Analysis of network traffic flow dynamics based on gravitational field theory*

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For further research on the gravity mechanism of the routing protocol in complex networks, we introduce the concept of routing awareness depth, which is represented by ρ. On this basis, we define the calculation formula of the gravity of the transmission route for the packet, and propose a routing strategy based on the gravitational field of the node and the routing awareness depth. In order to characterize the efficiency of the method, we introduce an order parameter, η, to measure the throughput of the network by the critical value of phase transition from free flow to congestion, and use the node betweenness centrality, B, to test the transmission efficiency of the network and congestion distribution. We simulate the network transmission performance under different values of the routing awareness depth, ρ. Simulation results show that if the value of the routing awareness depth ρ is too small, then the gravity of the route is composed of the attraction of very few nodes on the route, which cannot improve the capacity of the network effectively. If the value of the routing awareness depth ρ is greater than the network's average distance $\langle l \rangle$, then the capacity of the network may be improved greatly and no longer change with the sustainable increment of routing awareness depth ρ, and the routing strategy performance enters into a constant state. Moreover, whatever the value of the routing awareness depth ρ, our algorithm always effectively balances the distribution of the betweenness centrality and realizes equal distribution of the network load.

Keywords: routing strategy, congestion, gravitation field, complex networks

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1. Introduction

Congestion is a dynamics phenomenon that exists in many real complex systems, and is assumed to be a phase transition from free-flow to congestion.[1,2] When the number of new generated packets is greater than that of accomplished packets, network congestion begins to form, and if no effective measure is adopted, the congestion will deteriorate continuously and gradually spread to all the other nodes in the network. In order to reduce traffic jams and improve the ability of the network to resist congestion, the research mainly focuses on two aspects. One is to improve the topological structure of the network, and some studies have proved that the network structure may have a large influence on traffic jams and their spread.[3] The other is to explore efficient routing strategies. For many real complex systems such as transportation networks, power networks, and so on, it is too costly to update the structure of the network and this also has too great an effect on the surroundings. Echenique *et al.*[4] found in numerical simulations that the nature of the congestion transition depends on the type of routing rule. In recent years, the dynamic characteristics of traffic flow were analyzed and an efficient routing strategy was explored, which has become one of the main topics of complex networks.[5–7]

Routing strategies can be roughly divided into three types: global, local, and mixed. The global routing strategy requires global information about the network structure[8,9] and chooses the path with the lowest travel costs from the original node to the destination node, such as the shortest path algorithm.[6,10] The local routing strategy only considers the degree, traffic flow, and other aspects of the node, and usually transfers the packet to a neighbor at a local minimum cost or certain probability, like random walks[11,12] or the local routing strategy based on dynamical traffic load.[13] The mixed routing strategy may consider the influence of the path length, the queues of the neighbors, and other factors for packet transmission, and can also transfer the packet at minimum cost or a certain probability, such as the traffic awareness routing algorithm.[14] Previous research has shown that one way of quantifying the effects of congestion is to measure the betweenness centrality,[15,16] and that the transmission capacity of the network is inversely proportional to the maximum betweenness of the nodes.[17] When the scale of the network is kept constant, reducing the maximum betweenness will improve the capacity of the network. Inspired by this argument, some scholars began to optimize routing strategies by minimizing the maximum betweenness of the nodes. Danila *et*

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al.[18] proposed a heuristic algorithm of the optimization of transport, which balances the network traffic by minimizing the maximum betweenness. And compared with the traditional shortest path algorithm, this heuristic algorithm significantly reduces the average travel time on congested networks with only a negligible increase in the average travel time at low loads. Then, Danila *et al*.[19] considered a network with a heterogeneous distribution of link capacities and with a heterogeneous transport load, and found that overall traffic congestion and average travel time can be significantly reduced through a judicious use of slower and smaller capacity links, which can also improve the bearing capacity of the urban street network.

To a certain extent, the betweenness reflects the connection ability of the node. From the point of view of gravitation theory, we can regard betweenness as the attraction of the node for the packets, and the larger the betweenness of the node, the greater the gravity of the node for most nodes on the network.[20] Therefore, if the routing strategy can dynamically reduce the gravity of the congested nodes in the transmission process of packets, it may improve the throughput of the network to a certain extent. Based on such consideration, Liu and Li[20] described the gravitational field produced by the node through the use of gravitational field theory, defined the gravitational field equation of the node, and established the gravitation formula of the path for the packet based on the gravity of the node. They also proposed an efficient routing algorithm, which chooses the route of the shortest paths from the neighbors to the destination with maximum gravity, thereby ensuring that the packets are always transferred along a short distance and comparatively unblocked route. In Ref. [20], the mechanism of gravitational action in the network transmission was uncovered to some extent, and the proposed routing algorithm relieved the network congestion and improved the network capacity. In Ref. [20], all the nodes on the shortest path from the neighbor to the destination are considered, which means that the flow information of all the nodes is transparent to the packets. For some real complex systems, such as transportation systems, the packet cannot easily get into the congestion situations of all the nodes, and the drivers find it difficult to obtain the traffic flows of all the roads.

With this in mind, in this paper we further study the dynamic characteristics of network traffic flow based on gravitation field theory. We suppose that the packet cannot necessarily perceive the traffic flows of all the nodes on the path in transmission, so we introduce the concept of routing awareness depth and redefine the gravity formula of the transmission path for the packets. On this basis, we propose a new routing strategy under routing awareness depth. Our strategy considers four factors: routing awareness depth, capacity of the node, congestion of the node, and path length.

2. Routing strategy

2.1. Gravitational field of the node

In Ref. [20] the gravitational field of the node at length was described, and the gravitational field equation was defined by using the node capacity, the unblocked situation of the node, and the path length:

$$F_i = \begin{cases} \kappa \dfrac{c_i^{2\alpha}}{d_{ij}^{\gamma}}, & q_i = 0, \\ \kappa \dfrac{(c_i/q_i)^{\alpha} c_i^{\alpha}}{d_{ij}^{\gamma}}, & q_i > 0, \end{cases} \tag{1}$$

where F_i is the gravity of node i for the packet; κ is a constant; c_i is the transmission capacity of node i; q_i is the number of packets in the queue of node i; c_i/q_i can be considered as the unblocked reliability of node i; d_{ij} is the shortest path length between node i and node j; α and γ are two tunable parameters, which are used to adjust the dependences on the unblocked reliability of the node, the capacity of the node, and the path length, and $\alpha > 0$, $\gamma > 0$. In light of the gravitational field equation, the gravity of the node for packets is proportional to the α power of the product of the transmission capacity and the unblocked reliability of the node, but inversely proportional to the γ power of the shortest path length between the node and the packet.

