The voltage–current relationship and equivalent circuit implementation of parallel flux-controlled memristive circuits*

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A flux-controlled memristor characterized by smooth cubic nonlinearity is taken as an example, upon which the voltage–current relationships (VCRs) between two parallel memristive circuits – a parallel memristor and capacitor circuit (the parallel MC circuit), and a parallel memristor and inductor circuit (the parallel ML circuit) – are investigated. The results indicate that the VCR between these two parallel memristive circuits is closely related to the circuit parameters, and the frequency and amplitude of the sinusoidal voltage stimulus. An equivalent circuit model of the memristor is built, upon which the circuit simulations and experimental measurements of both the parallel MC circuit and the parallel ML circuit are performed, and the results verify the theoretical analysis results.

Keywords: flux-controlled memristor, memristor and capacitor (MC) circuit, memristor and inductor (ML) circuit, equivalent circuit

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1. Introduction

The memristor, studied by Chua in 1971[1,2] and realized by Strukov *et al.*[3] in 2008, is the fourth fundamental electronic component besides the resistor, capacitor, and inductor. Since the successful fabrication of the memristor, much research interest has been aroused by the potential applications of memristors in computers, neural networks, analog circuitries, and so on.[4–8] As the Hewlett Packard (HP) memristor is still not easily available to researchers, any verification of proposed memristive circuits is mainly based on simulations, emulations or equivalent circuits.[7–12] Recently, a number of memristor models have been proposed.[7–11] The models are more or less based on the HP memristor, which is not an ideal element but a generalized memristive circuit.[2]

In the past few years, the physical realization of various memristors,[3,13] the modeling and analysis of the basic memristor circuit,[8,14] and the design and analysis of memristor-based application circuits,[9–11] etc, have attracted much attention in the electrical and electronic engineering communities. Research work to explore the characteristics of the memristor is important and has attracted much attention.[15,16] Until now, very few research results, especially circuit simulation results and experimental observations,[9–11] have been reported on the memristor and its basic constituting circuits.[12] In this paper, a flux-controlled memristor characterized by a smooth cubic nonlinearity[8,11] is taken as an example, and the voltage–current relationships (VCRs) of both parallel MC and parallel ML circuits are studied. The dependences of the VCR of these two parallel memristive circuits on the circuit parameters, and the frequency and amplitude of the sinusoidal voltage stimulus, are obtained. An equivalent circuit model of a flux-controlled memristor is built, based on which, circuit simulations and experimental measurements of the two parallel memristive circuits are performed.

2. Flux-controlled memristor model

A memristor consisting of a flux-controlled memristor and a charge-controlled memristor is a two-terminal element characterized by a relation of the type $f(\phi,q)=0$. A flux-controlled memristor characterized by the following smooth continuous cubic nonlinearity is taken as an example to perform time-domain analysis of some basic memristive circuits:[8,11]

$$q(\varphi)=a\varphi+b\varphi^3, \tag{1}$$

where both a and b are positive constants.

From Eq. (1), the memductance $W(\varphi)$ of this flux-controlled memristor is obtained as

$$W(\varphi)=\frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi}=a+3b\varphi^2. \tag{2}$$

If we define $u(t)$ and $i(t)$ as the voltage across the memristor and the current flowing through the memristor, respectively, then we will have

$$i(t)=W(\varphi)u(t)=\left(a+3b\varphi^2\right)u(t), \tag{3}$$

where $\varphi(t)=\int_{-\infty}^{t}u(\tau)\mathrm{d}\tau$.

If we consider an external sinusoidal voltage stimulus

$$u(t)=U_{\mathrm{m}}\sin\omega t=U_{\mathrm{m}}\sin(2\pi ft) \tag{4}$$

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to be applied across the element, where $U_{\rm m}$ is the amplitude, ω is the radian frequency, and f is the frequency, then the flux-linkage is given by

$$\begin{aligned}\varphi(t) &= \int_{-\infty}^{t} u(\tau)\mathrm{d}\tau \\ &= \varphi_0 + \int_0^t U_{\rm m}\sin\omega\tau\mathrm{d}\tau \\ &= \varphi_0 + \frac{U_{\rm m}}{\omega}(1-\cos\omega t),\end{aligned} \tag{5}$$

where $\varphi_0 = \int_{-\infty}^{0} u(\tau)\mathrm{d}\tau$ is the internal initial state of the flux-controlled memristor.

