Robust finite-time stabilization of unified chaotic complex systems with certain and uncertain parameters*

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This paper deals with the finite-time stabilization of unified chaotic complex systems with known and unknown parameters. Based on the finite-time stability theory, nonlinear control laws are presented to achieve finite-time chaos control of the determined and uncertain unified chaotic complex systems, respectively. The two controllers are simple, and one of the uncertain unified chaotic complex systems is robust. For the design of a finite-time controller on uncertain unified chaotic complex systems, only some of the unknown parameters need to be bounded. Simulation results for the chaotic complex Lorenz, Lü and Chen systems are presented to validate the design and analysis.

Keywords: finite-time stability, uncertain parameters, unified chaotic complex systems

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1. Introduction

In recent years, the theory and methods of the controlling chaotic system have received a great deal of attention among scientists, and have also been extensively studied due to potential applications in secure communication, modeling brain activity, chemical reactions, ecological systems, etc.[1–4] Several approaches to chaos control have been proposed in existing works. For example, active control,[5] nonlinear control,[6] adaptive control,[7] sliding mode control,[8] and the backstepping method.[9] These control approaches can stabilize the chaotic system asymptotically.

However, in the actual engineering process, we may need to stabilize the system as quickly as possible. Therefore, it is important to study finite-time stabilization of the chaotic system. Attaining finite-time stability of the controlled systems requires finite time control techniques, which demonstrate better robustness as well as disturbance rejection properties.[10] Finite-time stability, i.e., the solution of an asymptotic system converging to a desired target (in general, referring to the origin) in a finite time, which is basically concerned with finite time control, implies optimality in stable time.[11]

Many research works on finite time chaos control have been performed.[12–14] Wang *et al.*[15] proposed terminal slide mode control for finite-time synchronization of the Duffing chaotic system. Based on the finite-time stability theory and Lyapunov function, Yu[16] investigated the stabilization of three-dimensional chaotic systems in a finite time. Guo and Vincent[17] proposed a single control input approach for stabilizing three-dimensional chaotic systems in a finite time, and Gao *et al.*[18] realized the robust finite time synchronization of chaotic systems.

There also exist, however, interesting cases of dynamical systems where the main variables of the dynamical systems are complex. For example, the complex Lorenz equations were used to describe and simulate the physics of a detuned laser and the thermal convection of liquid flows.[19,20] The electric field amplitude and the atomic polarization amplitude are both complex; for details see Ref. [21] and the references therein. Another example is when the synchronization of chaotic complex systems is used to transmit information in secure communications, where the complex variable (the doubling number of real variables) may increase the message content and enhance the security of the transmitted information.[22,23] In recent years, several nonlinear systems with complex variables have been introduced, and their chaotic behaviors analyzed.[19,24] In particular, the control of chaotic complex systems with known and unknown parameters has been studied well, and many methods have been applied to achieve the synchronization and stability of chaotic complex systems.[25–27] However, the finite-time stabilization of chaotic complex systems with known and unknown parameters still need to be studied further.

Motivated by the above discussion, this paper addresses the finite-time stabilization of unified chaotic complex systems from the so-called unified chaotic systems presented in Ref. [28], which are a generalized form of the Lü, Chen, and Lorenz systems. Based on the finite-time stability theory of a cascade-connected system, we present two continuous controllers to realize finite-time chaos control of the general chaotic complex system with certain and uncertain parameters, respectively. The controllers are simple to construct and robust to parameter uncertainties.

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2. Preliminary definitions and lemmas and system description

We give the precise definition of finite-time stability for chaotic complex systems and introduce some necessary lemmas. Consider the following dynamic complex system:

$$\dot{z}(t) = f(z), \tag{1}$$

where $z(t) \in R^n$ is the system state. If there exists a constant $T > 0$ [T may depend on the initial state $z(0)$], such that

$$\lim_{t\to T} \|z(t)\| = \lim_{t\to T} \sqrt{\|z^{\mathrm{r}}(t)\|^2 + \|z^{\mathrm{i}}(t)\|^2} = 0$$

and $\|z(t)\| = \sqrt{\|z^{\mathrm{r}}(t)\|^2 + \|z^{\mathrm{i}}(t)\|^2} \equiv 0$, if $t \geq T$, then the complex system (1) is finite-time stable.

