Enhancing stationary optomechanical entanglement with the Kerr medium*

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We theoretically investigate the stationary entanglement of a optomechanical system with an additional Kerr medium in the cavity. There are two kinds of interactions in the system, photon–mirror interaction and photon–photon interaction. The optomechanical entanglement created by the former interaction can be effectively controlled by the latter one. We find that the optomechanical entanglement is suppressed by Kerr interaction due to photon blockage. We also find that the Kerr interaction can create the stationary entanglement and induce the resonance of entanglement in the small detuning regime. These results show that the Kerr interaction is an effective control for the optomechanical system.

Keywords: optomechanics, Kerr medium, entanglement

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1. Introduction

Entanglement not only plays a basic role in quantum theory, but also is a key resource in quantum information processing.[1] The entanglement beyond the microscopic level inspires long-standing interest. Its existence has been widely accepted, providing that decoherence induced by environment[2] can be sufficiently suppressed. Generally, the experimental proposals for the observation of macroscopic quantum entanglement are often based on the principle of Schrödinger's cat.[3,4]

Optomechanics[5] provides another promising approach in this context. Its standard model is composed by a Fabry–Pérot cavity with a movable micromirror. The center-of-mass motion of a mechanical oscillator can be manipulated by the radiation pressure force of the cavity.[6] The variant beyond the standard optomechanics is abundant, such as a semitransparent membrane within a standard Fabry–Pérot cavity,[7] nanoelectromechanical systems formed by a microwave cavity capacitively coupled to a nanoresonator,[8] and ultracold atoms interacting with an optical cavity.[9,10] Optomechaincs may possibly be applied to sense very small masses or forces with the precision of quantum limits,[11] realize the quantum interfaces for quantum information processing,[12] cool the mirror down to the ground state,[13] even entangle photon and micro- or nano-mechanical oscillator.[14–21]

All these interesting properties of optomechanics rely on the coupling between the light and mirror, which closely depends on the radiation pressure of photon. This inspires us to study the possibility to manipulate the system using effects of the photon statistics. Actually, it was reported that type-I optical parametric amplifier inside a cavity can achieve cooling of a movable mirror to sub-Kelvin temperatures.[22] The Kerr medium in the cavity effectively inhibits the normal mode splitting due to the photon blockade mechanism.[23] Kerr interaction would manipulate the energy band loops of ultracold atoms in the cavity.[24]

In this paper, we consider a standard optomechanical system and the cavity additionally fills with the Kerr medium, which gives rise to a strong nonlinear interaction between photons. Specifically, we investigate the effect of Kerr interaction on the stationary optomechanical entanglement. With Kerr interaction, the photon in a cavity can block the injection of the other photon due to photon blockade effect.[25] We find that the large Kerr interaction can suppress the optomechanical entanglement due to this blockade effect. More interestingly, in the small detuning regime, we also find that the Kerr interaction can create the stationary optomechanical entanglement, compared with the case without Kerr interaction. And the optomechanical entanglement also shows the resonance-like peak with respect to Kerr interaction in this regime. The reason for these behaviors may be that Kerr interaction can enhance the fluctuational correlation of photon field. All these results show that Kerr interaction is an effective control for the optomechanical system.

2. The model of system

In this paper, we consider a standard optomechanical system with additionally optical Kerr medium in the cavity, as

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shown in Fig. 1. One pump laser with frequency $\omega_{\rm p}$ is injected into a Fabry–Pérot cavity with frequency $\omega_{\rm c}$. The cavity is composed by two mirrors: a fixed partially transmitting mirror and a movable totally reflecting mirror in contact with a thermal bath at temperature T. The movable mirror is treated as a quantum mechanical harmonic oscillator with effective frequency $\omega_{\rm m}$ and energy decay rate $\gamma_{\rm m}$.

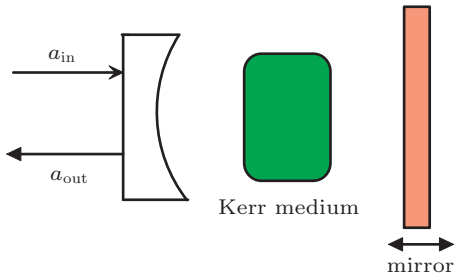


Fig. 1. (color online) The sketch of the optomechanical system. The cavity contains Kerr medium.