Substituting Eqs. (4) and (5) into Eq. (3) and considering the following relationships

$$2\sin\omega t\cos\omega t = \sin 2\omega t,$$

and

$$4\cos^2\omega t\sin\omega t = \sin 3\omega t + \sin\omega t,$$

we have

$$\begin{aligned}i(t) &= \left[a + \frac{3b}{\omega^2}(\varphi_0\omega + U_{\rm m} - U_{\rm m}\cos\omega t)^2\right]U_{\rm m}\sin\omega t \\ &= c_1\sin(\omega t) + c_2\sin 2\omega t + c_3\sin 3\omega t \\ &= \sum_{k=1}^{3} c_k\sin k\omega t,\end{aligned} \tag{6}$$

where

$$c_1 = \left(a + 3b\varphi_0^2 + \frac{6b}{\omega}\varphi_0 U_{\rm m} + \frac{15b}{4\omega^2}U_{\rm m}^2\right)U_{\rm m},$$

$$c_2 = -\frac{3b}{\omega^2}(\varphi_0\omega + U_{\rm m})U_{\rm m}^2,$$

and

$$c_3 = \frac{3b}{4\omega^2}U_{\rm m}^3.$$

From Eq. (6), it follows that the current flowing through this memristor consists of the fundamental component, the second harmonic component, and the third harmonic component, which are related to the circuit parameters, memristor initial flux-linkage,[17,18] and the frequency and amplitude of the sinusoidal voltage stimulus.

3. The parallel memristor and capacitor (MC) circuit

A parallel memristive circuit consisting of a flux-controlled memristor and capacitor is shown in Fig. 1.[16] Applying Kirchhoff's current law to the circuit, we obtain

$$i(t) = C\frac{\mathrm{d}u(t)}{\mathrm{d}t} + \left\{a + 3b\left[\int_{-\infty}^{t} u(\tau)\mathrm{d}\tau\right]^2\right\}u(t). \tag{7}$$

When the sinusoidal voltage stimulus of Eq. (4) is applied to the input port of the parallel MC circuit, the resulting current is

$$i(t) = \omega C U_{\rm m}\cos\omega t + \sum_{k=1}^{3} c_k\sin k\omega t. \tag{8}$$

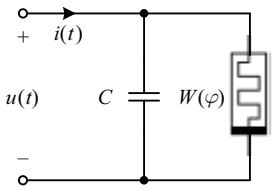


Fig. 1. Parallel memristive circuit of a memristor and capacitor.

It should be noted that only the fundamental component of the port current flowing through the parallel memristive circuit is affected by the parallel capacitor. Therefore, the properties of the parallel memristive circuit are similar to those of the memristor, which are related to the circuit parameters, the memristor initial flux-linkage, the frequency and the amplitude of the sinusoidal voltage stimulus.

Based on Eq. (8), the simulation study of the parallel MC circuit can be performed. Let $a = 0.6667$ mS, $b = 194.4945$ S/Wb^2, $U_{\rm m} = 3$ V, and $\varphi_0 = 0$ Wb be fixed,[9] and the two parameters f and C be varied. Figure 2(a) shows the

