Absorption cross section of black holes with global monopole*

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We study the absorption problem for a massless scalar field propagating in general static spherically-symmetric black holes with a global monopole. The absorption cross section expression is provided using a partial-wave method, which permits us to make an elegant and powerful resummation of the absorption cross section, and to extract the physical information encoded in the sum over the partial-wave contributions.

Keywords: global monopole, absorption cross section, massless scalar wave

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1. Introduction

Over the last four decades, the physics of particle scattering from different kinds of black holes has been one of the most active topics of strong gravitational fields. Apart from the quasinormal modes which, in principle, can be identified as the poles of the corresponding black hole scattering matrix, the behavior of the cross section with respect to the scattering angle is one of the most interesting features in this area. The key issues around which the physics of black hole scattering centers, are related to phenomena such as glory, orbiting, rainbow and super-radiant scattering. Scattering by black holes is of foundational interest in both black-hole physics and scattering theory. The presence of black holes can be inferred only through indirect methods. One of the most useful and efficient ways to study the properties of black holes is by scattering matter waves off them. During the last 40 years, studies on the absorption of waves and particles by black holes and analogous higher dimensional objects have received considerable attention, because this topic is directly associated with numerous fundamental aspects of classical and quantum black-hole physics. It also helps us to enrich our understanding of spacetime properties (see, e.g., Refs. [1]–[12] and references therein). Using a numerical method, Okawa *et al.*[13] argued that in a scattering by two black holes in the five-dimensional (5D) spacetime, a visible domain whose curvature radius is much shorter than the Planck length can be formed. Décanini *et al.*[14] showed that the fluctuations of the high-energy absorption cross section are totally and very simply described from the properties of the waves trapped near the photon sphere. Batic *et al.*[15] reported that the orbiting scattering of particles with a massless spin of 0, 1, and 2 from Schwarzschild black holes can be characterized by a sudden rise in $|R^l|$ at a critical angular momentum. Monopoles, as a result of gauge-symmetry breaking, are similar to elementary particles. Such monopoles have Goldstone fields with the energy density decreasing with r^{-2}, and the large energy in the Goldstone field surrounding global monopoles suggests that they can produce strong gravitational fields. Research on monopoles is important for the topological defects in the early universe and other physical effects.[16–20] When the Schwarzschild black hole swallows a global monopole, forming a black-hole global-monopole system, it possesses a solid deficit angle, which makes it quite different topologically from that of a Schwarzschild black hole alone. In this work, we compute the absorption cross section for massless scalar waves with the arbitrary frequencies of a black hole with a global monopole.[21]

2. The partial-wave method

2.1. Massless scalar field equation

The most general static spherically-symmetric metric can be written as

$$\mathrm{d}s^2 = f(r)\,\mathrm{d}t^2 - f^{-1}(r)\,\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2, \tag{1}$$

where $f(r) = 1 - 8\pi G\eta^2 - 2GM/r$, the parameter M is the mass of a black hole that carries a global monopole charge, and η is the symmetry breaking scale when the global monopole is formed during the early universe. Unless otherwise stated, we use the units of $G = c = 1$.

The Klein–Gordon equation can be written in spacetime as

$$\frac{1}{f}\frac{\partial^2}{\partial t^2}\Psi - \frac{1}{r^2}\frac{\partial}{\partial r}\left(fr^2\frac{\partial}{\partial r}\Psi\right) + \frac{1}{r^2}\nabla^2\Psi = 0, \tag{2}$$

where $\nabla^2 = \eta^{ij}\nabla_i\nabla_j$, and i and j denote angular variables θ and φ. The positive-frequency solution of Eq. (2) is in the

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form of

$$\Psi = \frac{\psi_{\omega l}(r)}{r} Y_{lm}(\theta,\varphi)\, e^{-i\omega t}, \tag{3}$$

with $Y_{lm}(\theta,\varphi)$ being the scalar spherical harmonics, and l and m being the corresponding angular momentum quantum numbers. Substituting Eq. (3) into Eq. (2), we obtain the radial equation as

$$f\frac{d}{dr}\left(f\frac{d\psi_{\omega l}}{dr}\right) + [\omega^2 - V_{\rm eff}(r)]\psi_{\omega l} = 0, \tag{4}$$

with the effective scattering potential given by

$$V_{\rm eff}(r) = f(r)\left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2}\right). \tag{5}$$