In the adiabatic limit, the cavity approximates as a single mode because the scatterings of photons to other modes can be ignored if the free spectral range of the cavity $c/2L$ is much larger than $\omega_{\rm m}$, with c being the velocity of light in vacuum and L being the length of the cavity. In the rotating frame with the pump frequency, the Hamiltonian of the total system is[26]

$$H/\hbar = H_0 + H_{\rm I} \tag{1}$$

with free terms

$$H_0 = \Delta_0 a^\dagger a + {\rm i}\varepsilon_{\rm p}(a^\dagger - a) + \frac{1}{2}\omega_{\rm m}(p^2+q^2) \tag{2}$$

and the interaction parts

$$H_{\rm I} = -G_0 a^\dagger a q + \frac{1}{2}\chi a^{\dagger 2} a^2. \tag{3}$$

Here, $a^\dagger$ is the creation operator of cavity with the commutation relation $[a, a^\dagger] = 1$. The strength of the pump laser is $\varepsilon_{\rm p} = \sqrt{2\kappa P/\hbar\omega_{\rm p}}$. The position operator q and momentum operator p of the mechanical resonator satisfy the standard commutation $[p,q] = {\rm i}$. $G_0 = \beta q_0$ is coupling parameter between the cavity and the mechanical resonator with $\beta = \partial\omega_{\rm c}/\partial x$ and $q_0 = \sqrt{\hbar/2\omega_{\rm m} m}$ being the zero-point motion of the mechanical resonator. Giant optical Kerr nonlinearities are obtained by placing a $\xi^{(3)}$ medium inside a cavity,[25] with $\chi = 3\omega_{\rm c}^2 {\rm Re}[\xi^{(3)}]/2\varepsilon_0 V_{\rm c}$, ε_0 being the dielectric constant of the medium, $V_{\rm c}$ being the volume of the cavity, and $\xi^{(3)}$ being the third-order nonlinear susceptibility. Here, $\Delta_0 = \omega_{\rm c} - \omega_{\rm p}$. The dissipation process of the cavity is consider as $H_{\rm d} = -{\rm i}\kappa a^\dagger a$, with κ being the decay of the cavity.

The equations of motion derived from Heisenberg operator equations in addition to dissipation processes reduce to

$$\frac{{\rm d}a}{{\rm d}t} = -(\kappa + {\rm i}\Delta_0)a - {\rm i}\chi a^\dagger a^2 + {\rm i}G_0 q a + \varepsilon_{\rm p} + \sqrt{2\kappa}a_{\rm in}, \tag{4}$$

$$\frac{{\rm d}p}{{\rm d}t} = -\gamma_{\rm m} p - \omega_{\rm m} q + g_{\rm m} a^\dagger a + \xi(t), \tag{5}$$

$$\frac{{\rm d}q}{{\rm d}t} = \omega_{\rm m} p, \tag{6}$$

where $a_{\rm in}$ is the input vacuum noises with zero mean $\langle a_{\rm in}\rangle = 0$ and correlation $\langle a_{\rm in}(t) a_{\rm in}^\dagger(t')\rangle = \delta(t-t')$. The mechanical mode is affected by a viscous force with damping rate $\gamma_{\rm m}$ and a Brownian stochastic force $\xi(t)$. Generally, this Brownian force is non-Markovian. At present, the experimentally realizable quality factor $Q_{\rm m} = \omega_{\rm m}/\gamma_{\rm m}$ of the movable mirror is very high, up to 10^6. In this limit, the correlation functions can be approximately written as $\langle\xi(t)\rangle = 0$, $\langle\xi_{\rm in}(t)\xi_{\rm in}(t')\rangle = \gamma_{\rm m}(2n_{\rm th}+1)\delta(t-t')$ with $n_{\rm th} = [\exp(\hbar\omega_{\rm m}/k_{\rm B}T) - 1]^{-1}$.

