Adaptive function projective synchronization of uncertain complex dynamical networks with disturbance*

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We investigate the problem of function projective synchronization (FPS) in drive–response dynamical networks with non-identical nodes. An adaptive controller is proposed for the FPS of complex dynamical networks with uncertain parameters and disturbance. Not only are the unknown parameters of the networks estimated by the adaptive laws obtained from the Lyapunov stability theory and Taylor expansions, but the unknown bounded disturbances are also simultaneously conquered by the proposed control. Finally, a numerical simulation is provided to illustrate the feasibility and effectiveness of the obtained result.

Keywords: complex networks, function projective synchronization, disturbance adaptive control

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1. Introduction

Over the past few decades, there has been increasing interest in the study of complex dynamics networks.[1–9] As a typical kind of dynamics, synchronization in complex networks has received a great deal of attention, and many types of synchronization phenomena have been reported, such as complete synchronization, anti-synchronization, phase synchronization, lag synchronization, intermittent lag synchronization, generalized synchronization, intermittent generalized synchronization, time scale synchronization, and projective synchronization.[10–22]

Projective synchronization (PS) means that the drive and response systems can be synchronized up to a scaling factor. PS was first reported by Mainieri and Rehacek[23] in partially linear systems. Later, Chen *et al.*[24,25] put forward another new PS called function projective synchronization (FPS). FPS is characterized by master and slave systems that can be synchronized up to a scaling function, not a constant. Obviously, setting the scaling function to constant or unity will obtain projective or complete synchronization, respectively. So FPS is a more general definition of PS. The FPS of chaotic systems has been widely studied because the unpredictability of the scaling functions can additionally enhance the communication security.[26,27] Therefore, it is necessary to develop FPS methods. Very recently, FPS has been investigated in a two-cell quantum-CNN chaotic oscillator,[28,29] coupled partially linear chaotic systems,[30] and some non-partially linear systems.[31,32] Zhang *et al.*[33] investigated the FPS in a drive–response dynamical network (DRDN), and it was shown that FPS with a desired scaling function can be realized in a DRDN by using a simple control law. Moreover, the scaling function does not need to be differentiable, bounded, and nonzero at all times. However, the assumption of identical nodes is not realistic for many real-world networks. For example, the nodes in the metabolic networks and the internet are distinct in general. The behavior of dynamical networks with non-identical nodes is much more complicated than that in identical-node cases.[34] Du[35] investigated the problem of FPS in DRDNs with non-identical nodes. Based on the adaptive open-plus-closed-loop (AOPCL) method, a general method of FPS, which is robust to limited data accuracy and noise effects, was derived. However, artificially adding disturbances can make the message transmission more complicated, and hence it is very difficult to realize drive–response FPS. On the other hand, in practice, the parameter mismatch is unavoidable, and this can destroy the FPS of the drive–response systems. However, so far, few works have considered the FPS of drive–response systems in which the time-delay coupled complex dynamics networks have uncertain parameters and disturbance.

Motivated by the above discussion, in this paper, we propose a general FPS scheme for complex dynamical networks with uncertain parameters and disturbance. A simple adaptive controller is designed to achieve the FPS. Not only are the unknown parameters of the networks estimated by the adaptive laws obtained from the Lyapunov stability theory and Taylor expansions, but the unknown bounded disturbances are also simultaneously conquered by the use of the updating laws. For the coupling matrix, we do not assume it to be symmetric or irreducible. In addition, numerical simulations are performed

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to verify the effectiveness of the theoretical results.

The rest of this paper is organized as follows. The network model is introduced and some necessary definitions, lemmas, and hypotheses are given in Section 2. The FPS for uncertain complex networks with non-identical nodes is discussed in Section 3, and examples and their simulations are given in Section 4. Finally, the conclusions are drawn in Section 5.