2.2. Effective scattering potential

We plot the effective scattering potential $V_{\rm eff}$ in Fig. 1, from which we can see that it tends to be zero as $r \to r_{\rm H}$ and $r \to \infty$. The effective potential $V_{\rm eff}$ acts as the scattering barrier, and increases with the angular momentum quantum number l. In terms of the Wheeler-type coordinate $x = x(r)$ defined by $d/dx \equiv f\, d/dr$, equation (4) can be rewritten as

$$d^2\psi_{\omega l}/dx^2 + [\omega^2 - V_{\rm eff}(x)]\psi_{\omega l} = 0. \tag{6}$$

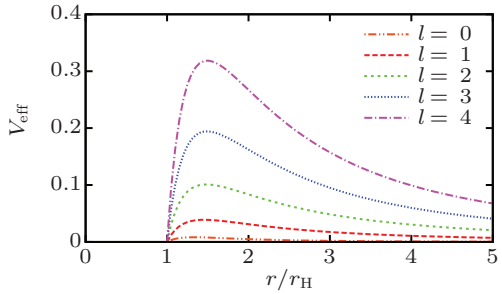


Fig. 1. (color online) The scattering potential $V_{\rm eff}$ for different values of angular momentum l with fixed parameters $M = 1$ and $\eta = 0.1$. It vanishes at the horizon. As $r \to \infty$, $V_{\rm eff}$ falls as $1/r^3$ for $l = 0$ and as $1/r^2$ for $l > 0$.

In Fig. 2, we plot $V_{\rm eff}$ as a function of the Wheeler-type coordinate x. It is obvious that the larger the value of l, the higher the scattering barrier.

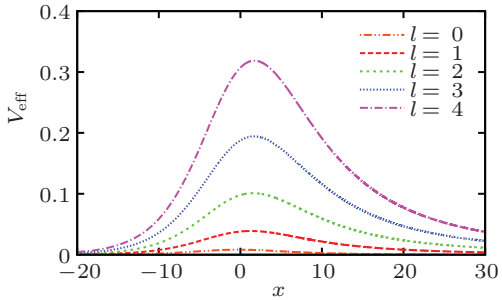


Fig. 2. (color online) The scattering potential $V_{\rm eff}$ for different values of the angular momentum l with fixed parameters $M = 1$ and $\eta = 0.1$ in the Wheeler-type coordinate. The larger the value of l, the higher the scattering barrier.

The asymptotic form of the incoming modes from the past infinity can be written as

$$\psi_{\omega l} \approx \begin{cases} A^{\rm tr}_{\omega l}\, e^{-i\omega x}, & x \to -\infty, \\ A^{\rm in}_{\omega l}\, e^{-i\omega x} + A^{\rm out}_{\omega l}\, e^{i\omega x}, & x \to +\infty, \end{cases} \tag{7}$$

with the conserved relation $|A^{\rm tr}_{\omega l}|^2 + |A^{\rm out}_{\omega l}|^2 = |A^{\rm in}_{\omega l}|^2$. The phase shift δ_l is defined by

$$e^{2i\delta_l} = (-1)^{l+1} A^{\rm out}_{\omega l}/A^{\rm in}_{\omega l}. \tag{8}$$

3. Absorption cross sections

Based on quantum-mechanics theory, the total absorption cross section can be written as

$$\sigma_{\rm abs} = \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)(1 - |e^{2i\delta_l}|^2). \tag{9}$$

We can obtain

$$\sigma_{\rm abs} = \sum_{l=0}^{\infty}\sigma^{(l)}_{\rm abs} = \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)|T_{\omega l}|^2, \tag{10}$$

where $|T_{\omega l}|$ is the transmission coefficient.

In order to obtain the absorption cross sections of scalar field waves from black-hole spacetime, we will solve the radial equation (6) under the boundary conditions (7), and compute the coefficients $A^{\rm in}_{\omega l}$ and $A^{\rm out}_{\omega l}$ by matching into Eq. (8) to obtain the numerical phase shift. The partial and total absorption cross sections of scalar waves from black holes with global monopoles can be simulated immediately.

The partial absorption cross sections for $l = 0, 1$ and $\eta = 0.100$, 0.105, and 0.110 are plotted in Fig. 3. It is obvious that the partial absorption cross section tends to be zero as ωM increases. Comparing the curves with different values of η, we can see that a larger value of η leads to a larger partial absorption cross section. It also shows that the difference in the curves is numerically small relative to their amplitude with larger values of ωM.