Lemma 1[29] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -cV^{\eta} \quad \forall t \geq t_0, \ V(t_0) \geq 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), \quad t_0 \leq t \leq t_1 \tag{3}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1 \tag{4}$$

with

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof 1 Consider the following differential equation:

$$\dot{X}(t) = -cX^{\eta}(t), \quad X(t_0) = V(t_0). \tag{6}$$

Although this differential equation does not satisfy the global Lipschitz condition, the unique solution of Eq. (6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{7}$$

Therefore, from the comparison Lemma,[30] one obtains

$$V^{1-\eta}(t) = V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), \quad t_0 \leq t \leq t_1 \tag{8}$$

and

$$V(t) = 0, \quad \forall t \geq t_1$$

with t_1 given in Eq. (5).

Lemma 2 When a, b, and $c < 1$ are all positive numbers, the following inequality holds:

$$(a+b)^c \leq a^c + b^c. \tag{9}$$

Proof 2 It is straightforward to prove that the function $f(\lambda) := \lambda^c + (1-\lambda)^c$ arrives at its minimal value 1 at $\lambda = 1/2$ when $0 < \lambda < 1$ and $0 < c < 1$. Thus, $f(\lambda) \geq 1$. Since a and b are positive numbers, namely, $a + b \neq 0$, set $\lambda = a/(a+b)$. Then

$$f(a,b) = \Big(\frac{a}{a+b}\Big)^c + \Big(1 - \frac{a}{a+b}\Big)^c = \frac{a^c}{(a+b)^c} + \frac{b^c}{(a+b)^c} \geq 1.$$

Hence, $(a+b)^c \leq a^c + b^c$.

Referring to the unified chaotic system,[28] we consider the following general chaotic complex system:

$$\begin{cases} \dot{z}_1 = (25\alpha + 10)(z_2 - z_1), \\ \dot{z}_2 = (28 - 35\alpha)z_1 + (29\alpha - 1)z_2 - z_1 z_3, \\ \dot{z}_3 = \dfrac{1}{2}(\tilde{z}_1 z_2 + z_1 \tilde{z}_2) - \dfrac{(8+\alpha)z_3}{3}, \end{cases} \tag{10}$$

where z_1, z_2, and z_3 are state variables and $\alpha \in [0,1]$ is the system parameter. System (10) is called the unified chaotic complex system because system (10) is chaotic for any $\alpha \in [0,1]$. When $0 \leq \alpha < 0.8$, system (10) is called the generalized complex Lorenz system, and the complex Lorenz system ($\alpha = 0$) has been discussed in Ref. [24]. When $\alpha = 0.8$, system (10) is called the complex Lü system.[25] When $0.8 < \alpha \leq 1$, system (10) is called the generalized complex Chen system, and the complex Chen system ($\alpha = 1$) has also been discussed in Ref. [24]. In the following, we consider the finite-time stability of unified chaotic complex systems.

3. Finite-time stability of determined unified chaotic complex systems

Consider the controlled chaotic complex system

$$\begin{cases} \dot{z}_1 = (25\alpha + 10)(z_2 - z_1) + u_1, \\ \dot{z}_2 = (28 - 35\alpha)z_1 + (29\alpha - 1)z_2 - z_1 z_3 + u_2, \\ \dot{z}_3 = \dfrac{1}{2}(\tilde{z}_1 z_2 + z_1 \tilde{z}_2) - \dfrac{(8+\alpha)z_3}{3} + u_3. \end{cases} \tag{11}$$

Next, we will design a complex controller vector to stabilize the unified chaotic complex system (11) in a finite time. The design procedure also consists of two steps.