2. Model description

We consider the following DRDNs and mismatched terms:

$$\dot{\boldsymbol{x}}^{\mathrm{d}}(t) = \boldsymbol{f}(\boldsymbol{x}^{\mathrm{d}}(t)) + \boldsymbol{\Delta}_{\mathrm{d}}, \tag{1}$$

$$\begin{aligned}\dot{\boldsymbol{x}}_i^{\mathrm{r}}(t) &= \boldsymbol{g}_i(\boldsymbol{x}_i^{\mathrm{r}}(t)) + \boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\boldsymbol{\theta}_i + c\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{x}_j^{\mathrm{r}}(t) \\ &\quad + c\sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}_2\boldsymbol{x}_j^{\mathrm{r}}(t-d_i) + \boldsymbol{\Delta}_i + \boldsymbol{u}_i,\end{aligned} \tag{2}$$

where superscripts d and r stand for the drive system and the response networks, respectively. In Eq. (1), $\boldsymbol{x}^{\mathrm{d}}(t) = (x_1^{\mathrm{d}}(t),\ x_2^{\mathrm{d}}(t),\ \ldots,\ x_n^{\mathrm{d}}(t))^{\mathrm{T}} \in \mathbf{R}^n$ denotes the state vector of the drive system, $\boldsymbol{f} : \mathbf{R}^n \to \mathbf{R}^n$ is a continuously differentiable vector function determining the dynamic behavior of the drive system, and $\boldsymbol{\Delta}_{\mathrm{d}}$ contains the mismatched terms, including the modeling errors, parameter perturbations, noise disturbance, etc, which could exist in many control systems. If there are no mismatched terms in drive system (1), $\boldsymbol{\Delta}_{\mathrm{d}}$ is a zero vector. In Eq. (2), $\boldsymbol{x}_i^{\mathrm{r}}(t) = (x_{i1}^{\mathrm{r}}(t),\ x_{i2}^{\mathrm{r}}(t),\ \ldots,\ x_{in}^{\mathrm{r}}(t))^{\mathrm{T}} \in \mathbf{R}^n$ denotes the state vector of the i-th node, $\boldsymbol{g}_i : \mathbf{R}^n \to \mathbf{R}^n$ and $\boldsymbol{G}_i : \mathbf{R}^n \to \mathbf{R}^{n\times m_i}$ are known continuous nonlinear function matrices, $\boldsymbol{\theta}_i \in \mathbf{R}^{m_i}$ is an unknown constant parameter vector, $\boldsymbol{d}_i \geq 0$ is an unknown coupling delay, $\boldsymbol{\Delta}_i \in \mathbf{R}^n$ is the disturbance, $\boldsymbol{u}_i \in \mathbf{R}^n$ is the control input, and c is the coupling strength. $\boldsymbol{\Gamma}_1 = \mathrm{diag}(r_1, r_2, \ldots, r_n)$ and $\boldsymbol{\Gamma}_2 = \mathrm{diag}(\varsigma_1, \varsigma_2, \ldots, \varsigma_n)$ are the inner coupling matrices with $r_i = 1$ and $\varsigma_i = 1$ for the i-th state. This means that two nodes are coupled via the i-th state variable, i.e., $\boldsymbol{\Gamma}_1$ and $\boldsymbol{\Gamma}_2$ determine the variables with which the systems in the nodes are coupled. $\boldsymbol{A} = (a_{ij})_{N\times N} \in \mathbf{R}^{N\times N}$ and $\boldsymbol{B} = (b_{ij})_{N\times N} \in \mathbf{R}^{N\times N}$ are the weight configuration matrices, representing the topological structure of the network. If there is a connection from node i to node j $(j \neq i)$, then $b_{ij} \neq 0, a_{ij} \neq 0$; otherwise, $b_{ij} = a_{ij} = 0$ $(j \neq i)$. The diagonal elements of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, \quad b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}, \quad i = 1, 2, \ldots, N. \tag{3}$$

Definition 1 (FPS)[35] For the DRDN model with mismatched terms, it is said that the DRDN model (1) and (2) is FPS if there exists a continuously differentiable scaling function $\alpha(t)$ such that $\lim_{t\to\infty} \left\| \boldsymbol{x}_i^{\mathrm{r}} - \alpha(t)\boldsymbol{x}^{\mathrm{d}} \right\| = 0$, $i = 1, 2, \ldots, N$.

Let the error term be

$$\boldsymbol{e}_i = \boldsymbol{x}_i^{\mathrm{r}} - \alpha(t)\boldsymbol{x}^{\mathrm{d}}, \quad i = 1, 2, \ldots, N, \tag{4}$$

where $\alpha(t)$ is the scaling function. Our task is to find an appropriate controller so that the DRDN achieves synchronization, i.e., $\lim_{t\to\infty} \|\boldsymbol{e}_i\| = 0$, $i = 1, 2, \ldots, N$.