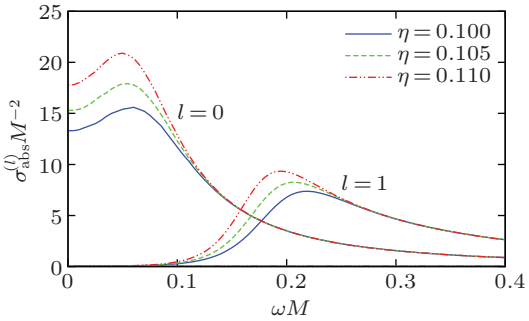


Fig. 3. (color online) Partial absorption cross section as a function of parameter ωM for $l = 0, 1$ and $\eta = 0.1$, 0.105, and 0.110.

In Fig. 4, we plot the partial absorption cross section $\sigma^{(l)}_{\rm abs}$ for $l = 0, 1, \ldots, 6$. We see that the partial wave ($l = 0$) contribution is responsible for the nonvanishing cross section in the zero-energy limit. Moreover, for $l > 0$, the corresponding partial absorption cross section starts from zero, reaches a maximum value $\sigma^{(l)\max}_{\rm abs}$, and decreases asymptotically. With the

increase in the value of l, the corresponding value of $\sigma_{\rm abs}^{(l)\max}$ decreases, and the value of ωM associated with $\sigma_{\rm abs}^{(l)\max}$ increases. This is all compatible with the fact that the scattering potential $V_{\rm eff}$ is larger for larger values of l. We can also see that the total absorption cross section $\sigma_{\rm abs}$ oscillates around the limit of geometrical optics. In the regime $\omega M \sim 0.1$, the total absorption section creates a regular oscillation pattern, and when $\omega M \gg 1$, the absorption cross section approaches the geometric-optical limit: $\sigma_{\rm abs}^{\rm hf} = \pi b_{\rm c}^2 \approx 9$, where $b_{\rm c} = r_{\rm c}/\sqrt{f(r_{\rm c})}$.

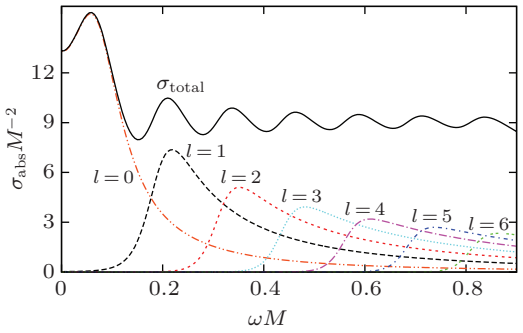


Fig. 4. (color online) Partial and total absorption cross sections as functions of parameter ωM at fixed parameters $M = 1$ and $\eta = 0.1$. The summation in Eq. (10) is performed up to $l = 6$.

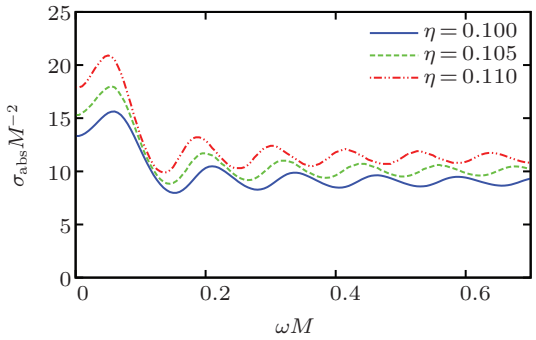


Fig. 5. (color online) The total absorption cross section for a massless scalar field propagating in the black hole with a global monopole for three values of η. The $\eta = 0.1$ case has been illustrated in Fig. 4.

In Fig. 5 we plot the total absorption cross section $\sigma_{\rm abs}$ for $l = 0, 1, \ldots, 6$ with $\eta = 0.100$, 0.105, and 0.110. We can see that with the increase in the value of η, the corresponding value of $\sigma_{\rm abs}$ increases, the value of ωM associated with the extremum decreases, and the larger is the corresponding value the total absorption falls approaching.

4. Conclusions

In this paper, we have investigated the absorption cross section of a massless scalar wave from the most general static spherically-symmetric black hole with a global monopole using analytic and numerical methods. We found that the influence of the coupling constant η on the absorption cross section becomes more obvious in the low-frequency region, and the partial absorption cross section tends to unity when the parameter ωM is large enough. In addition, the absorption cross section becomes larger, while the coupling parameter η becomes stronger. The opposite properties can be seen for the Reisser–Nordström black holes.[22] These properties imply that the coupling parameter makes the absorption of black holes stronger.

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