Step 1 Let the complex controller $u_1 = -(25\alpha + 10)z_2 - [(z_1^{\mathrm{r}})^{\beta} + \mathrm{j}(z_1^{\mathrm{i}})^{\beta}]$, where $\beta = q/p$ and $p > q$ are positive odd integers. Introducing the controller u_1, the first equation of Eq. (11) is

$$\dot{z}_1 = -(25\alpha + 10)z_1 - [(z_1^{\mathrm{r}})^{\beta} + \mathrm{j}(z_1^{\mathrm{i}})^{\beta}]. \tag{12}$$

Consider the candidate Lyapunov function $V_1 = z_1 \tilde{z}_1/2$, where $\tilde{z}$ denotes the conjugate of variable z, so $V_1 = [(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2]/2$. The time derivative of V_1 along the trajectory of Eq. (12) is

$$\begin{aligned} \dot{V}_1 &= z_1^{\mathrm{r}}[-(25\alpha + 10)z_1^{\mathrm{r}} - (z_1^{\mathrm{r}})^{\beta}] \\ &\quad + z_1^{\mathrm{i}}[-(25\alpha + 10)z_1^{\mathrm{i}} - (z_1^{\mathrm{i}})^{\beta}] \\ &= -(25\alpha + 10)(z_1^{\mathrm{r}})^2 - (z_1^{\mathrm{r}})^{\beta+1} \\ &\quad - (25\alpha + 10)(z_1^{\mathrm{i}})^2 - (z_1^{\mathrm{i}})^{\beta+1} \\ &\leq -(z_1^{\mathrm{r}})^{\beta+1} - (z_1^{\mathrm{i}})^{\beta+1} \end{aligned}$$

$$= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left[\frac{1}{2}(z_1^{\mathrm{r}})^2\right]^{(1+\beta)/2} - \left(\frac{1}{2}\right)^{-(1+\beta)/2}\left[\frac{1}{2}(z_1^{\mathrm{i}})^2\right]^{(1+\beta)/2}$$
$$= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left\{\left[\frac{1}{2}(z_1^{\mathrm{r}})^2\right]^{(1+\beta)/2} + \left[\frac{1}{2}(z_1^{\mathrm{i}})^2\right]^{(1+\beta)/2}\right\}. \quad (13)$$

According to Lemma 2, we have

$$\left[\frac{1}{2}(z_1^{\mathrm{r}})^2\right]^{(1+\beta)/2} + \left[\frac{1}{2}(z_1^{\mathrm{i}})^2\right]^{(1+\beta)/2} \geq \left\{\frac{1}{2}[(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2]\right\}^{(1+\beta)/2},$$

then

$$-\left[\frac{1}{2}(z_1^{\mathrm{r}})^2\right]^{(1+\beta)/2} - \left[\frac{1}{2}(z_1^{\mathrm{i}})^2\right]^{(1+\beta)/2} \leq -\left\{\frac{1}{2}[(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2]\right\}^{(1+\beta)/2}.$$

Hence, we obtain

$$\dot{V}_1 = -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left\{\left[\frac{1}{2}(z_1^{\mathrm{r}})^2\right]^{(1+\beta)/2} + \left[\frac{1}{2}(z_1^{\mathrm{i}})^2\right]^{(1+\beta)/2}\right\}$$
$$\leq -\left\{\frac{1}{2}[(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2]\right\}^{-(1+\beta)/2}\left(\frac{1}{2}\right)^{-(1+\beta)/2}$$
$$= -\left(\frac{1}{2}\right)^{-(1+\beta)/2} {V_1}^{(1+\beta)/2}. \quad (14)$$

Since $0 < \beta < 1$, then $0 < (1+\beta)/2 < 1$. From Lemma 1, state z_1 will reach $z_1 = 0$ at a finite time $T_1 = [z_1^{\mathrm{r}}(0) + z_1^{\mathrm{i}}(0)]/(1-\beta)$.