2.2. Routing awareness depth

In Ref. [20], an efficient routing strategy was given: calculating the gravity of the shortest path from each neighbor to the destination, and choosing the neighbor on the shortest path with maximum gravity as the next routing node. All the nodes on the shortest path from the neighbor to the destination are transparent to the packets, which ensures that the packets are always delivered along a short distance and comparatively unblocked route. In Ref. [20], whatever the length of the path is, the packet can perceive the traffic flow information of all the nodes on the shortest path and the gravities of all the nodes that are used to calculate the gravity of the path. For the packet, it would be ideal to perceive the congestion situation of all the nodes. However, for packets on real complex networks, for instance urban traffic networks, in nature of the perception ability, the driver always knows several selectable paths to the destination, but cannot ensure which path saves more time because the driver can only perceive the congestion of nearby roads; the traffic flows of far-away roads are unclear.

For the further study of the gravitation mechanism of packet transmission, we introduce the concept of routing awareness depth, which means the maximum length of perceiving the packet, represented by ρ. As shown in Fig. 1, the packet is at node v, the shortest distance from each node of $\{a, b, c, d, e, f, g, h, i, j, k\}$ to v is

$\{1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3\}$. When $\rho = 1$, there are a, b, c nodes which are in one unit from v, so the packet can only perceive the traffic flow of a, b, c; when $\rho = 2$, the packet can perceive nine nodes of $a \sim i$; and when $\rho = 3$, the packet can perceive the congestion of 11 nodes of $a \sim k$.

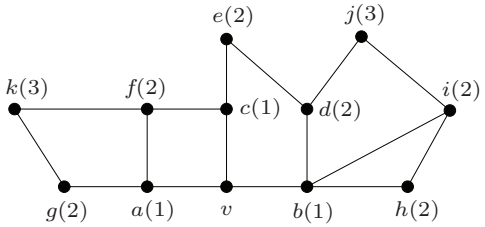


Fig. 1. Routing awareness depth. $a \sim k$, v represent the nodes of the network and v is the node that the packet belongs to. The numbers in parentheses of each node represent the distance from the node to v.

2.3. Proposed routing strategy

In order to investigate the region of R_c of the network, the traffic model is described as follows: suppose that all nodes have the functions of receiving, sending, and routing, and the initial network load is zero; at each time step, there are R packets generated in the system with randomly chosen sources and destinations, and all nodes can deliver in most c_i packets towards their destinations. The queue length of each node is assumed to be unlimited and the first in first out (FIFO) discipline is applied at each queue. During transmission, packets are always delivered from the current node to one of its neighbors. Once the packet arrives at its destination, it will be removed from the system. Otherwise, the packet proceeds forward towards a neighbor according to a given routing strategy.

The routing algorithm in Ref. [20] uses all the nodes in the shortest path from the neighbor to the destination to evaluate the gravity of the shortest path. With the concept of routing awareness depth introduced in this paper, we should modify the routing process to some extent: the gravity of the shortest path from the neighbor to the destination is defined as the combined action of the gravities of all the nodes within the routing awareness depth ρ on the shortest path. As shown in Fig. 2, suppose that the original node s, destination node t, current node v and its neighbors constitute a set N_v, then all the nodes in the shortest path from neighbor i $(i \in N_v)$ to t will constitute a set N_{it}. Here, N_{it} contains i but not t, because t is the destination whose traffic flow has no effect on the transmission performance of the packet at node v, but other nodes on the path can directly affect the transmission of the packet. Because each node in N_{it} can explode a gravitational field, this means that each node has an attraction for the packet. To some extent, the packet may possibly be delivered to node i $(i \in N_v)$, but this probability cannot be determined by some other node in the path, and it is instead the result of joint action of all the nodes in the path. In this regard, we define the gravity of the shortest path for the packet as the average of the gravities of all the nodes on the path within the routing awareness depth ρ, which is expressed as

$$F_{it} = \frac{1}{\rho}(F_1 + F_2 + \cdots + F_j) = \frac{1}{\rho} \sum_{j \le \rho, j \in N_{it}} F_j, \tag{2}$$

where F_{it} is the gravity of the shortest path for the packet from i to t, and ρ is the routing awareness depth. From the point of view of the gravitational field theory, we consider that the greater the gravity F_{it}, the greater the probability that the packet transfers along the path. Therefore, we design the routing process as follows.

(i) Obtain the neighbor set N_v of current node v.

(ii) For each neighbor i $(i \in N_v)$, if the packet has not passed through the neighbor i, extract the shortest path from i to t, and then generate the node set N_{it}, and calculate the gravity of all the nodes for the packet within the routing awareness depth ρ, so as to calculate the gravity F_{it} of the shortest path from i to t.

(iii) Choose the neighbor such that the gravity of the shortest path from this neighbor to t is maximum ($F = \max\{F_{it}\}$) as the next routing node of the packet. If several neighbors experience maximum gravity, choose the neighbor with the shortest distance. If several neighbors still exist, then randomly choose one node within these neighbors as the next routing node.

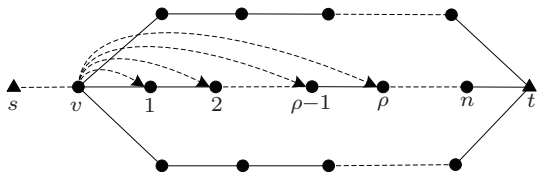


Fig. 2. The principle of the routing strategy.

3. Simulation

3.1. Evaluation indices

For evaluating the performance of the routing strategy, we introduce the order parameter η [21] and betweenness centrality B.[22] The order parameter is used to describe the phase transition from free-flow state to jammed state, which is defined as follows:

$$\eta(R) = \lim_{t \to \infty} \frac{1}{R} \frac{W(t)}{t}, \tag{3}$$

where $W(t)$ is the number of packets at time t; R is the number of new packets generated at each time step; and $\eta(R)$ is the ratio of the number of remaining packets to that of all generated packets until time t, which reflects the capacity of delivering the network packets. For $R < R_c$, $W(t) = 0$, and $\eta(R) = 0$, indicating that the system is in the free state with no traffic congestion. Otherwise for $R > R_c$, $W(t) > 0$, and $\eta(R)$, which increases abruptly and $\eta(R) \to 1$, then the system will ultimately collapse. Therefore, a phase transition occurs at

$R = R_c$ and R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. Thus the maximal handling and delivering capacity of the system is measured by R_c.

The betweenness centrality reflects the aggregation of the connection capacity of the node,[23,24] and we use the betweenness centrality measurement method in Ref. [20]. $b_i^{(s,t)}$ is the number of packets passing through node i with respect to the original node s and destination node t, so the number of all packets passing through node i over time window $[t_0, T+t_0]$ is

$$b_i = \sum_{t_0}^{T+t_0} \sum_{s,t=1, s\neq t}^{N} b_i^{(s,t)}. \tag{4}$$

Therefore, the betweenness centrality of node i is defined as

$$B_i = \frac{b_i}{\sum_{j=1}^{N} b_j}, \tag{5}$$

where B_i is the betweenness centrality of node i, and N is the number of network nodes. B_i reflects the aggregation of the connection capacity of i, and the greater the value of B_i, the more the nodes passing through node i, so congestion will be easily triggered.