Assumption 1[36] For any positive constant ε_i, the time-varying disturbance $\boldsymbol{\Delta}_i(t)$ is bounded, i.e., $\|\boldsymbol{\Delta}_i(t)\| \leq \varepsilon_i$.

3. The controller design for FPS

In this section, we propose an adaptive method for the FPS between the uncertain complex dynamical network (2) with delayed coupling and the drive system (1).

Theorem 1 For a given synchronization scaling function $\alpha(t)$ and any initial conditions $\boldsymbol{x}_{\mathrm{d}}(0)$, $\boldsymbol{x}_{\mathrm{r}}(0)$, the DRDN (1) and (2) can realize FPS with the following control law:

$$\begin{aligned}\boldsymbol{u}_i &= \dot{\boldsymbol{y}}(t) - \boldsymbol{g}_i(\boldsymbol{y}(t)) - q_i\boldsymbol{e}_i(t) \\ &\quad - \boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\hat{\boldsymbol{\theta}}_i(t) - \beta_i(t)\mathrm{sgn}(\boldsymbol{e}_i(t)),\end{aligned} \tag{5}$$

$$\dot{q}_i = k_3\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t), \tag{6}$$

$$\dot{\hat{\boldsymbol{\theta}}}_i(t) = k_1\boldsymbol{G}_i^{\mathrm{T}}(\boldsymbol{x}_i^{\mathrm{r}}(t))\boldsymbol{e}_i(t), \tag{7}$$

$$\dot{\beta}_i(t) = k_2\boldsymbol{e}_i^{\mathrm{T}}(t)\mathrm{sgn}(\boldsymbol{e}_i(t)), \tag{8}$$

where k_1, k_2, k_3 are positive constants and $\boldsymbol{y}(t) = \alpha(t)\boldsymbol{x}^{\mathrm{d}}$. In Eq. (5), $\hat{\boldsymbol{\theta}}_i(t)$ is the estimated parameter for network (2).

Proof The time derivative of Eq. (4) is

$$\dot{\boldsymbol{e}}_i(t) = \dot{\boldsymbol{x}}_i^{\mathrm{r}}(t) - \dot{\boldsymbol{y}}_i(t), \quad i = 1, 2, \ldots, N. \tag{9}$$

Substituting Eq. (2) into Eq. (9), we have

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) &= \boldsymbol{g}_i(\boldsymbol{x}_i^{\mathrm{r}}(t)) + \boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\boldsymbol{\theta}_i + c\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{x}_j^{\mathrm{r}}(t) \\ &\quad + c\sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}_2\boldsymbol{x}_j^{\mathrm{r}}(t-d_i) + \boldsymbol{\Delta}_i + \boldsymbol{u}_i - \dot{\boldsymbol{y}}(t).\end{aligned} \tag{10}$$

The vector function $\boldsymbol{g}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))$ is linearized as follows in the neighborhood of the goal value via the Taylor expansions:

$$\begin{aligned}\boldsymbol{g}_i(\boldsymbol{x}_i^{\mathrm{r}}(t)) &= \boldsymbol{g}_i(\boldsymbol{y}(t)) + \frac{\partial \boldsymbol{g}_i(\boldsymbol{y}(t))}{\partial \boldsymbol{y}(t)}(\boldsymbol{x}_i^{\mathrm{r}}(t) - \boldsymbol{y}(t)) + \cdots, \\ &\quad i = 1, 2, \ldots, N.\end{aligned} \tag{11}$$

Keeping the first-order terms in Eq. (11) and substituting in Eq. (10), we have

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) &= \boldsymbol{J}_i\boldsymbol{e}_i(t) - \boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\tilde{\boldsymbol{\theta}}_i(t) - \beta_i(t)\mathrm{sgn}(\boldsymbol{e}_i(t)) \\ &\quad + c\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{e}_j(t) + c\sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}_2\boldsymbol{e}_j(t-d_i) \\ &\quad + \boldsymbol{\Delta}_i - q_i\boldsymbol{e}_i(t), \quad i = 1, 2, \ldots, N,\end{aligned} \tag{12}$$

where $\boldsymbol{J}_i = \partial \boldsymbol{g}_i(\boldsymbol{y}(t))/\partial \boldsymbol{y}(t)$ is the Jacobian matrix of $\boldsymbol{g}_i(\boldsymbol{y}(t))$ with respect to $\boldsymbol{y}$.

