system (5) converge to the state of the leader stochastically, if there exist positive definite matrices $X\in\mathbb{R}^{n\times n}$, $\bar{Q}_i\in\mathbb{R}^{n\times n}$ $(i=1,2,3)$, $\bar{R}_i\in\mathbb{R}^{n\times n}$ $(i=1,\ldots,6)$, any symmetric matrices $\bar{Z}_i\in\mathbb{R}^{n\times n}$ $(i=1,2,3)$, and any matrices $\bar{S}\in\mathbb{R}^{n\times n}$, $Y\in\mathbb{R}^{l\times n}$ satisfying the following LMIs:

$$\bar{\Phi}+\Pi\bar{\Upsilon}+(\Pi\bar{\Upsilon})^{\mathrm{T}}<0_{5Nn},\tag{27}$$

$$\begin{bmatrix}\bar{R}_2 & \bar{S}\\ \star & \bar{R}_2\end{bmatrix}\ge0_n,\tag{28}$$

$$\begin{bmatrix}\bar{R}_3 & \bar{Z}_1\\ \star & \bar{R}_4\end{bmatrix}>0_n,\ \begin{bmatrix}\bar{R}_5 & \bar{Z}_2\\ \star & \bar{R}_6\end{bmatrix}>0_n,\ \begin{bmatrix}\bar{R}_5 & \bar{Z}_3\\ \star & \bar{R}_6\end{bmatrix}>0_n,\tag{29}$$

where $\bar{\Phi}$, Π, and $\bar{\Upsilon}$ are defined in Eq. (26). Then, system (5) under the consensus protocol gain $K=YX^{-1}$ is asymptotically stable.

Proof Let us define $Z_4=\gamma_1 P$ and $Z_5=\gamma_2 P$ in Eq. (25). With the same Lyapunov–Krasovskii functional candidate in Eq. (10), by using the similar method in Eqs. (11)–(20), and considering the zero equation in Eq. (25), a sufficient condition guaranteeing the stability of system (5) can be obtained as

$$\begin{aligned}&\Phi+\begin{bmatrix}I_N\otimes Z_4\\ 0_{3Nn\times Nn}\\ I_N\otimes Z_5\end{bmatrix}\Upsilon+\left\{\begin{bmatrix}I_N\otimes Z_4\\ 0_{3Nn\times Nn}\\ I_N\otimes Z_5\end{bmatrix}\Upsilon\right\}^{\mathrm{T}}\\ &=\Phi+\underbrace{\begin{bmatrix}I_N\otimes\gamma_1 I_n\\ 0_{3Nn\times Nn}\\ I_N\otimes\gamma_2 I_n\end{bmatrix}}_{\Pi}(I_N\otimes P)\\ &\quad\times\underbrace{[(I_N\otimes F-\mathcal{B}\otimes GK),0_{Nn},-(\mathcal{L}\otimes GK),0_{Nn},-I_{Nn}]}_{\Upsilon}\\ &\quad+\{\Pi(I_N\otimes P)\Upsilon\}^{\mathrm{T}}\\ &<0_{5Nn},\end{aligned}\tag{30}$$

where $\boldsymbol{\Phi}$, $\boldsymbol{\Upsilon}$, and $\boldsymbol{\Pi}$ are defined in Eqs. (6) and (26), respectively.

To obtain the consensus protocol gain, we define

$$\begin{aligned}\boldsymbol{X} &= \boldsymbol{P}^{-1},\ \boldsymbol{Y} = \boldsymbol{K}\boldsymbol{X},\\ \bar{\boldsymbol{Q}}_i &= \boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}_i\boldsymbol{X},\ \bar{\boldsymbol{R}}_j = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{R}_j\boldsymbol{X},\\ \bar{\boldsymbol{Z}}_i &= \boldsymbol{X}^{\mathrm{T}}\boldsymbol{Z}_i\boldsymbol{X},\ \bar{\boldsymbol{S}} = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{X},\end{aligned}$$

where $i = 1,2,3$ and $j = 1,\ldots,6$. Then, pre- and post-multiplying inequality (30) by matrix $(\boldsymbol{I}_N \otimes \mathrm{diag}\{\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{X}\})^{\mathrm{T}}$ and $\boldsymbol{I}_N \otimes \mathrm{diag}\{\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{X}\}$ leads to