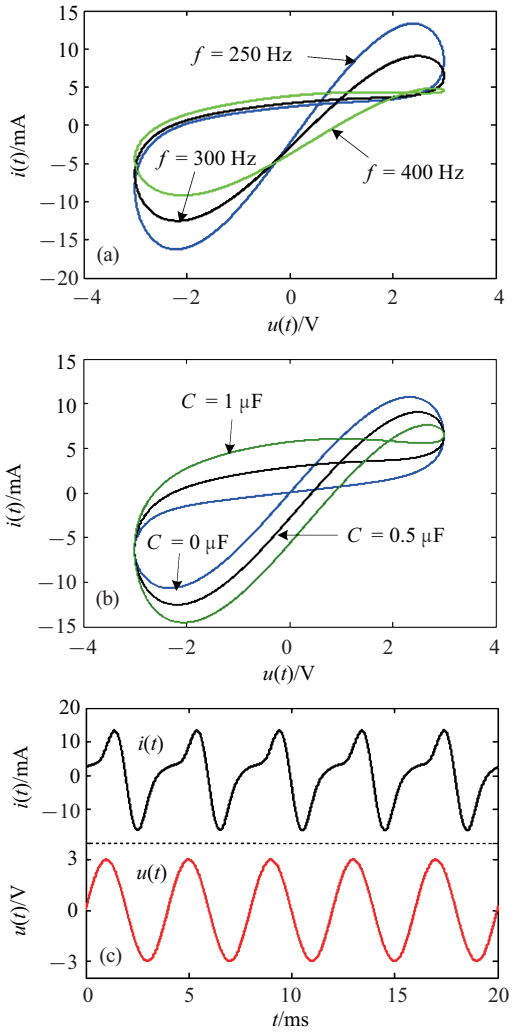


Fig. 2. (color online) The VCR curves and time-domain waveforms of the parallel memristor and capacitor circuit: (a) VCR curves under different frequencies with $C = 0.5$ μF; (b) VCR curves under different capacitances with $f = 300$ Hz; (c) time-domain waveforms with $C = 0.5$ μF and $f = 300$ Hz.

VCR curves of the parallel MC circuit for sinusoidal voltage stimuli at three different frequencies in the case $C = 0.5$ μF. Figure 2(b) indicates the VCR curves of the parallel MC circuit for three different capacitances with $f = 300$ Hz, and figure 2(c) demonstrates the time-domain waveforms of the sinusoidal voltage stimulus $u(t)$ and the resulting port current $i(t)$ when $C = 0.5$ μF and $f = 300$ Hz. As observed from Fig. 2, it is obvious that when the lower frequency sinusoidal voltage stimulus is applied or smaller capacitance is connected, then the parallel MC circuit typically behaves as a pinched hysteresis loop characteristic.[8]

4. The parallel memristor and inductor (ML) circuit

Another parallel memristive circuit consisting of a flux-controlled memristor and inductor is shown in Fig. 3.[16] Similarly, we easily obtain

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(\tau)\mathrm{d}\tau + \left\{a + 3b\left[\int_{-\infty}^{t} u(\tau)\mathrm{d}\tau\right]^2\right\}u(t). \quad (9)$$

For the sinusoidal voltage stimulus of Eq. (4), the port current is

$$i(t) = I_0 + \frac{U_\mathrm{m}}{\omega L} - \frac{U_\mathrm{m}}{\omega L}\cos\omega t + \sum_{k=1}^{3} c_k \sin k\omega t, \quad (10)$$

where I_0 is the initial inductor current.

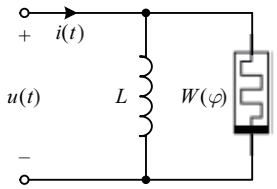


Fig. 3. The parallel memristive circuit of the memristor and inductor.

It can be found from Eq. (10) that, besides the fundamental and harmonic components, the port current of the parallel ML circuit contains a direct current component, and both the direct current component and fundamental component of the port current are affected by the parallel inductor.

Based on Eq. (10) and the same circuit parameters of the flux-controlled memristor and the amplitude of the sinusoidal voltage stimulus, figure 4(a) shows the VCR curves of the parallel ML circuit with sinusoidal voltage stimuli at three different frequencies when $L = 0.4$ H. Figure 4(b) shows the VCR curves of the parallel ML circuit with three different inductances when $f = 300$ Hz, and figure 4(c) shows the time-domain waveforms of $u(t)$ and $i(t)$ when $L = 0.4$ H and $f = 300$ Hz. Here, the initial inductor current I_0 is assumed to be zero. Obviously, when a lower frequency sinusoidal voltage stimulus is applied or larger inductance is connected, the parallel ML circuit typically behaves as a pinched hysteresis loop characteristic.[8] Additionally, the current of the parallel ML circuit has a direct current component. If $L = 0.4$ H and $f = 300$ Hz, then the direct current component calculated from Eq. (10) will be 3.9789 mA.