Step 2 Let $u_2 = -L_1 z_2 - [(z_2^{\mathrm{r}})^\beta + \mathrm{j}(z_2^{\mathrm{i}})^\beta]$, where $L_1 \geq 29\alpha - 1$ and $u_3 = -L_2 z_3 - z_3^\beta$, where $L_2 \geq -(8+\alpha)/3$. Then when $t > T_1$, the last two equations of the system (11) become

$$\begin{cases} \dot{z}_2 = (29\alpha - 1)z_2 - L_1 z_2 - [(z_2^{\mathrm{r}})^\beta + \mathrm{j}(z_2^{\mathrm{i}})^\beta], \\ \dot{z}_3 = -\left(\dfrac{8+\alpha}{3}\right) z_3 - L_2 z_3 - z_3^\beta. \end{cases} \quad (15)$$

Choose the Lyapunov function for Eq. (15) as follows

$$V_2 = \frac{1}{2}(z_2\tilde{z}_2 + z_3^2) = \frac{1}{2}[(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2 + z_3^2].$$

The derivative of V_2 along the trajectories of Eq. (15) is

$$\dot{V}_2 = \dot{z}_2^{\mathrm{r}} z_2^{\mathrm{r}} + \dot{z}_2^{\mathrm{i}} z_2^{\mathrm{i}} + \dot{z}_3 z_3$$
$$= [(29\alpha - 1)z_2^{\mathrm{r}} - L_1 z_2^{\mathrm{r}} - (z_2^{\mathrm{r}})^\beta] z_2^{\mathrm{r}} + [(29\alpha - 1)z_2^{\mathrm{i}} - L_1 z_2^{\mathrm{i}} - (z_2^{\mathrm{i}})^\beta] z_2^{\mathrm{i}} + \left[-\left(\frac{8+\alpha}{3}\right) z_3 - L_2 z_3 - z_3^\beta\right] z_3$$
$$= -[L_1 - (29\alpha - 1)][(z_2^{\mathrm{r}})^2 + (z_2^{\mathrm{i}})^2] - \left[L_2 + \frac{8+\alpha}{3}\right] z_3^2 - (z_2^{\mathrm{r}})^{\beta+1} - (z_2^{\mathrm{i}})^{\beta+1} - z_3^{\beta+1}. \quad (16)$$

Also, $L_1 \geq 29\alpha - 1$ and $L_2 \geq -(8+\alpha)/3$. Hence

$$\dot{V}_2 \leq -(z_2^{\mathrm{r}})^{\beta+1} - (z_2^{\mathrm{i}})^{\beta+1} - z_3^{\beta+1}$$
$$= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}\left[\frac{1}{2}(z_2^{\mathrm{r}})^2\right]^{(1+\beta)/2} - \left(\frac{1}{2}\right)^{-(1+\beta)/2}\left[\frac{1}{2}(z_2^{\mathrm{i}})^2\right]^{(1+\beta)/2} - \left(\frac{1}{2}\right)^{-(1+\beta)/2}\left(\frac{1}{2}z_3^2\right)^{(1+\beta)/2}. \quad (17)$$

Similarly, according to Lemma 2, we have

$$\dot{V}_2 \leq -\left\{\frac{1}{2}[(z_2^{\mathrm{r}})^2 + (z_2^{\mathrm{i}})^2 + z_3^2]\right\}^{(1+\beta)/2}\left(\frac{1}{2}\right)^{-(1+\beta)/2}$$
$$= -\left\{\frac{1}{2}[z_2\tilde{z}_2 + z_3^2]\right\}^{(1+\beta)/2}\left(\frac{1}{2}\right)^{-(1+\beta)/2}$$
$$= -\left(\frac{1}{2}\right)^{-(1+\beta)/2} {V_2}^{(1+\beta)/2}. \quad (18)$$

Since $0 < \beta < 1, 0 < (\beta+1)/2 < 1$. From Lemma 1, the complex state variables z_2 and z_3 will converge to $z_2 = 0, z_3 = 0$ at a finite time T_2. Then after T_2, the complex state variables of the system (11) will stay at $z_1 = 0, z_2 = 0, z_3 = 0$. This means that controlled unified chaotic systems with certain parameters are finite-time stable by the controller $u_2 = -L_1 z_2 - [(z_2^{\mathrm{r}})^\beta + \mathrm{j}(z_2^{\mathrm{i}})^\beta]$ and $u_3 = -L_2 z_3 - z_3^\beta$.