In order to test the efficiency of our routing strategy, we choose the Barabási–Albert (BA) network model[25] with a network size of $N = 100$ and $m_0 = m = 4$ for simulation. The results are obtained from transport simulations over 10000 time steps by choosing the average of results of the last 1000 time steps to calculate the critical capacity R_c and the betweenness centrality B_i. We conduct experiments on the routing strategy under different values of ρ, α, and γ. For further evaluating the performance of the routing strategy, we make some statistical analyses of the maximum travel time $\langle T_{max} \rangle$,[26] average travel time $\langle T_{avg} \rangle$, and average path length $\langle L_{avg} \rangle$ of the packets. In this paper, the average path length $\langle L_{avg} \rangle$ is defined as the average of the path length over all packets that have arrived at the destinations. Without loss of generality, we study the traffic dynamics when $c = 1$ and $c = k$.

3.2. Results

Figure 3 shows the relations of the order parameter η, maximum travel time $\langle T_{max} \rangle$, average travel time $\langle T_{avg} \rangle$, and average path length $\langle L_{avg} \rangle$ change with R under the traditional shortest path algorithm (SP) and node capacity $c = 1$. The results show that for $R > 5$, η, $\langle T_{max} \rangle$, and $\langle T_{avg} \rangle$, all increase abruptly, which means that in the traffic state, a phase transition occurs when $R = 5$, so the critical point of the network capacity is $R_c = 5$. The average path length $\langle L_{avg} \rangle$ decreases with increasing R. Because with the increase in R, the congestion situation of the network becomes more serious, it is more difficult for packets with a longer travel distance to arrive at the destination, which leads to the decrease in the average path length. Figure 4 shows the results of our routing strategy under different values of the routing awareness depth ρ, $\alpha = 1$, and $\gamma = 2$. The simulations indicate that for $\rho = 1$, the critical point of the network capacity is $R_c = 3$, and the average path length $\langle L_{avg} \rangle$ presents the tendency for "reduce-increase-reduce" change with increasing R; for $\rho = 2$, η, $\langle T_{max} \rangle$, and $\langle T_{avg} \rangle$ all increase abruptly when $R > 8$, which means that the network is in the region until $R_c = 8$. The relation between the average travel time $\langle L_{avg} \rangle$ and R is similar to that in the case of $\rho = 1$; for $\rho \geq 3$, the change in routing awareness depth ρ will not affect the relation between η and R, which also displays in the relation between $\langle T_{max} \rangle$ and R, the relation between $\langle T_{avg} \rangle$ and R. Whatever the value of ρ, the critical network capacity is approximately equal to 16, and $R_c \approx 16$. The average path length $\langle L_{avg} \rangle$ first increases and then deceases with increasing R, showing the maximum $\langle L_{avg} \rangle = 6.25$ at $R = R_c$. The reason for this is that if R is small, then the traffic flow on the network is in the free state, and the packets mainly spread along the shortest path. With the increase in R and $R < R_c$, a small number of packets begin to gather in the queues of some of the nodes, and the routing choice is more inclined to the free nodes, which results in the increase of average path length $\langle L_{avg} \rangle$. For $R > R_c$, the network will become jammed rapidly, and more and more nodes will turn to enter into the congestion state, which finally leads to serious congestion of the network. Under this situation, the packets will no longer avoid some of the congested nodes, and the average path length will decrease with increasing R.

By analyzing the principle of the proposed routing strategy, we can see that if the routing awareness depth ρ is very small, then the transmission of the packet can only perceive the traffic flow of some nearby nodes. Take $\rho = 1$ for example, only the neighbors are perceived, and the gravity of each shortest path is determined by the congestion situation of a neighbor but not restricted by the path length, which results in the most unblocked neighbors being the next routing nodes. The lack of inducement of path length in the transfer direction of the packet leads to the excessive roundabout of the transfer direction, too much transmission time and low network capacity. Otherwise, if the routing awareness depth ρ is greater, then the traffic of more nodes on the path is perceived, and the routing process considers the restrictions of node congestion and path length, so the strategy improves the network capacity efficiently. In addition, the results also reveal that when the routing awareness depth ρ is greater than a value, like $\rho \geq 3$ in our simulations, then the network transmission performance will no longer change with ρ further increasing.

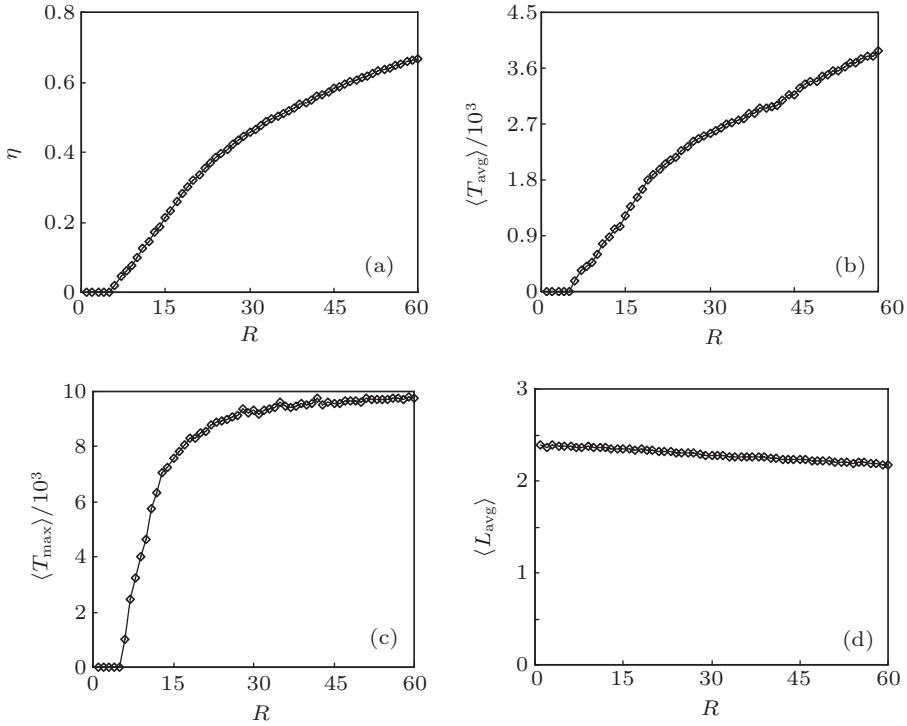


Fig. 3. Results of the shortest routing strategy, showing (a) order parameter η versus R; (b) average travel time $\langle T_{\rm avg}\rangle$ versus R; (c) maximum travel time $\langle T_{\rm max}\rangle$ versus R; and (d) average path length $\langle L_{\rm avg}\rangle$ versus R. Other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = 1$.