In the mean-field approximation, the steady state is given by

$$\langle p\rangle = 0, \quad \langle q\rangle = G_0 n_{\rm ph}/\omega_{\rm m}, \quad \langle a\rangle = \alpha_{\rm s} = \frac{\varepsilon_{\rm p}}{\kappa + {\rm i}\Delta} \tag{7}$$

with $\Delta = \Delta_0 - G_0\langle q\rangle + \chi n_{\rm ph}$ and $n_{\rm ph} = |\alpha_{\rm s}|^2$.

Formally, the equations of motion are very similar to the counterparts in Ref. [14], except for the additional Kerr term. Actually, they are very different as the photon number is partly determined by the Kerr term. Taking the interaction between the photon and the mechanical resonator into account, Kerr interaction of photon will play a non-trivial role in the properties of the system. In the following, we study the effect of Kerr interaction on the stationary optomechanical entanglement.

3. Stationary entanglement of optomechanical system

3.1. Theoretical frame

The stationary entanglement in the optomechanical system implies the strong quantum correlation between photon and phonon modes. One way to achieve this is making use of the strong coupling, which can be realized by a strong driving field and a high-fineness cavity. In this case, the cavity can be considered as in a classically coherent state, and the quantum entanglement can be characterized by the fluctuational correlations between photon and phonon modes.

Denoting the operators as the sum of their mean values and the fluctuational operators, $a = \langle a\rangle + \delta a$, $p = \langle p\rangle + \delta p$, and $q = \langle q\rangle + \delta q$, the quantum Langevin equations are linearized around the mean values. The equations of the fluctuational operators read

$$\begin{aligned}\dot{\delta a} = {} & -(\kappa + {\rm i}\Delta)\delta a - {\rm i}\chi(\alpha_{\rm s}^2\delta a^\dagger + |\alpha_{\rm s}|^2\delta a) \\ & + {\rm i}G\delta q + \sqrt{2\kappa}a_{\rm in},\end{aligned} \tag{8}$$

$$\dot{\delta p} = -\gamma_{\rm m}\delta p - \omega_{\rm m}\delta q + G^*\delta a + G\delta a^\dagger + \xi(t), \tag{9}$$

$$\dot{\delta q} = \omega_{\rm m}\delta p \tag{10}$$

with $G = G_0\alpha_{\rm s}$. The second order terms, such as $\delta a^\dagger\delta a$, have been omitted as they make a small contribution compared with $\langle a^\dagger\rangle\delta a$. It is convenient to rewrite the linearized equation

in terms of field quadrature fluctuations $\delta X = [\delta a \exp(\mathrm{i}\theta) + \text{h.c.}]/\sqrt{2}$ and $\delta Y = [\delta a \exp(\mathrm{i}\theta) - \text{h.c.}]/\mathrm{i}\sqrt{2}$

$$\dot{\delta q} = \omega_{\mathrm{m}} \delta p, \tag{11}$$

$$\dot{\delta p} = -\gamma_{\mathrm{m}} \delta p - \omega_{\mathrm{m}} \delta q + G_2 \delta X + \xi(t), \tag{12}$$

$$\dot{\delta X} = -\kappa \delta X + \Delta \delta Y + \sqrt{2\kappa} X_{\mathrm{in}}, \tag{13}$$

$$\dot{\delta Y} = -\kappa \delta Y - \Delta_2 \delta X + G_2 \delta q + \sqrt{2\kappa} Y_{\mathrm{in}}, \tag{14}$$

with $\theta = \arctan[\Delta/\kappa]$, $\Delta_2 = \Delta + 2\chi|\alpha_{\mathrm{s}}|^2$, and $G_2 = \sqrt{2}G$.

These equations can be rewritten compactly as

$$\dot{u}(t) = Au(t) + n(t). \tag{15}$$

Here, $u = (\delta q,\ \delta p,\ \delta X,\ \delta Y)^{\mathrm{T}}$, $n(t) = (0,\ \xi,\ \sqrt{2\kappa}X_{\mathrm{in}},\ \sqrt{2\kappa}Y_{\mathrm{in}})$, and

$$A = \begin{pmatrix} 0 & \omega_{\mathrm{m}} & 0 & 0 \\ -\omega_{\mathrm{m}} & -\gamma_{\mathrm{m}} & G_2 & 0 \\ 0 & 0 & -\kappa & \Delta \\ G_2 & 0 & -\Delta_2 & -\kappa \end{pmatrix}. \tag{16}$$