Choose the following Lyapunov function candidate:

$$\begin{aligned} V(t) = & \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_3}(q_i - q^*)^2 \\ & + \frac{1}{2}\int_{t-d_i}^{t}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(\xi)\boldsymbol{e}_i(\xi)\,\mathrm{d}\xi + \frac{1}{2k_1}\sum_{i=1}^{N}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(t)\tilde{\boldsymbol{\theta}}_i(t) \\ & + \frac{1}{2k_2}\sum_{i=1}^{N}\tilde{\beta}_i^2(t), \end{aligned}$$

where $\tilde{\beta}_i(t) = \beta_i(t) - \beta_i^*$, $\tilde{\boldsymbol{\theta}}_i(t) = \hat{\boldsymbol{\theta}}_i(t) - \boldsymbol{\theta}_i$, and β_i^* are the positive constants designed later.

The time derivative of $V(t)$ along the trajectories of Eq. (12) is

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\dot{\boldsymbol{e}}_i(t) + \sum_{i=1}^{N}\frac{1}{k_3}(q_i - q^*)\dot{q}_i \\ & + \frac{1}{2}\sum_{i=1}^{N}[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-d_i)\boldsymbol{e}_i(t-d_i)] \\ & + \frac{1}{k_1}\sum_{i=1}^{N}\dot{\hat{\boldsymbol{\theta}}}_i^{\mathrm{T}}(t)\tilde{\boldsymbol{\theta}}_i(t) + \frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t). \end{aligned}$$

By the application of control input (5) to error dynamics $\dot{\boldsymbol{e}}_i(t)$, we have

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)[\boldsymbol{J}_i\boldsymbol{e}_i(t) - \boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\tilde{\boldsymbol{\theta}}_i(t) \\ & - \beta_i(t)\mathrm{sgn}(\boldsymbol{e}_i(t)) + c\sum_{j=1}^{N}a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{e}_j(t) \\ & + c\sum_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}_2\boldsymbol{e}_j(t-d_i) + \boldsymbol{\Delta}_i \\ & - q_i\boldsymbol{e}_i(t)] + \sum_{i=1}^{N}\frac{1}{k_3}(q_i - q^*)\dot{q}_i \\ & + \frac{1}{2}\sum_{i=1}^{N}[\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \boldsymbol{e}_i^{\mathrm{T}}(t-d_i)\boldsymbol{e}_i(t-d_i)] \\ & + \frac{1}{k_1}\sum_{i=1}^{N}\dot{\hat{\boldsymbol{\theta}}}_i^{\mathrm{T}}(t)\tilde{\boldsymbol{\theta}}_i(t) + \frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t) \\ = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{J}_i\boldsymbol{e}_i(t) - \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{G}_i(\boldsymbol{x}_i^{\mathrm{r}}(t))\tilde{\boldsymbol{\theta}}_i(t) \\ & - \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\beta_i(t)\mathrm{sgn}(\boldsymbol{e}_i(t)) + c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{e}_j(t) \\ & + c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\Gamma_2\boldsymbol{e}_j(t-d_i) + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i \\ & - \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)q_i\boldsymbol{e}_i(t) + \sum_{i=1}^{N}\frac{1}{k_3}(q_i - q^*)\dot{q}_i \\ & + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-d_i)\boldsymbol{e}_i(t-d_i) \\ & + \sum_{i=1}^{N}\frac{1}{k_1}\dot{\hat{\boldsymbol{\theta}}}_i^{\mathrm{T}}(t)\tilde{\boldsymbol{\theta}}_i(t) + \frac{1}{k_2}\sum_{i=1}^{N}\dot{\beta}_i(t)\tilde{\beta}_i(t). \end{aligned}$$