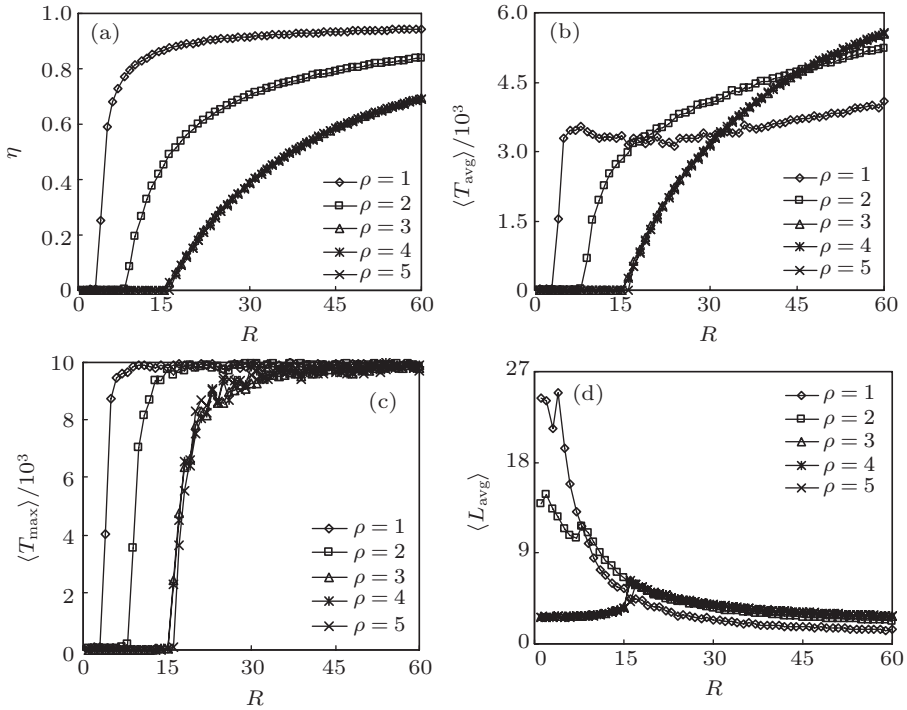


Fig. 4. Results of the proposed routing strategy for different values of ρ, showing (a) order parameter η versus R; (b) average travel time $\langle T_{\rm avg}\rangle$ versus R; (c) maximum travel time $\langle T_{\rm max}\rangle$ versus R; and (d) average path length $\langle L_{\rm avg}\rangle$ versus R. Other parameters are network size $N = 100$, $m_0 = m = 4$, $\alpha = 1$, $\gamma = 2$, and $c = 1$.

In order to analyze the effect of the routing awareness depth on the network capacity, we simulate the $R_{\rm c}$ versus ρ for different values of α and γ. The results are shown in Fig. 5. The simulations show that whatever the values of α and γ are, with the increase in ρ, the network capacity always increases first and then enters into a steady state, and the critical value of the routing awareness depth is $\rho = 3$. Moreover, the network average distance $\langle l\rangle = 2.3772$. We can see that when

$\rho < \langle l \rangle$, the network capacity increases with ρ increasing, and when $\rho > \langle l \rangle$, the network capacity will not change with the increase of ρ. In Ref. [20], we used all the nodes in the shortest path from neighbor to destination for measuring the gravity of the path, which means that the transmission process considers the total length of the path. For all the packets on the network, when the network traffic state is in the free-flow state, the average path length $\langle L_{\rm avg} \rangle$ of the packet transfer is approximately equal to the average distance $\langle l \rangle$ of the network, and we can see that the critical capacity in Ref. [20] is equal to the critical capacity of our routing strategy with $\rho > \langle l \rangle$. Thus, for this routing strategy based on the gravitational field theory, we can conclude an important dynamic phenomenon that the transmission capacity of the network reaches the maximum and enters into a steady state when the routing awareness depth ρ is greater than the average distance $\langle l \rangle$ of the network.

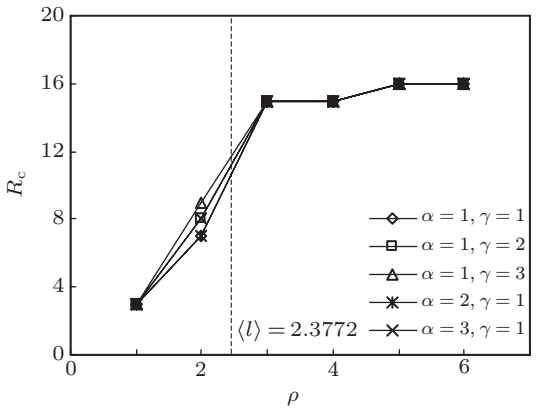


Fig. 5. Plots of critical capacity R_c versus ρ. The other parameters are network size $N = 100$, $m_0 = m = 4$, $c = 1$, and the network's average distance is $\langle l \rangle$.

In order to study the dynamic characteristics of the traffic flow under different values of α and γ, we simulate the variations of η, $\langle T_{\rm max} \rangle$, $\langle T_{\rm avg} \rangle$, and $\langle L_{\rm avg} \rangle$ with R for different values of α and γ. Figures 6 and 7 show the results when $\rho = 2$ and $\rho = 3$, respectively. The simulations show that, i) for $\rho = 2$, the network capacity R_c maintains a consistent change neither with the value of α nor with the value of γ. With α being constant, R_c will increase with the value of γ increasing, and the reason for this lies in the value of ρ. Because ρ is small, the transmission is lacking in the guide of the path length, and the increase in γ leads to an enhancement in the dependence of the transmission on path length to some extent, thereby resulting in the improvement of the network capacity. Otherwise, if γ is constant, the increase in α is equivalent to the reduction in the dependence on path length, which may reduce the network capacity. The average path length $\langle L_{\rm avg} \rangle$ first decreases with R increasing, and then abruptly increases at $R = R_c$, and then reduces abruptly for $R > R_c$ and decreases gradually with the further increase in R. According to our routing strategy, because of the routing awareness depth $\rho = 2$, the transfer process will not strongly depend on the path length. If R is smaller, the transmission of the packets will be more inclined to the free nodes, which causes the path's circuity to some extent, so the average path length $\langle L_{\rm avg} \rangle$ is larger. With the increase in R, the number of free nodes on the network is reduced, which correspondingly strengthens the dependence on the path length, and thus the average path length $\langle L_{\rm avg} \rangle$ decreases gradually. When $R = R_c$, a large number of nodes start to collect some of the packets, and a number of packets in the node queues start to present a non-uniform distribution to some extent, but no node is in a serious jam state. Then the packet will choose some relative free nodes to transfer again, which results in the abrupt increase of the average path length $\langle L_{\rm avg} \rangle$. For $R > R_c$, the network will be in a serious congestion state rapidly. In addition, there is no free node, and thus the packet will not avoid the congested nodes. Therefore, the average path length $\langle L_{\rm avg} \rangle$ decreases with R further increasing. ii) For $\rho = 3$, whatever the value of α and γ are, the network capacity R_c is always kept unchanged. This indicates that when $\rho > \langle l \rangle$, the efficiency of the routing strategy will rely on neither α nor γ, and the algorithm can always ensure a high transmission capacity of the network. For $R < R_c$, the average path length $\langle L_{\rm avg} \rangle$ increases rapidly with R increasing; for $R > R_c$, $\langle L_{\rm avg} \rangle$ decreases gradually with R further increasing, and reaches the maximum at $R = R_c$. According to the principle of our algorithm, when $\rho = 3$, each packet delivery almost considers the traffic states of all the nodes on the path. If R is too small, then the traffic flow of the network is in a free-state, the routing process is similar to that of the traditional shortest routing algorithm, and the packets will transfer along the shortest path. With the increase in R and $R < R_c$, some nodes begin to collect a few packets, and then the packets will choose a more unobstructed path and avoid those congested nodes, which results in the increase of the average path length $\langle L_{\rm avg} \rangle$. The avoiding strategy designed in this paper reduces the congestion of some nodes and effectively prevents the congestion from deteriorating, so the network will not fall into congestion. With the further increase in R, the congestion of the nodes will be completely out of control, which will cause more and more nodes to enter into a congestion state, and finally the whole network will sink into congestion. During this transmission, the packets no longer avoid some of the congested nodes, and the average path length $\langle L_{\rm avg} \rangle$ will decrease gradually with R increasing.