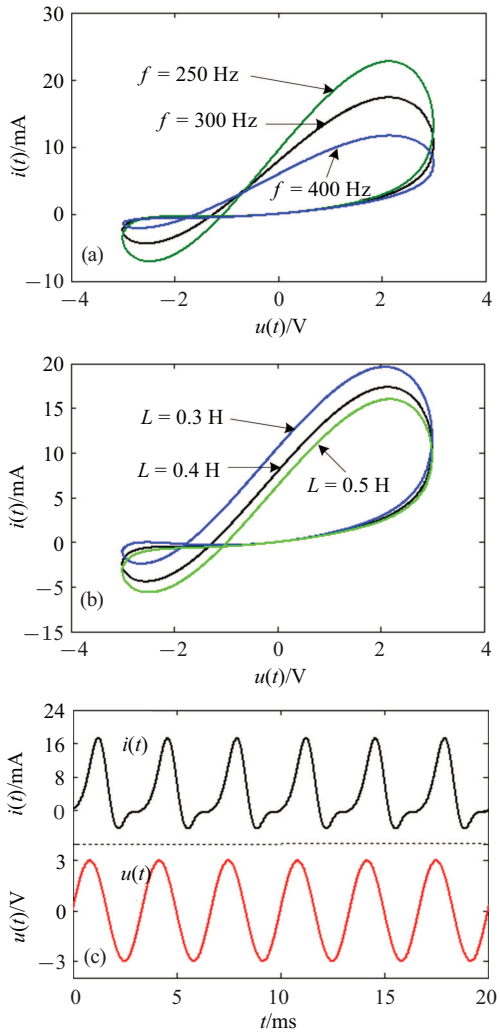


Fig. 4. (color online) The VCR curves and time-domain waveforms of the parallel memristor and inductor circuit: (a) VCR curves under different frequencies with $L = 0.4$ H; (b) VCR curves under different inductances with $f = 300$ Hz; (c) time-domain waveforms with $L = 0.4$ H and $f = 300$ Hz.

5. Verification by circuit simulation and experiment

Figure 5 shows the circuit schematic diagram used to realize the memristor characterized by Eq. (1).[9] The operational amplifier U_1 is used to avoid the loading effect. The operational amplifier U_2 connected to the resistor R and capacitor C is an integrator, and is used to realize the following relationship

$$u_\mathrm{a}(t) = -\frac{1}{R_0C_0}\int_{-\infty}^{t} u(t)\mathrm{d}\tau = -\frac{1}{R_0C_0}\varphi(t). \quad (11)$$

The output of the multiplier M_2 in Fig. 5 is given as

$$u_\mathrm{b}(t) = g_1g_2[u_\mathrm{a}(t)]^2u(t), \quad (12)$$

where g_1 and g_2 are the two scale factors of the multipliers M_1 and M_2, respectively. The operational amplifier U_3 connecting

to R_2, R_3, and R_4 is the current–inverter that implements the following relationship[9] (when $R_3 = R_4$):

$$i_1(t) = -\frac{u(t) - u_b(t)}{R_2} = \left\{-\frac{1}{R_2} + \frac{g_1 g_2 [u_a(t)]^2}{R_2}\right\} u(t). \quad (13)$$

Thus, the resulting port current can be obtained as

$$i(t) = \left\{\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{g_1 g_2 [u_a(t)]^2}{R_2}\right\} u(t). \quad (14)$$

Substituting Eq. (11) into Eq. (14), we obtain the expression of equivalent memductance as

$$W(\varphi) = \frac{1}{R_1} - \frac{1}{R_2} + \frac{g_1 g_2}{R_2 (R_0 C_0)^2} \varphi^2. \quad (15)$$

Comparing Eq. (2) with Eq. (15) we have the following relationships

$$a = \frac{1}{R_1} - \frac{1}{R_2}, \quad b = \frac{g_1 g_2}{3R_2 (R_0 C_0)^2}. \quad (16)$$

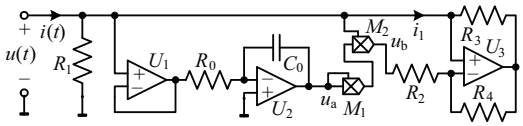


Fig. 5. The equivalent circuit of the memristor with smooth cubic nonlinearity.

In our equivalent circuit implementation of a flux-controlled memristor, the circuit parameters are listed in Table 1, with AD633JN serving as the analog multiplier, and AD711kN used as the operational amplifier, at operation voltages of ±15 V. Thus, the equivalent parameters of the flux-controlled memristor are $a = 0.6667$ mS and $b = 194.4945$ S/Wb^2.

Table 1. The circuit parameters of the memristor equivalent circuit.