To demonstrate the effectiveness of the proposed method, we present the simulation results for the finite-time stability of the unified certain chaotic system. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve systems with a time step size of 0.001. The initial condition of the chaotic system is adopted as $(z_1(0), z_2(0), z_3(0)) = (-5 + 3\mathrm{j}, 5 - 4\mathrm{j}, 10)$. The controller gains are chosen as $L_1 = 29\alpha + 1$, $L_2 = (8+\alpha)/3 + 1$, $\beta = 1/3$. Figures 1, 2, and 3 show the simulation results for the certain complex Lorenz, Lü and Chen

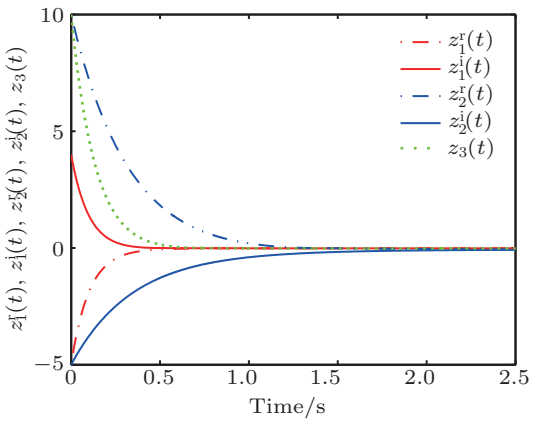


Fig. 1. (color online) The real and imaginary part responses of the complex state variables of the controlled complex Lorenz system ($\alpha = 0$).

systems, respectively. From the figures, we can see that the controlled chaotic complex systems are finite-time stable.

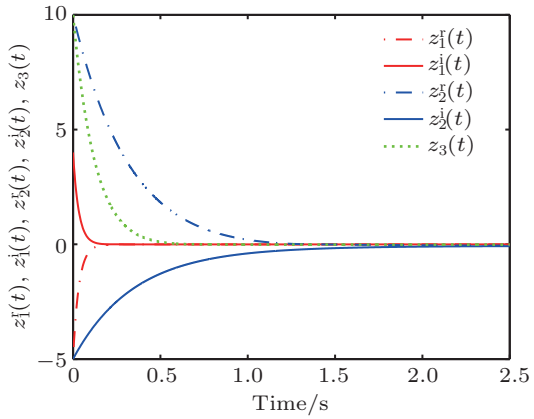


Fig. 2. (color online) The real and imaginary part responses of the complex state variables of the controlled complex Lü system ($\alpha = 0.8$).

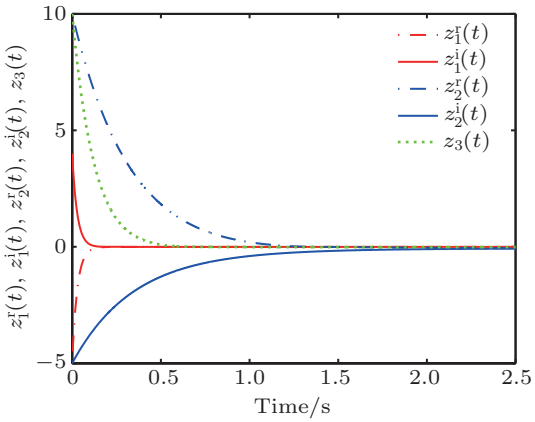


Fig. 3. (color online) The real and imaginary part responses of the complex state variables of the controlled complex Chen system ($\alpha = 1$).

4. Finite-time stability of the unified chaotic complex systems with unknown parameters

It is known that the parameters of chaotic complex systems may vary due to disturbance in practice, and their presence will lead to serious degradation of system performance. Therefore, their effect cannot be ignored in the system design. In this section, our aim is to design a control law that realizes finite-time stability for unified chaotic complex systems with unknown parameters.