By using the adaptation laws (6)–(8),

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{J}_i\boldsymbol{e}_i(t) + c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}a_{ij}\boldsymbol{\Gamma}_1\boldsymbol{e}_j(t) \\ & + c\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\sum_{j=1}^{N}b_{ij}\boldsymbol{\Gamma}_2\boldsymbol{e}_j(t-d_i) + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i \\ & - \sum_{i=1}^{N}q^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & - \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t-d_i)\boldsymbol{e}_i(t-d_i) - \sum_{i=1}^{N}\beta_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\mathrm{sgn}(\boldsymbol{e}_i(t)). \end{aligned}$$

Let $\boldsymbol{e}(t) = (\boldsymbol{e}_1^{\mathrm{T}}(t),\ \boldsymbol{e}_2^{\mathrm{T}}(t),\ \ldots,\ \boldsymbol{e}_N^{\mathrm{T}}(t))^{\mathrm{T}} \in \mathbf{R}^{nN}$, $\boldsymbol{Q} = (\boldsymbol{A}\otimes\boldsymbol{\Gamma}_1)$, $\boldsymbol{P} = (\boldsymbol{B}\otimes\boldsymbol{\Gamma}_2)$, where $\otimes$ represents the Kronecker product,

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{J}_i\boldsymbol{e}_i(t) + c\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}(t) + c\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t-d_i) \\ & + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i - \sum_{i=1}^{N}q^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t-d_i)\boldsymbol{e}(t-d_i) \\ & - \sum_{i=1}^{N}\beta_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\mathrm{sgn}(\boldsymbol{e}_i(t)). \end{aligned}$$

By the use of the fact that $2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \le \boldsymbol{x}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{x} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{S}^{-1}\boldsymbol{y}$ for any vectors $\boldsymbol{x},\boldsymbol{y} \in \mathbf{R}^m$ and a positive definite matrix $\boldsymbol{S} \in \mathbf{R}^{m\times m}$, we have

$$\begin{aligned} \dot{V}(t) \le & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{J}_i\boldsymbol{e}_i(t) + c\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}(t) + \frac{1}{2}c^2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}(t) \\ & + \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t-d_i)\boldsymbol{e}(t-d_i) + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i \\ & - \sum_{i=1}^{N}q^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & - \frac{1}{2}\boldsymbol{e}^{\mathrm{T}}(t-d_i)\boldsymbol{e}(t-d_i) - \sum_{i=1}^{N}\beta_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\mathrm{sgn}(\boldsymbol{e}_i(t)) \\ = & \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{J}_i\boldsymbol{e}_i(t) + c\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}(t) + \frac{1}{2}c^2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}(t) \\ & + \sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i - \sum_{i=1}^{N}q^*\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) \\ & + \frac{1}{2}\sum_{i=1}^{N}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t) - \sum_{i=1}^{N}\beta_i^*\boldsymbol{e}_i^{\mathrm{T}}(t)\mathrm{sgn}(\boldsymbol{e}_i(t)). \end{aligned}$$

Let $\boldsymbol{\Theta} = \mathrm{diag}(\boldsymbol{J}_1, \boldsymbol{J}_2, \ldots, \boldsymbol{J}_N)$, from Assumption 1, we have

$$\begin{aligned} \dot{V}(t) \le & \boldsymbol{e}^{\mathrm{T}}(t)\left(\boldsymbol{\Theta} + c\boldsymbol{Q} + \frac{1}{2}c^2\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P} + \frac{1}{2}\boldsymbol{I}\right)\boldsymbol{e}(t) \\ & - q^*\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) + \sum_{i=1}^{N}[\varepsilon_i - \beta_i^*]\,\|\boldsymbol{e}_i(t)\|. \end{aligned}$$

Let $\lambda = \lambda_{\max}(\boldsymbol{\Theta} + c\boldsymbol{Q} + \frac{1}{2}c^2\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P} + \frac{1}{2}\boldsymbol{I})$, we obtain

$$\dot{V}(t) \le (\lambda - q^*)\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) + \sum_{i=1}^{N}[\varepsilon_i - \beta_i^*]\,\|\boldsymbol{e}_i(t)\|.$$

Therefore, by taking appropriate q^* and β_i^* such that

$$(\lambda - q^*) < 0, \tag{13}$$

$$\varepsilon_i - \beta_i^* < 0, \quad \text{for} \quad i = 1,2,\ldots,N. \tag{14}$$

Based on the LaSalle invariance principle, we can obtain $\dot{V}(t) \leq 0$, which means that the FPS between network (2) and reference node (1) is achieved with the control input (5) and update laws (6)–(8). This completes the proof.