$$\bar{\boldsymbol{\Phi}} + \boldsymbol{\Pi}\underbrace{[(\boldsymbol{I}_N \otimes \boldsymbol{F}\boldsymbol{X} - \mathcal{B}\otimes\boldsymbol{G}\boldsymbol{Y}), \boldsymbol{0}_{Nn}, -(\mathcal{L}\otimes\boldsymbol{G}\boldsymbol{Y}), \boldsymbol{0}_{Nn}, -(\boldsymbol{I}_N\otimes\boldsymbol{X})]}_{\bar{\boldsymbol{\Upsilon}}} + (\boldsymbol{\Pi}\bar{\boldsymbol{\Upsilon}})^{\mathrm{T}} < \boldsymbol{0}_{5Nn}. \tag{31}$$

In conclusion, condition (31) is equivalent to LMIs (27).

As a special case, when the time delay is a constant, system (5) can be rewritten as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{F} - \mathcal{B}\otimes\boldsymbol{G}\boldsymbol{K})\boldsymbol{x}(t) - (\mathcal{L}\otimes\boldsymbol{G}\boldsymbol{K})\boldsymbol{x}(t-h). \tag{32}$$

Before deriving this, several matrices are defined for simplicity

$$\begin{aligned}\tilde{\boldsymbol{\zeta}}(t) &= [\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-h), \dot{\boldsymbol{x}}^{\mathrm{T}}(t)]^{\mathrm{T}},\\ \tilde{\boldsymbol{\Phi}} &= \tilde{\boldsymbol{e}}_1(\boldsymbol{I}_N\otimes\boldsymbol{X})\tilde{\boldsymbol{e}}_3^{\mathrm{T}} + \tilde{\boldsymbol{e}}_3(\boldsymbol{I}_N\otimes\boldsymbol{X})\tilde{\boldsymbol{e}}_1^{\mathrm{T}}\\ &\quad + \tilde{\boldsymbol{e}}_1(\boldsymbol{I}_N\otimes\tilde{\boldsymbol{Q}})\tilde{\boldsymbol{e}}_1^{\mathrm{T}} - \tilde{\boldsymbol{e}}_2(\boldsymbol{I}_N\otimes(\tilde{\boldsymbol{Q}})\tilde{\boldsymbol{e}}_2^{\mathrm{T}}\\ &\quad + \tilde{\boldsymbol{e}}_3(\boldsymbol{I}_N\otimes h^2\tilde{\boldsymbol{R}}_1)\tilde{\boldsymbol{e}}_3^{\mathrm{T}} - (\tilde{\boldsymbol{e}}_1-\tilde{\boldsymbol{e}}_2)(\boldsymbol{I}_N\otimes\tilde{\boldsymbol{R}}_1)(\tilde{\boldsymbol{e}}_1-\tilde{\boldsymbol{e}}_2)^{\mathrm{T}}\\ &\quad + \tilde{\boldsymbol{e}}_1(\boldsymbol{I}_N\otimes h\tilde{\boldsymbol{R}}_2 + \tilde{\boldsymbol{Z}}_1))\tilde{\boldsymbol{e}}_1^{\mathrm{T}} - \tilde{\boldsymbol{e}}_2(\boldsymbol{I}_N\otimes\tilde{\boldsymbol{Z}}_1)\tilde{\boldsymbol{e}}_2^{\mathrm{T}}\\ &\quad + \tilde{\boldsymbol{e}}_3(\boldsymbol{I}_N\otimes h\tilde{\boldsymbol{R}}_3)\tilde{\boldsymbol{e}}_3^{\mathrm{T}},\\ \tilde{\boldsymbol{\Upsilon}} &= [(\boldsymbol{I}_N\otimes\boldsymbol{F}\boldsymbol{X} - \mathcal{B}\otimes\boldsymbol{G}\boldsymbol{Y}), -(\mathcal{L}\otimes\boldsymbol{G}\boldsymbol{Y}), -(\boldsymbol{I}_N\otimes\boldsymbol{X})],\\ \tilde{\boldsymbol{\Pi}} &= [\gamma_1\boldsymbol{I}_{Nn}, \boldsymbol{0}_{Nn}, \gamma_2\boldsymbol{I}_{Nn}]^{\mathrm{T}},\end{aligned} \tag{33}$$