Consider the following unified chaotic complex systems with uncertain parameters:

$$\begin{cases} \dot{z}_1 = (25\alpha + 10)(z_2 - z_1) + u_1', \\ \dot{z}_2 = (28 - 35\alpha + \Delta_1)z_1 \\ \qquad + (29\alpha - 1 + \Delta_2)z_2 - z_1 z_3 + u_2', \\ \dot{z}_3 = \dfrac{1}{2}(\tilde{z}_1 z_2 + z_1 \tilde{z}_2) - \left(\dfrac{8+\alpha}{3} + \Delta_3\right) z_3 + u_3', \end{cases} \tag{19}$$

where Δ_i $(i = 1, 2, 3)$ denote the bounded uncertain parameters, namely, $|\Delta_i| \le \delta_i$ $(i = 1, 2, 3)$. In following, we will design a complex robust controller vector $U = (u_1', u_2', u_3')^{\mathrm{T}}$ to stabilize the uncertain unified chaotic complex systems (19) in a finite time. The design procedure also consists of two steps.

Step 1 Let the complex controller $u_1' = -(25\alpha + 10)z_2 - [(z_1^{\mathrm{r}})^\beta + \mathrm{j}(z_1^{\mathrm{i}})^\beta]$, where $\beta = q/p$ and $p > q$ are positive odd integers. Introducing the controller u_1', the first equation of Eq. (19) is

$$\dot{z}_1 = -(25\alpha + 10)z_1 - [(z_1^{\mathrm{r}})^\beta + \mathrm{j}(z_1^{\mathrm{i}})^\beta]. \tag{20}$$

According to the discussion of system (12), we know that system (20) is also finite-time stable, which means $z_1 \equiv 0$ after a finite time T_3.

Step 2 Let $u_2' = -L_2 z_2 - [(z_2^{\mathrm{r}})^\beta + \mathrm{j}(z_2^{\mathrm{i}})^\beta]$, where $L_2 \ge 29\alpha - 1 + \delta_2$ and $u_3' = -L_3 z_3 - z_3^\beta$, where $L_3 \ge -(8+\alpha)/3 + \delta_3$. Then when $t > T_3$, the last two equations of the system (19) become

$$\begin{cases} \dot{z}_2 = (29\alpha - 1 + \Delta_2)z_2 - L_2 z_2 - [(z_2^{\mathrm{r}})^\beta + \mathrm{j}(z_2^{\mathrm{i}})^\beta], \\ \dot{z}_3 = -\left(\dfrac{8+\alpha}{3} + \Delta_3\right) z_3 - L_3 z_3 - z_3^\beta. \end{cases} \tag{21}$$

Choose the Lyapunov function for Eq. (21) as

$$V_3 = \frac{1}{2}(z_2 \tilde{z}_2 + z_3^2) = \frac{1}{2}[(z_1^{\mathrm{r}})^2 + (z_1^{\mathrm{i}})^2 + z_3^2].$$