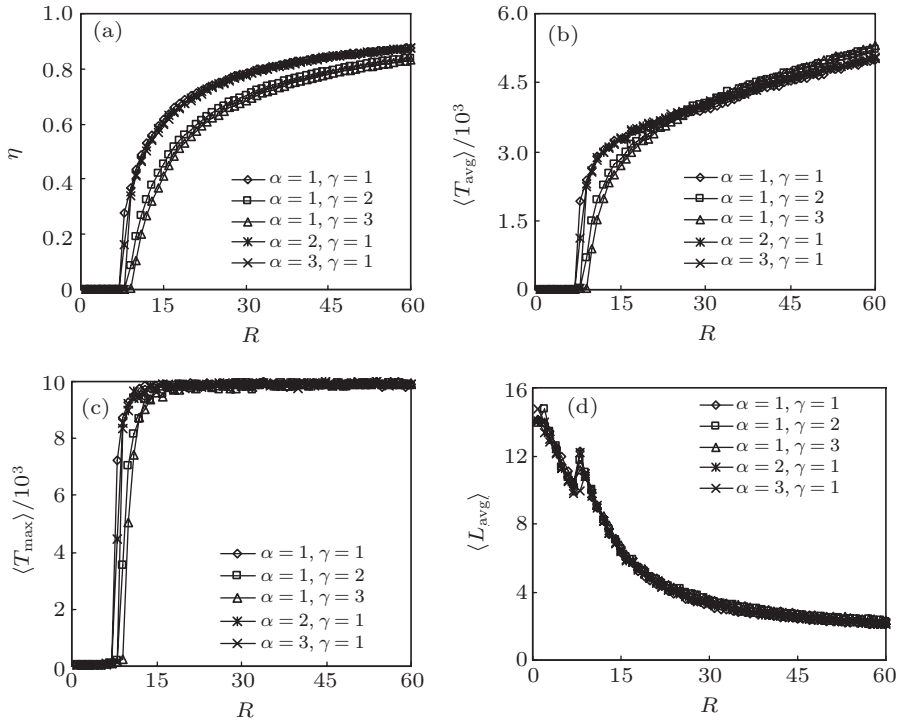


Fig. 6. Results of the proposed routing strategy for different values of α and γ when $\rho = 2$, showing (a) order parameter η versus R; (b) average travel time $\langle T_{\rm avg}\rangle$ versus R; (c) maximum travel time $\langle T_{\rm max}\rangle$ versus R; and (d) average path length $\langle L_{\rm avg}\rangle$ versus R. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = 1$.

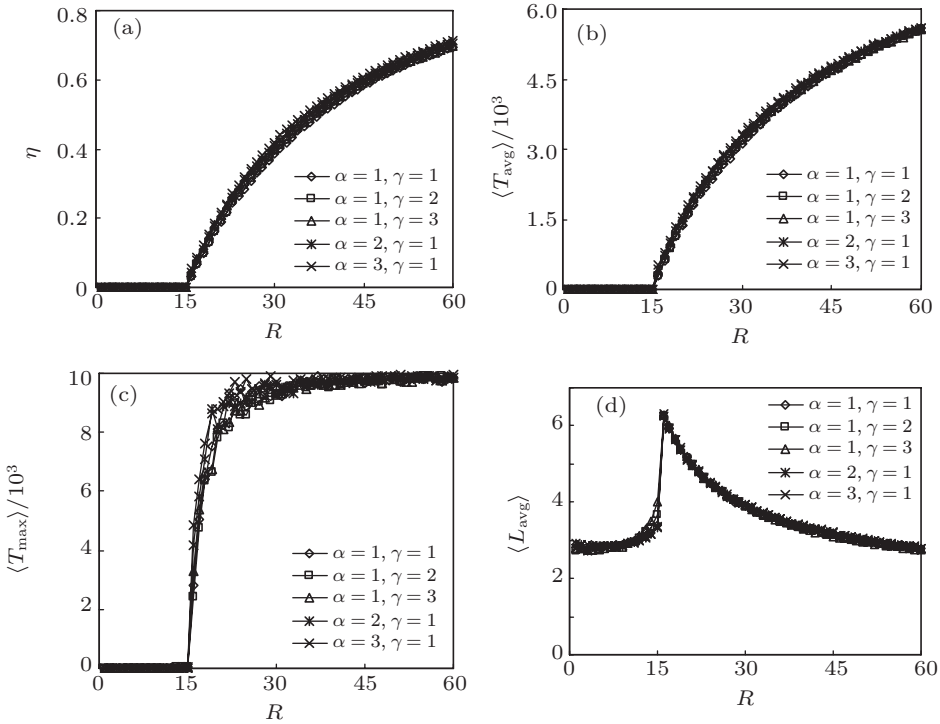


Fig. 7. The results of proposed routing strategy for different values of α and γ when $\rho = 3$, showing (a) order parameter η versus R; (b) average travel time $\langle T_{\rm avg}\rangle$ versus R; (c) maximum travel time $\langle T_{\rm max}\rangle$ versus R; and (d) average path length $\langle L_{\rm avg}\rangle$ versus R. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = 1$.