The steady state is a zero-mean Gaussian state due to the fact that the dynamics of the fluctuations is linearized and all noises are Gaussian. As a consequence, it is fully characterized by the 4×4 stationary correlation matrix (CM) V, with matrix elements

$$\begin{aligned} V_{ij} &= \frac{\langle u_i(\infty)u_j(\infty) + u_j(\infty)u_i(\infty)\rangle}{2} \\ &= \int_0^\infty \mathrm{d}s \int_0^\infty \mathrm{d}s' M_{ik}(s) M_{jl}(s') D_{kl}(s-s'), \end{aligned}$$

where $M = \exp(At)$ and

$$D_{kl}(s-s') = \frac{\langle n_k(\infty)n_l(\infty) + n_l(\infty)n_k(\infty)\rangle}{2} = D\delta(s-s')$$

being diffusion matrix with

$$D = \mathrm{diag}[0, \gamma_{\mathrm{m}}(2n_{\mathrm{th}}+1), \kappa, \kappa].$$

Therefore, $V = \int_0^\infty \mathrm{d}s M(s) D M(s)^{\mathrm{T}}$, which is equivalent to the following Lyapunov equation:[27]

$$AV + VA^{\mathrm{T}} = -D, \tag{17}$$

when the stability conditions satisfy $M(\infty) = 0$.

The correlation matrix V can be expressed as

$$V = \begin{pmatrix} V_A & V_C \\ V_C^{\mathrm{T}} & V_B \end{pmatrix}, \tag{18}$$

where V_A, V_B, and V_C are 2×2 matrices, with V_A associated to the mechanical resonator, V_B to the cavity mode, and V_C describing the optomechanical correlations. A convenient measure for CV entanglement is the logarithmic negativity,[28] given by

$$E_N = \max(0, -2\ln 2\eta^-), \tag{19}$$

where η^- is the symplectic eigenvalue of the bipartite system, defined as $\eta^- = (\Sigma(V)^2 - \sqrt{\Sigma(V) - 4\det V})^{1/2}/\sqrt{2}$, with $\Sigma(V) = \det(V_A) + \det(V_B) - 2\det(V_C)$.

3.2. Numerical results and discussions

In the following, we study the stationary optomechanical entanglement numerically. The stationary entanglement of the optomechanical system is well-defined in the regime where the system is in a single stable state. The steady state is reached when the system is stable, which occurs if and only if all the eigenvalues of A have a negative real part. These stability conditions can be obtained by using the Routh–Hurwitz criteria.[29] Its explicit expression is cumbersome and will not be presented here. However, it is easy to check numerically.

The parameters chosen to obtain the numerical results are similar to those used in recent experiments. For example, the parameter set of mechanical resonator is $\omega_{\mathrm{m}} = 2\pi \times 20$ MHz, $Q_{\mathrm{m}} = 2.0$ MHz. The coupling between photon and mirror, $G_0 = 1.0$ kHZ, which has achieved in the Fabry–Pérot cavity with length $L = 1$ mm and an mirror of effective mass $m \simeq 10$ ng.[30]

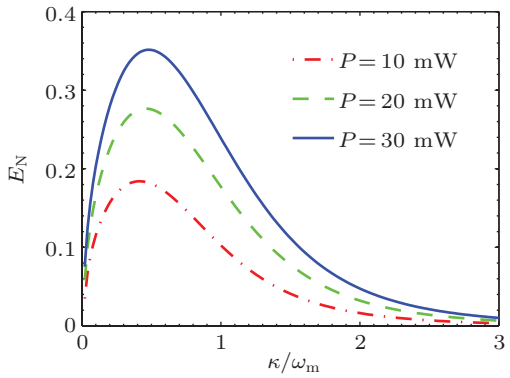


Fig. 2. (color online) The logarithmic negativity with respect to the cavity decay κ in unit of ω_{m}. Other parameters are $\Delta = \omega_{\mathrm{m}}$ and $T = 0.4$ K. In this figure, Kerr interaction is turned off $\chi = 0$.