The derivative of V_2 along the trajectories of Eq. (21) is

$$\begin{aligned} \dot{V}_3 &= \dot{z}_2^{\mathrm{r}} z_2^{\mathrm{r}} + \dot{z}_2^{\mathrm{i}} z_2^{\mathrm{i}} + \dot{z}_3 z_3 \\ &= [(29\alpha - 1 + \Delta_2)z_2^{\mathrm{r}} - L_2 z_2^{\mathrm{r}} - (z_2^{\mathrm{r}})^\beta] z_2^{\mathrm{r}} \\ &\quad + [(29\alpha - 1 + \Delta_2)z_2^{\mathrm{i}} - L_2 z_2^{\mathrm{i}} - (z_2^{\mathrm{i}})^\beta] z_2^{\mathrm{i}} \\ &\quad + \left[-\left(\frac{8+\alpha}{3} + \Delta_3\right) z_3 - L_3 z_3 - z_3^\beta \right] z_3 \\ &= (29\alpha - 1 + \Delta_2)(z_2^{\mathrm{r}})^2 - L_2 (z_2^{\mathrm{r}})^2 - (z_2^{\mathrm{r}})^{\beta+1} \\ &\quad + (29\alpha - 1 + \Delta_2)(z_2^{\mathrm{i}})^2 - L_2 (z_2^{\mathrm{i}})^2 \\ &\quad - (z_2^{\mathrm{i}})^{\beta+1} - \left(\frac{8+\alpha}{3} + \Delta_3\right) z_3^2 - L_3 z_3^2 - z_3^{\beta+1} \\ &= -[L_2 - (29\alpha - 1 + \Delta_2)][(z_2^{\mathrm{r}})^2 + (z_2^{\mathrm{i}})^2] \\ &\quad - \left[L_3 + \frac{8+\alpha}{3} + \Delta_3\right] z_3^2 - (z_2^{\mathrm{r}})^{\beta+1} \\ &\quad - (z_2^{\mathrm{i}})^{\beta+1} - z_3^{\beta+1}. \end{aligned} \tag{22}$$

In addition, $L_2 \ge 29\alpha - 1 + \delta_2$ and $L_3 \ge -(8+\alpha)/3 + \delta_3$, hence

$$\begin{aligned} \dot{V}_3 &\le -(z_2^{\mathrm{r}})^{\beta+1} - (z_2^{\mathrm{i}})^{\beta+1} - z_3^{\beta+1} \\ &= -\left(\frac{1}{2}\right)^{-(1+\beta)/2} \left[\frac{1}{2}(z_2^{\mathrm{r}})^2\right]^{(1+\beta)/2} \\ &\quad - \left(\frac{1}{2}\right)^{-(1+\beta)/2} \left[\frac{1}{2}(z_2^{\mathrm{i}})^2\right]^{(1+\beta)/2} \\ &\quad - \left(\frac{1}{2}\right)^{-(1+\beta)/2} \left(\frac{1}{2} z_3^2\right)^{(1+\beta)/2}. \end{aligned} \tag{23}$$

Similarly, according to Lemma 2, we have

$$\begin{aligned}\dot{V}_3 &\leq -\left\{\frac{1}{2}[(z_2^{\mathrm{r}})^2+(z_2^{\mathrm{i}})^2+z_3^2]\right\}^{(1+\beta)/2}\left(\frac{1}{2}\right)^{-(1+\beta)/2}\\ &= -\left\{\frac{1}{2}[z_2\tilde{z}_2+z_3^2]\right\}^{(1+\beta)/2}\left(\frac{1}{2}\right)^{(1+\beta)/2}\\ &= -\left(\frac{1}{2}\right)^{-(1+\beta)/2}V_2{}^{(1+\beta)/2}. \qquad (24)\end{aligned}$$

Since $0<\beta<1$, $0<(\beta+1)/2<1$. From Lemma 1, the complex state variables z_2 and z_3 will converge to $z_2=0$, $z_3=0$ at a finite time T_4. Then after T_4, the complex state variables of the system (19) will stay at $z_1=0$, $z_2=0$, $z_3=0$. This means that the controlled unified chaotic complex systems with uncertain parameters are finite-time stable by controller $u_2'=-L_2z_2-[(z_2^{\mathrm{r}})^\beta+\mathrm{j}(z_2^{\mathrm{i}})^\beta]$ and $u_3'=-L_3z_3-z_3^\beta$.

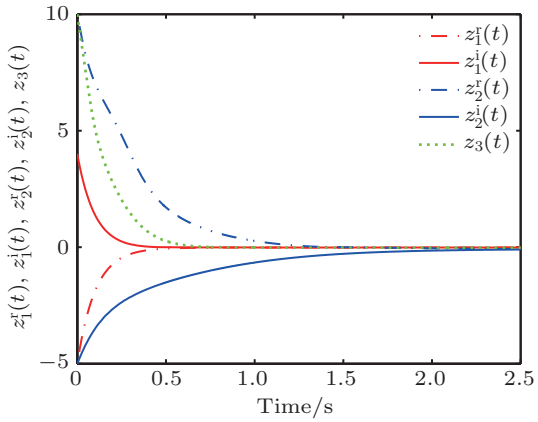


Fig. 4. (color online) The real and imaginary part responses of the complex state variables of the controlled uncertain complex Lorenz system ($\alpha=0$).