We then investigate the distribution of betweenness centrality as shown in Fig. 8 for the shortest path algorithm, and Fig. 9 for our routing strategy. The simulations show that for the shortest path algorithm, the distribution of betweenness centrality is most inhomogeneous when $R = 1$, and to some extent the inhomogeneous situation of distribution is alleviated

with the increase of R, but obvious homogeneity still exists. For our routing strategy, whatever the value of ρ is, the distribution of betweenness centrality always take on the trend from inhomogeneous to homogeneous and then back to inhomogeneous. When the distribution of betweenness centrality is most homogeneous, the betweenness centrality of each node is approximately equal to 0.01, and the corresponding R is slightly greater than the critical point R_c, and when taking $\rho = 1$ for example, the corresponding $R = 4$ but $R_c = 3$. When R is small and traffic is in the free-flow state, like $R = 1$, compared with the distribution of betweenness centrality under $\rho = 3$, the distribution of betweenness centrality is more homogeneous under $\rho = 1$ and $\rho = 2$, because when $\rho = 1$ and $\rho = 2$, the routing process mainly considers the congestion of the node and ignores the path length, which makes the transfer of packets tend to more unblocked nodes but not the nodes on the shortest or second shortest path. Therefore, the congestion situation of the nodes with a large degree is distinctly reduced.

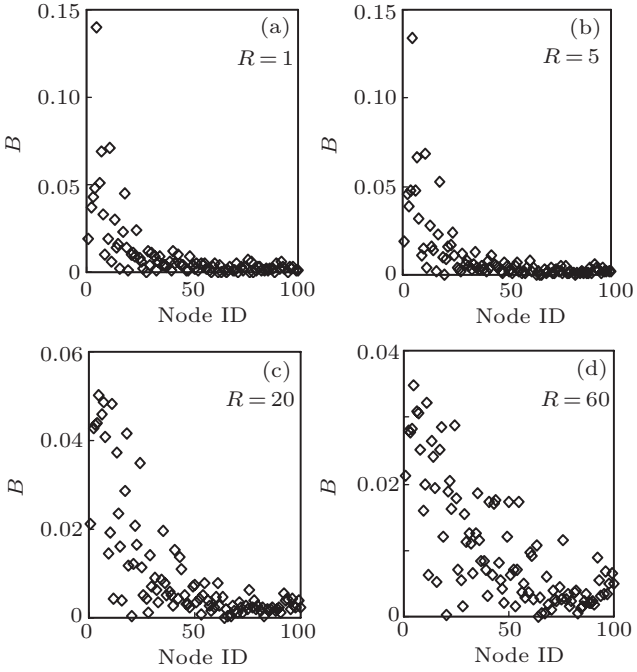


Fig. 8. Distributions of betweenness centrality for the shortest routing strategy under different values of R. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = 1$.

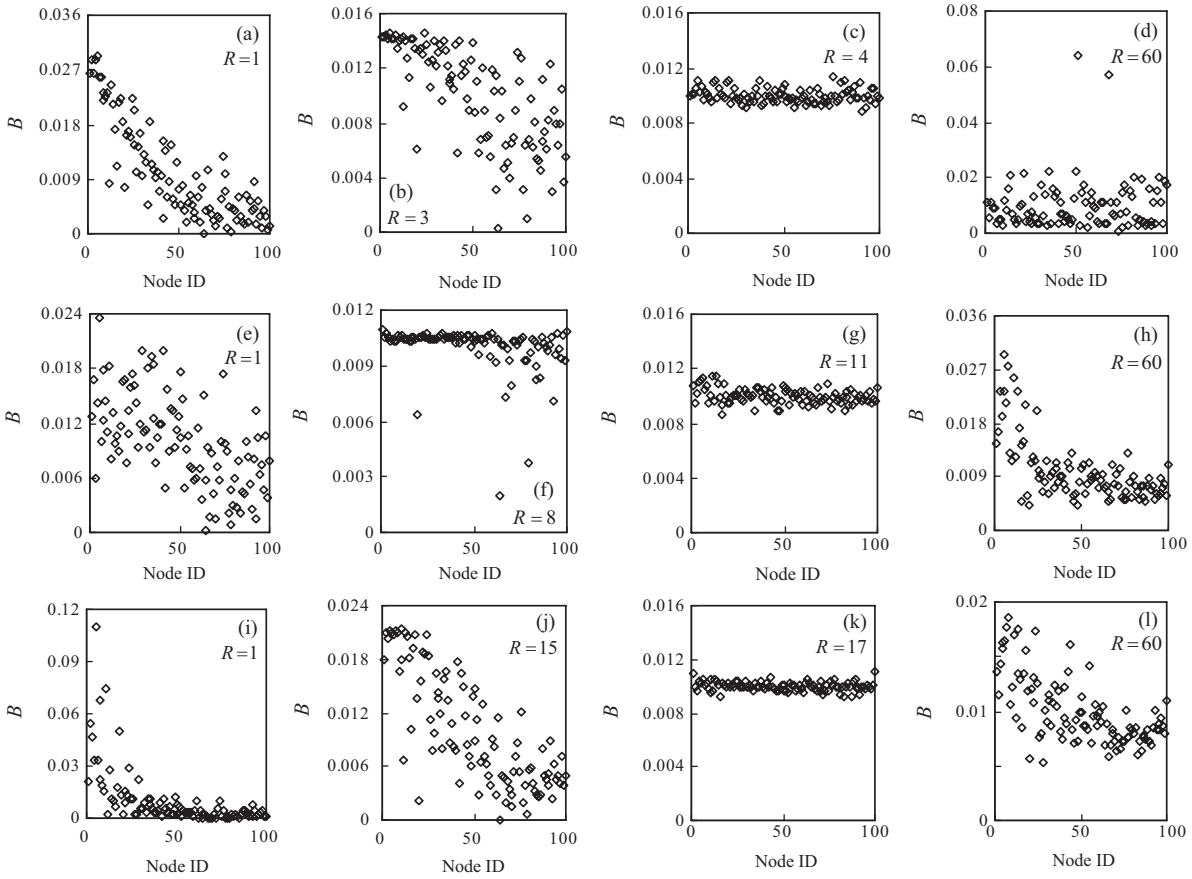


Fig. 9. (color online) Distributions of betweenness centrality for the proposed routing strategy under different values of R. Panels (a)–(d) are for $\rho = 1$; (e)–(h) for $\rho = 2$; and (i)–(l) for $\rho = 3$. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = 1$.