4. Illustrative example

In this section, we will study the FPS in the DRDN with uncertain terms. We take the Lorenz system as the drive system, which is described as follows:

$$\begin{cases} \dot{x}_1^{\mathrm{d}} = 10(x_2^{\mathrm{d}} - x_1^{\mathrm{d}}) + 2\cos(t), \\ \dot{x}_2^{\mathrm{d}} = (28 - x_3^{\mathrm{d}})x_1^{\mathrm{d}} - x_2^{\mathrm{d}} + 3\sin(t), \\ \dot{x}_3^{\mathrm{d}} = x_2^{\mathrm{d}} x_1^{\mathrm{d}} - (8/3)x_3^{\mathrm{d}} + 0.1\sin(t), \end{cases}$$

where $\boldsymbol{\Delta}_{\mathrm{d}} = \left[\, 2\cos(t) \;\; 3\sin(t) \;\; 0.1\sin(t) \,\right]$ denotes the uncertain terms.

The response network is described as follows:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{i1}^{\mathrm{r}}(t) \\ \dot{x}_{i2}^{\mathrm{r}}(t) \\ \dot{x}_{i3}^{\mathrm{r}}(t) \end{bmatrix} &= \begin{bmatrix} -x_{i2}^{\mathrm{r}}(t) - x_{i3}^{\mathrm{r}}(t) \\ x_{i1}^{\mathrm{r}}(t) \\ x_{i1}^{\mathrm{r}}(t)x_{i3}^{\mathrm{r}}(t) + 0.2 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ x_{i2}^{\mathrm{r}}(t) & 0 \\ 0 & -x_{i3}^{\mathrm{r}}(t) \end{bmatrix} \begin{bmatrix} \theta_{i1} \\ \theta_{i2} \end{bmatrix} \\ &\quad + c\sum_{j=1}^{N} a_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j^{\mathrm{r}}(t) + c\sum_{j=1}^{N} b_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j^{\mathrm{r}}(t - d_i) \\ &\quad + \boldsymbol{\Delta}_{\mathrm{r}}(t) + \boldsymbol{u}_i(t), \end{aligned}$$

where $i = 1,2,\ldots,N$, the unknown parameter vector $\boldsymbol{\theta}_{\mathrm{i}} = [\theta_{i1} \;\; \theta_{i2}]^{\mathrm{T}} = [2 \;\; 10]^{\mathrm{T}}$, $c = 0.1$, $d = 2$, $N = 5$, and $\boldsymbol{\Gamma} = \boldsymbol{I}_{3\times 3}$. The two coupling matrices are chosen as

$$\boldsymbol{A} = \boldsymbol{B} = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 1 & 1 & -3 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}.$$

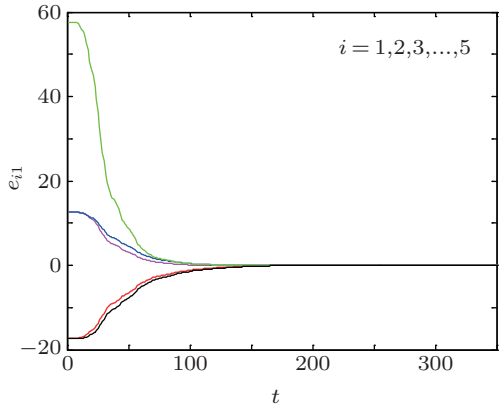


Fig. 1. (color online) The time evolution of errors e_{i1}.

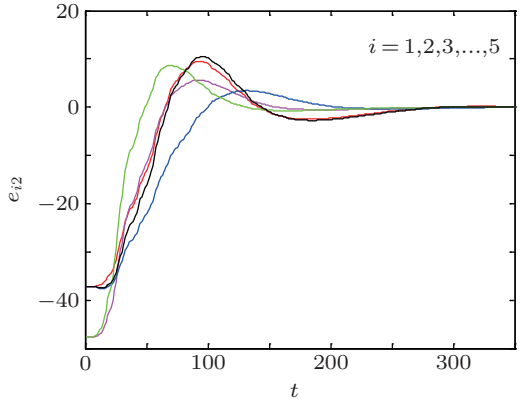


Fig. 2. (color online) The time evolution of errors e_{i2}.