Let us begin with the case of Kerr interaction $\chi = 0$. It is easy to argue that with the increase of the input laser power, the effectively optomechanical coupling G is enhanced, because the photon number in the cavity is increased. Then it is possible to obtain the bigger optomechanical entanglement. The dependence of entanglement on laser power P is presented in Fig. 2. It clearly displays the upper picture is plausible. The biggest entanglement is obtained near $\kappa/\omega_{\mathrm{m}} \simeq 0.5$. This is consistent with the previous result.[30] This figure also shows that with the sufficiently large cavity decay κ, the entanglement disappears completely. Actually, according to Eq. (7), the phonon number in the cavity is very small with a large κ, which may make the entanglement approach zero by weakening the effective coupling between the photon and phonon modes.

The effect of Kerr interaction on the optomechanical entanglement is shown in Fig. 3. Generally speaking, the optomechanical entanglement is suppressed by the large Kerr parameter χ. Specifically, both the largest value of entanglement and the regime having entanglement are decreased. These results can be understood in the following. The Kerr interaction of photon can induce the photon blockade. This block-

ade greatly lowers the number of the cavity photon. Then the effective coupling between the light and mirror is also weakened, which may spoil the optomechanical entanglement.

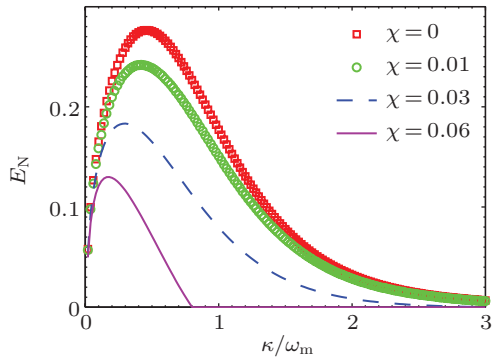


Fig. 3. (color online) The logarithmic negativity with respect to the cavity decay κ in unit of ω_m. Other parameters are $\Delta=\omega_m$, $P=20.0$ mW, and $T=0.4$ K.

Figure 4 plots the dependence of the entanglement of the optomechanical system on the effective detuning Δ of cavity. We find that there is no stable steady state when Δ is near the region $(0.2,0.7)$ ω_m when $\chi=0$. Hence, the optomechanical entanglement must be zero in this regime. However, with the increase of Kerr interaction, the optomechanical entanglement is created in the upper unstable regime. This result can be understood from Eq. (14). The Kerr interaction strengthens the correlation between δX and δY. This enhanced light correlation provides the possibility to create the entanglement of the optomechanical system. The entanglement should decrease if the Kerr interaction increases, as the photon blockade mechanism plays a role.

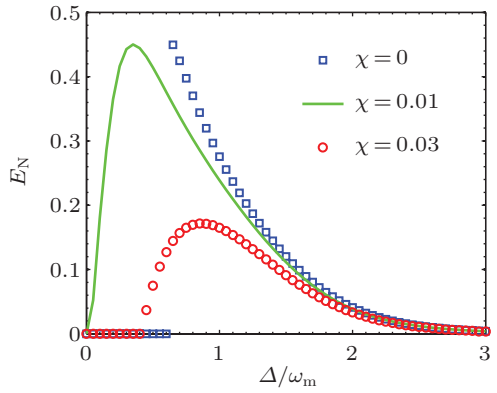


Fig. 4. (color online) The logarithmic negativity with respect to the effective cavity detuning Δ in unit of ω_m. Other parameters are $\kappa=0.5$ ω_m, $P=20.0$ mW, and $T=0.4$ K. For the case $\chi=0$, there is no stable steady state near the region $\Delta\in(0.2,0.7)$ ω_m.