In order to further investigate the efficiency of our routing strategy, we simulate the dynamics with the node capacity $c = k$, as shown in Fig. 10. The simulations show that whatever the value of ρ, compared with the routing efficiency of $c = 1$, the network capacity is improved greatly when $c = k$, which means that it can enhance the capacity of the network

by improving the node capacity. For $\rho = 1$, the values of η, $\langle T_{\max} \rangle$, and $\langle T_{\text{avg}} \rangle$ all mutate when $R \approx 50$, which indicates that the network is in the region until $R_c \approx 50$. The average path length $\langle L_{\text{avg}} \rangle$ decreases with increasing R. For $\rho = 2$, due to the enhancement of the control and guide of the path length over the transfer of the packets, the network capacity is further improved and $R_c \approx 70$. For $\rho \geq 3$, η, $\langle T_{\max} \rangle$, $\langle T_{\text{avg}} \rangle$, and $\langle L_{\text{avg}} \rangle$ are all insensitive to ρ, and the network capacity is almost equal for different values of ρ. Because the gravity of the route almost considers the gravities of all the nodes on the path, the routing strategy takes into account both the congestion of the node and the path length, so the network region is improved greatly and $R_c \approx 140$. With the increase in R, $\langle L_{\text{avg}} \rangle$ first increases and then decreases, and the maximum $\langle L_{\text{avg}} \rangle$ is 9.1167 at $R = 70$. From the results, we can see that the process of generating congestion is relatively slow when $\rho \geq 3$ and the congestion situation is weak. We also simulate the critical point R_c versus the routing awareness depth ρ, as shown in Fig. 11. The results show that when the node capacity $c = k$, the relation between R_c and ρ is similar to that in the case of $c = 1$, which means that when $\rho < \langle l \rangle$, the network capacity R_c is poor and increases with increasing ρ, but when $\rho > \langle l \rangle$, then the network capacity R_c abruptly reaches the maximum and no longer changes with increasing ρ. Therefore, we can conclude that no matter when $c = 1$ or $c = k$, as long as the routing awareness depth ρ is greater than the average distance $\langle l \rangle$ of the network, then the network capacity will always reach the maximum and remain unchanged.

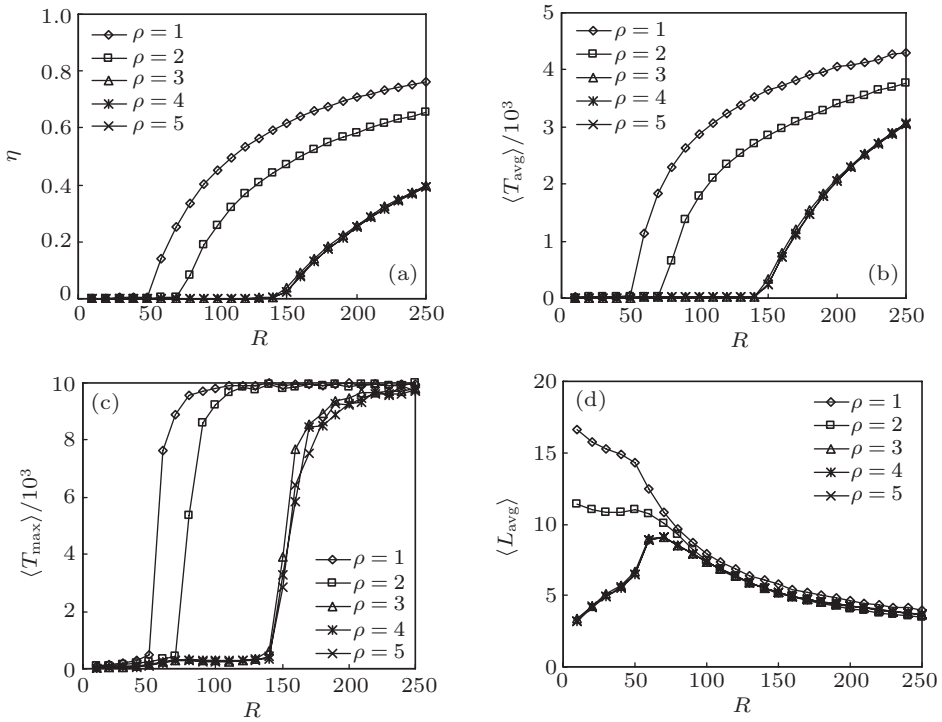


Fig. 10. Results of the proposed routing strategy for different values of ρ, showing (a) order parameter η versus R; (b) average travel time $\langle T_{\text{avg}} \rangle$ versus R; (c) maximum travel time $\langle T_{\max} \rangle$ versus R; and (c) average path length $\langle L_{\text{avg}} \rangle$ versus R. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = k$.

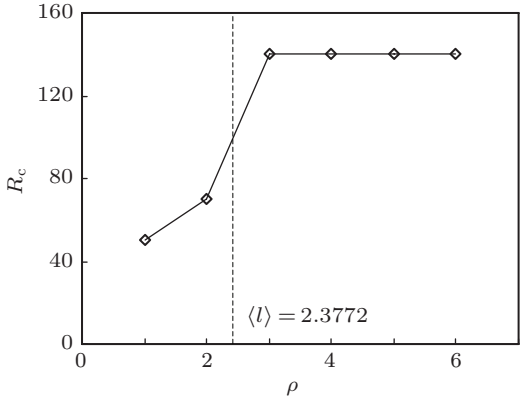


Fig. 11. Critical capacity R_c versus ρ. The other parameters are network size $N = 100$, $m_0 = m = 4$, $c = k$, $\alpha = 1$, $\gamma = 2$, and network average distance $\langle l \rangle$.

Figure 12 shows the distributions of betweenness centrality for different values of ρ when the node capacity $c = k$. The simulations indicate that with the increase in R, the distribution of betweenness centrality becomes more and more inhomogeneous when $\rho = 1$ or $\rho = 2$. For $\rho = 3$, the inhomogeneous situation first reduces and then deteriorates, but the situation is relatively good at $R \approx 30$. From the results, we can see that the distribution of betweenness centrality no longer represents homogeneous characteristics like those in the case of $c = 1$. When $c = k$, the processing capacities of the nodes with large degrees are great, which leads to large node gravities for the packets, and also increases the gravities of the paths containing these nodes, which results in a large number of packets choosing the path, so the betweenness centrality of nodes with

large degrees is great. But this does not show that the congestion of nodes with large degrees is very serious compared with the relative congestion (q/c) of nodes with small degrees, whose congestion is far less than that for the traditional shortest routing strategy, thereby reducing the network congestion effectively and improving the network capacity greatly.