Circuit parameters		Values
Capacitor C_0		47 nF
Resistor R_0		8.2 kΩ
Resistor R_1		750 Ω
Resistor R_2		1.5 kΩ
Resistors R_3, R_4		2 kΩ, 2 kΩ
Scale factor (M_1)	g_1	0.1
Scale factor (M_2)	g_2	1.3

Applying the equivalent circuit of the memristor in Fig. 5 to Fig. 1, the Multisim simulation results of the $u(t)$–$i(t)$ curves and time-domain waveforms of the parallel MC circuit with the same sinusoidal voltage stimuli and circuit parameters as those in Fig. 2, are depicted in Fig. 6. In Fig. 6(c), the current waveform has a positive bias with 16 mA and the voltage waveform has a negative bias with 8 V. The corresponding experimental measurements of the parallel MC circuit are shown in Fig. 7. Here, the digital oscilloscope Tektronix DPO3034 is employed to capture the measured waveforms.

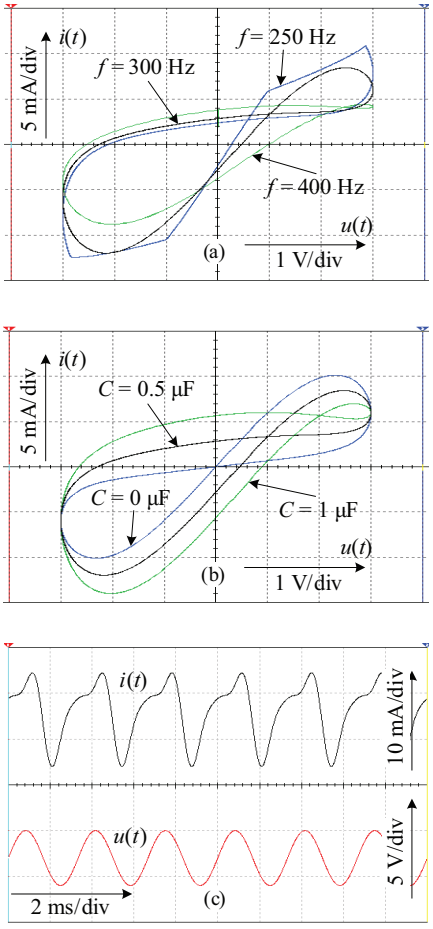


Fig. 6. (color online) The equivalent circuit simulation results of the $u(t)$–$i(t)$ curves under different parameters: (a) different frequencies with $C = 0.5$ μF; (b) different capacitances with $f = 300$ Hz; (c) time-domain waveforms with $C = 0.5$ μF and $f = 300$ Hz.

Similarly, applying the equivalent circuit of the memristor in Fig. 5 to Fig. 3, the corresponding simulation results of the $u(t)$–$i(t)$ curves of the parallel ML circuit with the same sinusoidal voltage stimuli and circuit parameters as those in Fig. 4, are plotted in Fig. 8. In Figs. 8(a) and 8(b), the $u(t)$–$i(t)$ curves have the same negative voltage bias with 1 div. In Fig. 8(c), the current waveform has a positive bias with 10 mA and the voltage waveform has a negative bias with 8 V. The corresponding experimental measurements of the parallel ML circuit are shown in Fig. 9.

Comparing the simulation results in Figs. 6 and 8 and the experimental measurements in Figs. 7 and 9 with the analysis results in Figs. 2 and 4, we can find that when the frequency of the sinusoidal voltage stimulus equals 250 Hz, the VCR curves of the parallel memristive circuits are not smooth in part of the applied voltage regions due to the nonlinearities of the operational amplifier and multiplier. Generally, the simulation results in Figs. 6 and 8 and the experimental measurements in Figs. 7 and 9 are consistent with the analysis results in Figs. 2 and 4.