where $\tilde{\boldsymbol{Q}} = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{X}$, $\tilde{\boldsymbol{R}}_i = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{X}$ $(i = 1,2,3)$, $\tilde{\boldsymbol{Z}}_1 = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{Z}_1\boldsymbol{X}$, and $\tilde{\boldsymbol{e}}_i \in \mathbb{R}^{3n\times n}$ are the block entry matrices, e.g., $\tilde{\boldsymbol{e}}_2 = [\boldsymbol{0}_{Nn}, \boldsymbol{I}_{Nn}, \boldsymbol{0}_{Nn}]^{\mathrm{T}}$.

Corollary 1 For given scalar h, γ_1, and γ_2, the agents in system (32) converge to the state of the leader stochastically, if there exist positive definite matrices $\boldsymbol{X} \in \mathbb{R}^{n\times n}$, $\tilde{\boldsymbol{Q}} \in \mathbb{R}^{n\times n}$, $\tilde{\boldsymbol{R}}_i \in \mathbb{R}^{n\times n}$ $(i = 1,2,3)$, any symmetric matrices $\tilde{\boldsymbol{Z}}_1 \in \mathbb{R}^{n\times n}$, and any matrix $\boldsymbol{Y} \in \mathbb{R}^{l\times n}$ satisfying the following LMIs:

$$\tilde{\boldsymbol{\Phi}} + \tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Upsilon}} + (\tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{\Upsilon}})^{\mathrm{T}} < \boldsymbol{0}_{3Nn}, \tag{34}$$

$$\begin{bmatrix}\tilde{\boldsymbol{R}}_2 & \tilde{\boldsymbol{Z}}_1\\ \star & \tilde{\boldsymbol{R}}_3\end{bmatrix} \geq \boldsymbol{0}_n. \tag{35}$$

Then, system (32) under the consensus protocol gain $\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1}$ is asymptotically stable.

Proof By constructing the following Lyapunov–Krasovskii functional and using some zero equalities

$$\begin{aligned}\boldsymbol{V} &= \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{P})\boldsymbol{x}(t)\\ &\quad + \int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Q})\boldsymbol{x}(s)\mathrm{d}s\\ &\quad + h\int_{t-h}^{t}\int_{s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N\otimes\boldsymbol{R}_1)\dot{\boldsymbol{x}}(u)\mathrm{d}u\mathrm{d}s\\ &\quad + \int_{t-h}^{t}\int_{s}^{t}\Big(\boldsymbol{x}^{\mathrm{T}}(u)(\boldsymbol{I}_N\otimes\boldsymbol{R}_2)\boldsymbol{x}(u)\\ &\quad + \dot{\boldsymbol{x}}^{\mathrm{T}}(u)(\boldsymbol{I}_N\otimes\boldsymbol{R}_3)\dot{\boldsymbol{x}}(u)\Big)\mathrm{d}u\mathrm{d}s,\end{aligned} \tag{36}$$

$$\begin{aligned}0 &= \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-h)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\boldsymbol{x}(t-h)\\ &\quad - 2\int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_1)\dot{\boldsymbol{x}}(s)\mathrm{d}s,\end{aligned} \tag{37}$$

$$\begin{aligned}0 &= 2(\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_2) + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)(\boldsymbol{I}_N\otimes\boldsymbol{Z}_3))\\ &\quad \times[(\boldsymbol{I}_N\otimes\boldsymbol{F} - \mathcal{B}\otimes\boldsymbol{G}\boldsymbol{K}), -(\mathcal{L}\otimes\boldsymbol{G}\boldsymbol{K}), -\boldsymbol{I}_{Nn}]\tilde{\boldsymbol{\zeta}}(t),\end{aligned} \tag{38}$$

we can obtain Corollary 1. The proof of Corollary 1 is very similar to that of Theorem 2, so it is omitted.