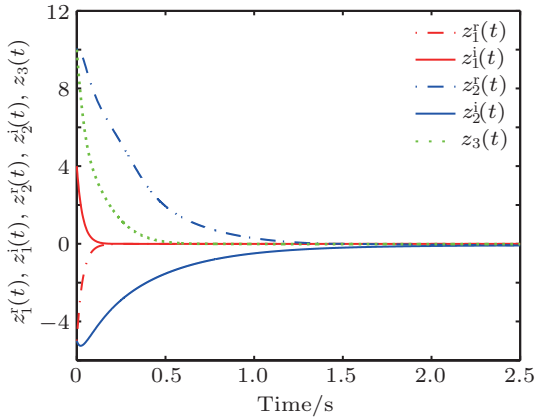


Fig. 5. (color online) The real and imaginary part responses of the complex state variables of the controlled uncertain complex Lü system ($\alpha=0.8$).

To demonstrate the effectiveness of the proposed method, we present the simulation results for the finite-time stability of the unified chaotic complex systems with uncertain parameters. The initial condition of the chaotic complex system is always adopted as $(z_1(0),z_2(0),z_3(0))=(-5+3\mathrm{j},5-4\mathrm{j},10)$. The uncertainties are selected as $\Delta_1=-0.5\sin(z_1^{\mathrm{r}})+0.5\mathrm{j}\cos(z_1^{\mathrm{i}})$, $\Delta_2=0.8\cos(z_2^{\mathrm{r}})-0.2\mathrm{j}\sin(z_2^{\mathrm{i}})$, $\Delta_3=\cos(z_3)$. Then the boundaries of the uncertainties are $\delta_1=\delta_2=\delta_3=1$. The controller gains are chosen as $L_2=29\alpha+1$, $L_3=(8+\alpha)/3+1$ and let $\beta=1/3$. Figures 4, 5, and 6 show the simulation results for the finite-time stability of the uncertain complex Lorenz, Lü and Chen systems, respectively. From the figures, we can see that the controller vector has strong robustness to the uncertainties. The controlled uncertain chaotic complex systems are finite-time stable.

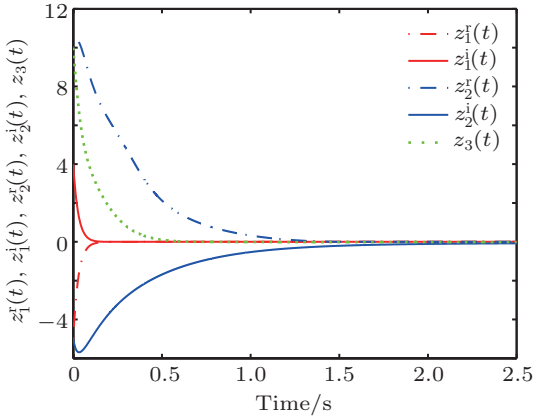


Fig. 6. (color online) The real and imaginary part responses of the complex state variables of the controlled uncertain complex Chen system ($\alpha=1$).

5. Conclusions

In this paper, the finite-time stability of unified chaotic complex systems with certain and uncertain parameters is considered. Based on finite-time stability theory, the controllers for the determined and undetermined systems are proposed to stabilize the unified chaotic complex systems, and the presented controllers for realizing finite-time stability have strong robustness to the uncertainties. For the design of the controllers, not all of the boundaries of the uncertainties are needed. From the proof process, we can see that the two methods can be extended effectively to other systems, such as the complex Liu system, the complex Duffing system, the hyperchaotic complex Lorenz system, and the hyperchaotic complex Lü system.

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