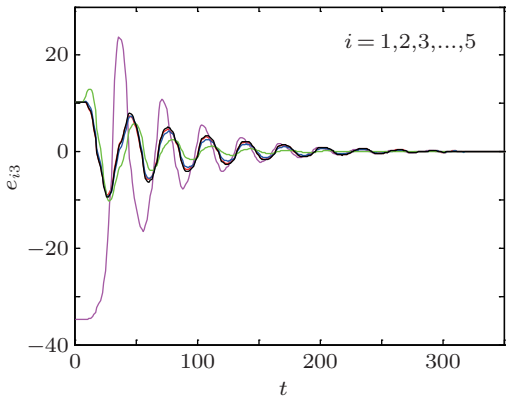


Fig. 3. (color online) The time evolution of errors e_{i3}.

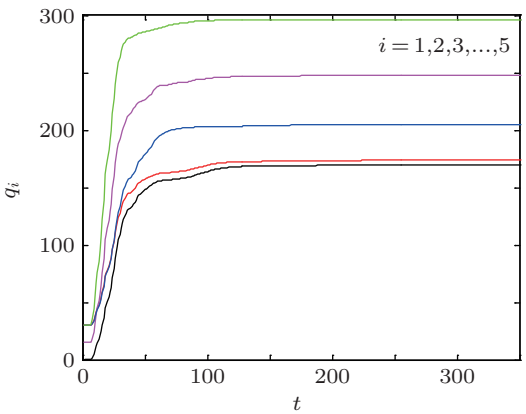


Fig. 4. (color online) The time evolution of feedback gains with mismatched terms.

We choose the disturbance $\boldsymbol{\Delta}_{\mathrm{r}}(t) = \begin{bmatrix} 0.3\cos(t)\sin(t) \\ 0.1\sin(t) \\ 0.5\cos(t) \end{bmatrix}$ and take $\alpha(t) = 2 + \sin(t)$. The controller $\boldsymbol{u}_i$ can be designed by Eq. (5) in Theorem 1 and $k_1 = 5$, $k_2 = 8$, $k_3 = 15$.

The simulation results are given in Figs. 1–6, and the numerical results show that the theoretical results are effective.

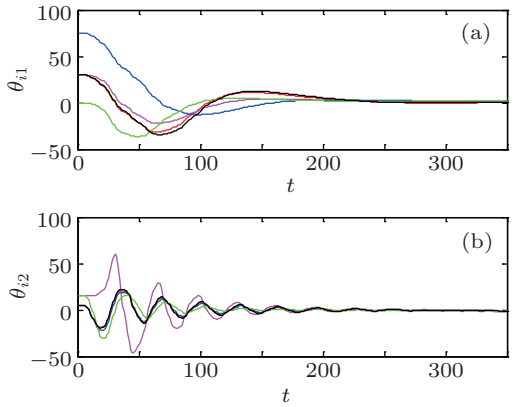


Fig. 5. (color online) The estimated parameters (a) θ_{i1} and (b) θ_{i2}, $i = 1, 2, \ldots, 5$.

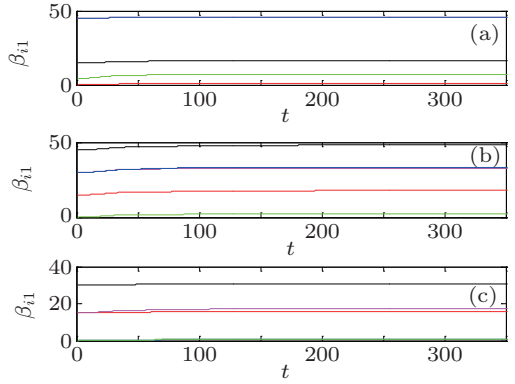


Fig. 6. (color online) The estimated parameters (a) β_{i1} and (b) β_{i2}, $i = 1, 2, \ldots, 5$.

5. Conclusion

In this paper, a general FPS scheme is proposed for complex dynamical networks with uncertain parameters and disturbance. A simple adaptive controller is designed to achieve the FPS. The unknown parameters are estimated using the adaptive laws obtained based on the Lyapunov stability theory and Taylor expansions. Even if there are unknown bounded disturbances, the proposed controller with the estimated parameters can also achieve the FPS of the drive–response network. In addition, numerical simulations are performed to verify the effectiveness of the theoretical results.