Remark 1 In Section 1, the mentioned references mainly used passivity-based architecture to solve the master–slave synchronization problem of BTSs. The LMIs in the proposed stability and stabilization criteria can be easily solved by various efficient convex algorithms. In this paper, we utilize Matlab's LMI control toolbox,[36] which implements the interior-point algorithm. This algorithm is significantly faster than the classical convex optimization algorithms.[30]

4. Illustrative example

In this section, one numerical example is given to illustrate the effectiveness of Theorem 2. Consider the NMTSs (5) with the interconnection topology described in Fig. 2 and the dynamics of the one-link arm obtained from Ref. [5], which means that each agent is contained by a one-link arm

$$\boldsymbol{F} = \begin{bmatrix}0 & 1\\ 0 & -B/J\end{bmatrix},\ \boldsymbol{G} = \begin{bmatrix}0\\ k/J\end{bmatrix}, \tag{39}$$

where J is the arm's inertia moment, B is the viscous friction coefficient of the arm's axis of rotation, and k is the motor's torque coefficient. These parameters are set to $J = 0.03$ kg·m^2, $B = 0.1$ kg·m^2/s, and $k = 0.5$ N·m/V.

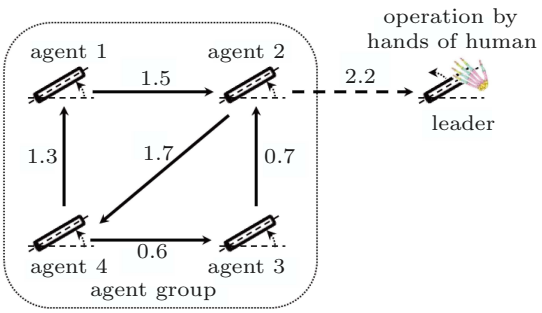


Fig. 2. (color online) Topology of the example.

From Fig. 2, $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{D}$ at the interconnection topology are

$$\mathcal{A} = \begin{bmatrix} 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 1.7 \\ 0 & 0.7 & 0 & 0 \\ 1.3 & 0 & 0.6 & 0 \end{bmatrix},$$
$$\mathcal{B} = \mathrm{diag}\{0, 2.2, 0, 0\},$$
$$\mathcal{D} = \mathrm{diag}\{1.5, 1.7, 0.7, 1.9\}.$$

For the system mentioned above, the consensus protocol gain $\boldsymbol{K}$ with fixed $\gamma_1 = 1.1$, $\gamma_2 = 1$, $h_\mathrm{m} = 1$, $h_\mathrm{M} = 3.4$, and $h_\mathrm{D} = 0.2$ obtained by Theorem 2 is

$$\boldsymbol{K} = [0.0046, 0.0014]. \tag{40}$$

In order to confirm this result, we simulate the states of the NMTSs with the dynamics of agent (39) for $h(t) = 1 + 2.4\sin^2(0.08t)$. The simulation results are given in Fig. 3.

Then, the simulation results for the state responses of the agents are shown in Figs. 3 and 4. It is found that all the agents follow the state of the leader under the obtained gain $\boldsymbol{K}$ in Eq. (40) for given initial states of the agents. Here, the trajectory of the leader is assumed to be $\boldsymbol{z}_0(t) = [\cos(t), \sin(t)]^\mathrm{T}$ in Fig. 3 and $\boldsymbol{z}_0(t) = [1, 2]^\mathrm{T}$ in Fig. 4. Since the real situation regarding tele-surgery should be considered, the assumption for the leader, which shows that the dynamics of the leader are different from those of the agents, should be included. Thus, the source of this discrepancy can be explained as follows. The leader robot is controlled by human hands to create the desired behavior. In particular, in Figs. 3(b) and 4(b), for the consensus or synchronization of the system dynamics, the necessity of the protocol gain $\boldsymbol{K}$ can be confirmed.