For clearly presenting the effect of Kerr interaction on the optomechanical entanglement, we plot E_N with respect to χ in Fig. 5. This figure shows E_N has two distinct behaviors with small and large cavity detuning Δ. In the small detuning regime, E_N has the resonance-like behavior with respect to χ. While with the large detuning, it is monotonous to χ. The upper results can be simply checked by frequency-locking mechanism using Eqs. (12) and (14). Setting frequency-locking between photon and mechanical resonator, one obtains

$$\chi_0=\frac{n\omega_m-\Delta}{2|\alpha_s|^2}$$

with n being integer and $|\alpha_s|^2\approx\varepsilon_p^2/\omega_m^2$. With $n=1$ (it is reasonable that the maximum entanglement is obtained in the fundamental frequency-locking) and the parameters in Fig. 5, one gets $\chi_0\approx0.11$ in small detunging regime. It is very close to the numerical value of Kerr parameter corresponding to the maximum entanglement in Fig. 5. However, the large Δ may make the frequency-locking between photon and mechanical resonator more easily, even without an additional frequency $\omega_\chi\propto\chi$ produced by Kerr interaction. Therefore, in this case, the maximum entanglement appears in $\chi=0$.

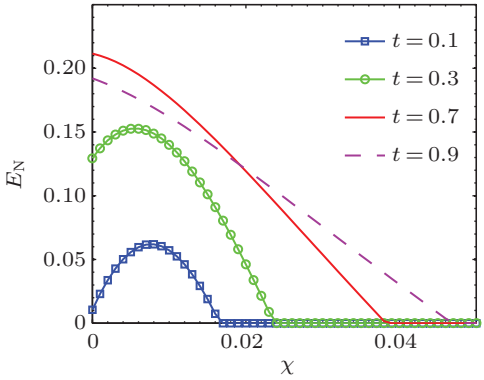


Fig. 5. (color online) The logarithmic negativity with respect to the Kerr parameter χ. $t\equiv\Delta/\omega_m$. Other parameters are $\kappa=\omega_m$, $P=20$ mW, and $T=0.4$ K.

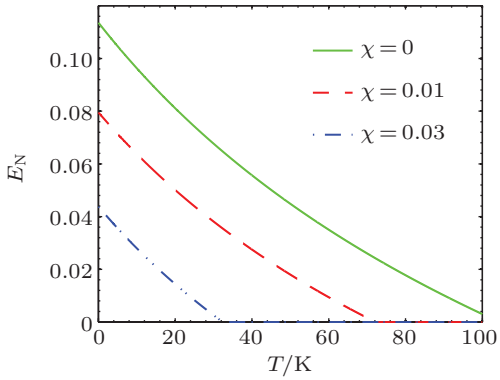


Fig. 6. (color online) The logarithmic negativity with respect to the temperature T (in unit of K). Other parameters are $\Delta=0.5$ ω_m, $\kappa=\omega_m$, and $P=10.0$ mW.

The robustness of the optomechanical entanglement with respect to the temperature T is shown in Fig. 6. Without the Kerr interaction, this entanglement is very robust with respect to T. With the increase of χ, the value and the critical value of temperature T_c decrease (T_c is defined as $T>T_c$, $E_N=0$).

Therefore, the robustness of entanglement is anti-correlation with Kerr parameter χ. This fact can also be understood similarly by photon blockage.

From the experimental point view, the giant optical Kerr nonlinearity can be obtained by placing a $\chi^{(3)}$ medium inside the cavity.[25] Another way is making use of a hybrid system consisting of a semiconductor quantum dot and the laser driven four-level atoms within a single mode cavity. Integrating out the degrees of freedom of atoms in some parameter regimes, the remaining photon field will possess Kerr interaction.[31,32] For the detection of the entanglement, one has to measure several quadrature correlations.[33] In our case, we can use the second Fabry–Pérot cavity adjacent to the first one, similar to the scheme proposed in Ref. [14]. The position and momentum of the mechanical mirror are directly measured by homodyning the output of the second cavity, whereas the quadratures are directly measured by homodyning output of the cavity. More technical details are shown in Ref. [34].

4. Conclusion

In summary, we have studied the effect of a Kerr medium on the stationary entanglement of the optomechanical system. The optomechanical entanglement is spoiled by Kerr interaction, which is due to weaker effective coupling between the photon and phonon modes induced by the photon blockade mechanism. More interestingly, Kerr interaction can create optomechanical entanglement, in contrast to the case without Kerr interaction, the optomechanics is just in the unstable state. Kerr interaction can also induce the resonance-like phenomenon with the small cavity detuning. All the results demonstrate that the Kerr interaction is a new handle to coherently control the dynamics of optomechanical system. Hence, it could be useful in the realization of tuneable quantum-mechanical devices in the future.

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