References

[1] Wen G H, Duan Z S, Li Z K and Chen G R 2012 *International Journal of Control* **85** 384
[2] Wen G H, Duan Z S, Yu W W and Chen G R 2012 *International Journal of Robust and Nonlinear Control* **23** 602
[3] Xiong X and Hu Y 2012 *Acta Phys. Sin.* **61** 150509 (in Chinese)
[4] Chou S W, Wang K and Liu Q 2012 *Acta Phys. Sin.* **61** 150201 (in Chinese)
[5] Hao C Q, Wang J and Deng B 2012 *Acta Phys. Sin.* **61** 148901 (in Chinese)
[6] Zhang C, Shen H Z and Li F 2012 *Acta Phys. Sin.* **61** 148902 (in Chinese)
[7] Wang D L and Yu Z G 2012 *Chin. Phys. B* **21** 080504
[8] Gao X Y, An H Z and Fang W 2012 *Acta Phys. Sin.* **61** 098902 (in Chinese)
[9] Wang Y Q and Yang X Y 2012 *Acta Phys. Sin.* **61** 090202 (in Chinese)
[10] Wang S G and Yao H X 2012 *Chin. Phys. B* **21** 050508
[11] Liang Y and Wang X Y 2012 *Acta Phys. Sin.* **61** 038901 (in Chinese)
[12] Chai Y, Lv L and Chen L Q 2012 *Chin. Phys. B* **21** 030506
[13] Yang C L and Tang K S 2011 *Chin. Phys. B* **20** 128901
[14] Rosenblum M G, Pikovsky A S and Kurths J 1996 *Phys. Rev. Lett.* **76** 1804
[15] Rosenblum M G, Pikovsky A S and Kurths J 1997 *Phys. Rev. Lett.* **78** 4193
[16] Xu S Y and Yang Y 2010 *Nonlinear Dynamics* **59** 485
[17] Wang J L and Wu H N 2012 *Nonlinear Dynamics* **67** 497
[18] Lü L, Meng L, Guo L, Zou J R and Yang M 2011 *Acta Phys. Sin.* **60** 124 (in Chinese)
[19] Abdurahman K, Wang X Y and Zhao Y Z 2011 *Acta Phys. Sin.* **60** 81 (in Chinese)
[20] Liu H, Chen J, Lu J and Cao M 2010 *Physica A* **389** 1759
[21] Guo W 2011 *Nonlinear Analysis: Real World Applications* **12** 2579
[22] Mahmoud G M and Mahmoud E E 2010 *Nonlinear Dynamics* **62** 875
[23] Mainieri R and Rehacek J 1999 *Phys. Rev. Lett.* **82** 3042
[24] Chen Y and Li X 2007 *Int. J. Mod. Phys. C* **18** 883
[25] Chen Y, An H L and Li Z B 2008 *Appl. Math. Comput.* **197** 96
[26] Du H, Zeng Q and Wang C 2009 *Chaos Soliton. Fract.* **42** 2399
[27] Sebastian S K and Sabir M 2010 *Phys. Lett. A* **374** 2017
[28] Yang W and Sun J 2010 *Phys. Lett. A* **374** 557
[29] Sudheer K S and Sabir M 2009 *Phys. Lett. A* **373** 1847
[30] Du H Y, Zeng Q S, Wang C H and Ling M X 2010 *Nonlinear Analysis: Real World Applications* **11** 705
[31] Du H Y, Zeng Q S and Wang C H 2008 *Phys. Lett. A* **372** 5402
[32] Shen L Q, Liu W Y and Ma J W 2009 *Chaos Soliton. Fract.* **42** 1292
[33] Zhang R, Yang Y Q, Xu Z Y and Hu M F 2010 *Phys. Lett. A* **374** 3025
[34] Solís-Perales G Ruiz-Velázquez E and Valle-Rodríguez D 2009 *Commun. Nonlinear Sci. Numer. Simulat.* **14** 2528
[35] Du H Y 2011 *Chaos Soliton. Fract.* **44** 510
[36] Ji D H, Jeong S C, Park J H, Lee S M and Won S C 2012 *Applied Mathematics and Computation* **218** 4872