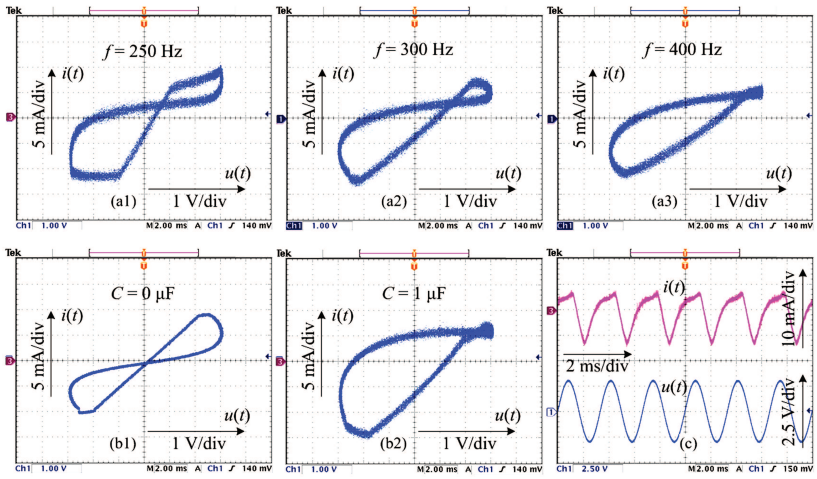


Fig. 7. (color online) The experimental results corresponding to Fig. 6: (a) different frequencies with $C = 0.5$ μF, where (a1) $f =$ 250 Hz, (a2) $f = 300$ Hz, and (a3) $f = 400$ Hz; (b) different capacitances with $f = 300$ Hz, where (b1) $C = 0$ μF and (b2) $C = 1$ μF; (c) time-domain waveforms with $C = 0.5$ μF and $f = 300$ Hz.

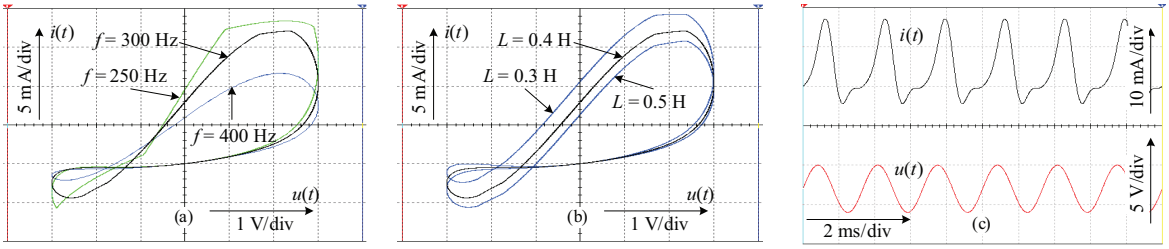


Fig. 8. (color online) The equivalent circuit simulation results of the $u(t)$–$i(t)$ curves under different parameters: (a) different frequencies with $L = 0.4$ H; (b) different inductances with $f = 300$ Hz; (c) time-domain waveforms with $L = 0.4$ H and $f = 300$ Hz.

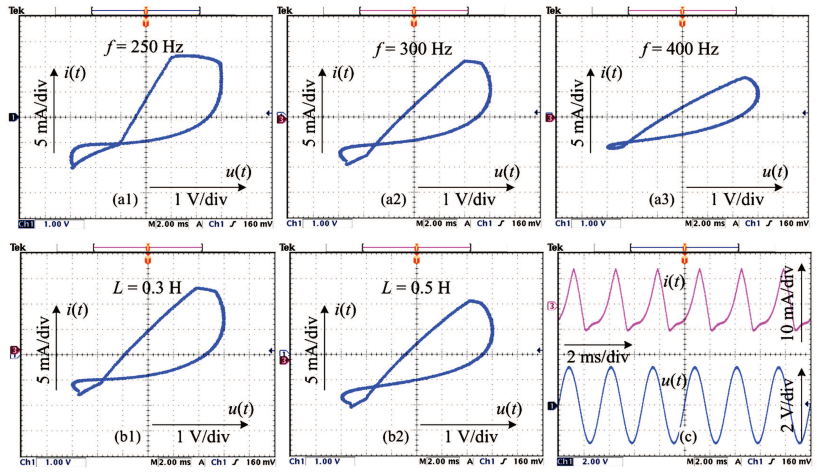


Fig. 9. (color online) The experimental results corresponding to Fig. 8: (a) different frequencies with $L = 0.4$ H, where (a1) $f = 250$ Hz, (a2) $f = 300$ Hz, and (a3) $f = 400$ Hz; (b) different inductances with $f = 300$ Hz, where (b1) $L = 0.3$ H and (b2) $L = 0.5$ H; (c) time-domain waveforms with $L = 0.4$ H and $f = 300$ Hz.

6. Conclusions

The parallel MC circuit and parallel ML circuit are two basic memristive circuits. To study the VCRs of these two parallel memristive circuits, a flux-controlled memristor characterized by smooth cubic nonlinearity is taken as an example. Theoretical analyses and numerical simulations, which are verified by circuit simulations and experimental observations based on an equivalent circuit model of the memristor, indicate that these two parallel memristive circuits are closely related to the circuit parameters, the frequency and the amplitude of the sinusoidal voltage stimulus. These results can help us to better understand the properties of memristive circuits.

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