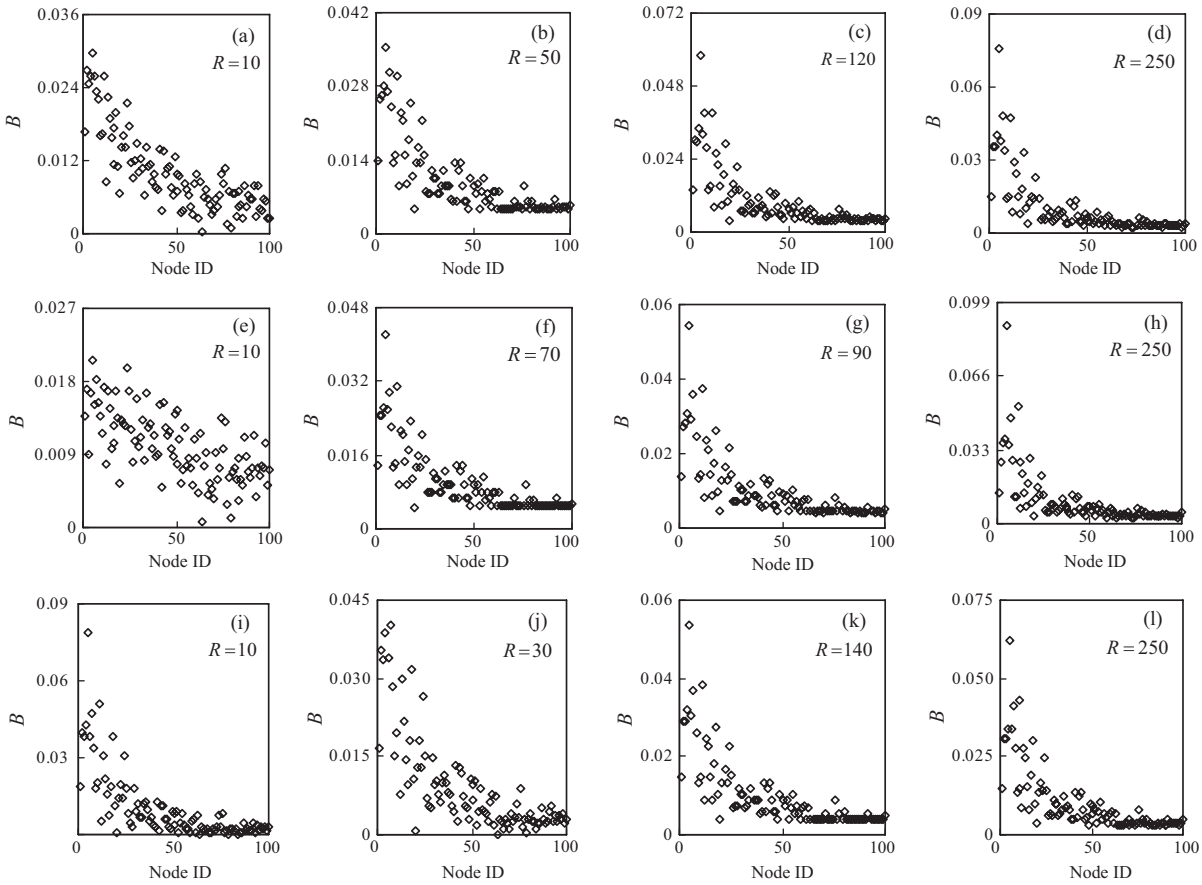


Fig. 12. Distributions of betweenness centrality for the proposed routing strategy under different values of R, showing panels (a)–(d) for $\rho = 1$; (e)–(h) for $\rho = 2$; and (i)–(l) for $\rho = 3$. The other parameters are network size $N = 100$, $m_0 = m = 4$, and $c = k$.

4. Conclusion

On the basis of the study in Ref. [20], we further investigate the dynamic characteristics of the routing strategy according to gravitational field theory. Considering the traffic processes of some real complex systems, we introduce the concept of routing awareness depth, and define the gravity of the shortest path from the neighbor to the destination as the combined action of the gravities of all the nodes within the routing awareness depth ρ on the shortest path. We establish the gravity formula of the path and propose a new routing strategy. We simulate the network transmission efficiencies under different values of ρ. The simulations show that if ρ is too small, then the strategy can only consider the gravities of individual nodes and cannot improve the transmission capacity of the network effectively, but if ρ is greater, then the strategy will consider more nodes in the path and the capacity of the network will be improved efficiently. Whatever the value of ρ is, the strategy can always balance the distribution of the node betweenness centrality and the network loads. Moreover, the results also uncover an important dynamic phenomenon, that when the value of ρ is greater than the average distance $\langle l \rangle$ of the network, then the routing strategy improves the region of the network, and the transmission efficiency will not change with increasing ρ.

References

[1] Daniele D M, Luca D A, Ginestra B and Matteo M 2007 *Phys. Rev. E* **79** 015101
[2] Perotti J I and Billoni O V 2012 *Phys. Rev. E* **86** 011120
[3] Li S B, Wu J J, Gao Z Y, Lin Y and Fu B B 2011 *Acta Phys. Sin.* **60** 050701 (in Chinese)
[4] Echenique P, Gomez-Gardenes J and Moreno Y 2005 *Europhys. Lett.* **71** 325
[5] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D U 2006 *Phys. Rep.* **424** 175
[6] Newman M E J 2010 *Networks: An Introduction* (Oxford: Oxford University Press)
[7] Ramascc J J, Lama M S L, Eduardo L and Boettcher S 2010 *Phys. Rev. E* **82** 036119
[8] Yan G, Zhou T, Hu B, Fu Z Q and Wang B H 2006 *Phys. Rev. E* **73** 046108
[9] Zhou S Y, Wang K, Zhang Y F, Pei W J, Pu C L and Li W 2011 *Chin. Phys. B* **20** 080501
[10] Shen Y, Pei W J, Wang K and Wang S P 2009 *Chin. Phys. B* **18** 3783
[11] Noh J D 2004 *Phys. Rev. Lett.* **92** 11

[12] Yang S J 2005 *Phys. Rev. E* **71** 016107
[13] Liu F, Zhao H, Li M, Ren F Y and Zhu Y B 2010 *Chin. Phys. B* **19** 040513
[14] Wang D, Yu H, Jing Y W, Jiang N and Zhang S Y 2009 *Acta Phys. Sin.* **58** 6802 (in Chinese)
[15] Goh K I, Kahng B and Kim D 2001 *Phys. Rev. Lett.* **87** 278701
[16] Meng Q K and Zhu J Y 2009 *Chin. Phys. B* **18** 3632
[17] Guimerà R, Díaz-Guilera A, Vega-Redondo F, Cabrales A and Arenas A 2002 *Phys. Rev. Lett.* **89** 248701
[18] Danila B, Yu Y, Marsh J A and Bassler K E 2007 *Chaos* **17** 026102
[19] Danila B, Sun Y D and Bassler K E 2009 *Phys. Rev. E* **80** 066116
[20] Liu G and Li Y S 2012 *Acta Phys. Sin.* **61** 248901 (in Chinese)
[21] Arenas A, Díaz-Guilera A and Guimerá R 2001 *Phys. Rev. Lett.* **86** 3196
[22] Crucitti P, Latora V and Porta S 2006 *Chaos* **16** 015113
[23] Li Q Q, Zeng Z, Yang B S and Li B J 2010 *Geomatics and Information Science of Wuhan University* **35** 37
[24] Freeman L G 1977 *Sociometry* **40** 35
[25] Barabási A L and Albert R 1999 *Science* **286** 509
[26] Echenique P, Gomez-Gardenes J and Moreno Y 2004 *Phys. Rev. E* **70** 056105