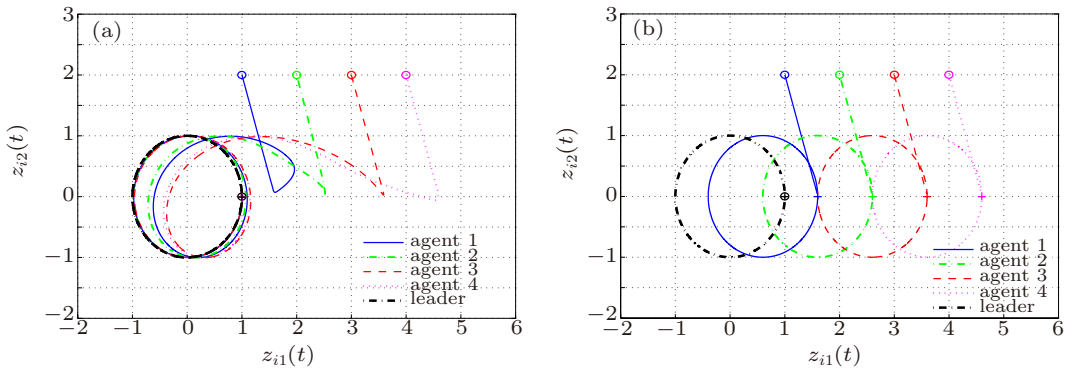


Fig. 3. (color online) The trajectories of the varying leader and each agent: (a) with the obtained gain $\boldsymbol{K}$ in Eq. (40), and (b) without the gain $\boldsymbol{K}$.

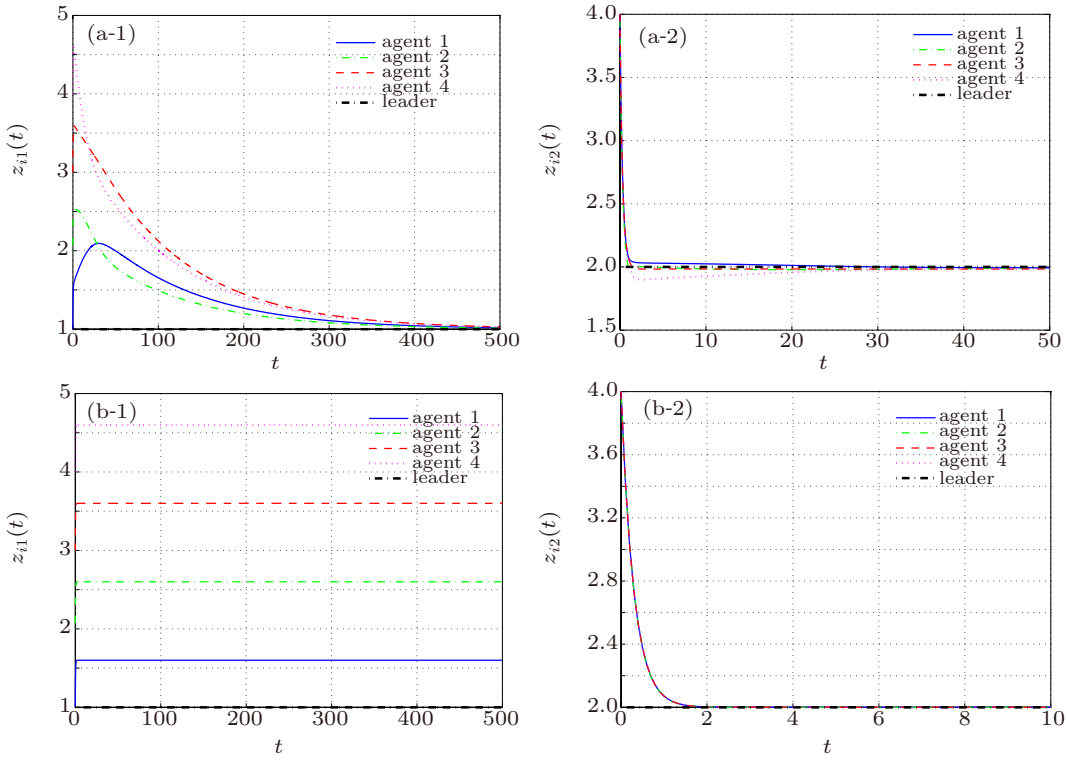


Fig. 4. (color online) The trajectories of the static leader and each agent: (a) with the obtained gain $\boldsymbol{K}$ in Eq. (40), and (b) without the gain $\boldsymbol{K}$.

5. Conclusion

In this paper, the delay-dependent leader–following consensus control problem for NMTSs with interval time-varying communication delays is studied. To do this, a suitable Lyapunov–Krasovskii functional and the reciprocally convex approach are used to investigate the feasible region of consensus criterion in Theorem 1. Based on the result of Theorem 1, the design procedure for a protocol gain of the concerned system was derived as Theorem 2. One illustrative example was given to show the effectiveness and usefulness of the presented criteria. It should be pointed out that the proposed method in this paper can also be extended to networks with time delays between the agents and leader.

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