Atom dynamical dissipation in the laser cavity with quasimode*

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In this paper we study the atom dissipation effect in a laser cavity. The cavity field mode is described by the Fox–Li quasimode due to the leakiness of the cavity. Our results show that the atom decay rate versus the decay rate of the quasimode is a Lorentz type. Effects of the atom–cavity detuning as well as cavity size are also discussed.

Keywords: quasimode, dissipative cavity, decay rate

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1. Introduction

The electromagnetic field spectrum in free space is continuous. However, there is no continuous spectrum when the cavity wall appears. Special physical effects of the field as well as the atom in the cavity occur, such as the Casimir effect[1–3] and Purcell effect.[4] Actually, quantum systems are never perfectly isolated from the environments, and in some cases they must be treated as open systems.[5–9] For example, we consider the electromagnetic field in the optical cavity; when the cavity wall is dissipative, there is energy exchange between the field and the environment. In such a case, the normal field modes do not exist in the cavity any longer. Therefore, the quasimode is usually introduced to describe the electromagnetic field. Especially, Fox and Li used a kind of quasimode to describe the field mode in the dissipative cavity with least diffraction dissipation. In our early work,[10] we applied the Fox–Li quasimode theory to study the Casimir–Polder force between the two-level system and the cavity wall.

In this paper, we focus on the atomic decay[4,11,12] in such a dissipative laser cavity. In our consideration, the two-level system is located in the optical cavity, and the multimode electromagnetic field in the cavity serves as the environment. We calculate the "population inversion" to discuss the problem. The population inversion and the relative physical phenomenon is widely applied in laser science.[13–15] Actually, the same problem is discussed by others applying the master equation and other approaches. Here, we utilize the Fox–Li quasimode theory to describe the field in the laser cavity.

The organization of the paper is as follows. In Section 2, we establish our model in a dissipative laser cavity and describe the cavity field by Fox–Li quasimode. In Section 3, we calculate the atom dynamical population inversion by making use of the Heisenburg equation. In Section 4, we show our results and give some discussions. We plot the atom decay rate as a function of the decay rate at different detunings. In Section 5, we draw our conclusion.

2. Model and Hamiltonian

The system in our consideration is schematically illustrated in Fig. 1; a two-level atom is put into a laser cavity[16] with a semitransparent wall which is imbedded in a large ideal cavity. The mirrors at $x = 0$ and $x = L$ are completely reflective, while the one at $x = l$ represents a semitransparent one with large dielectric constant. As an idealization of such a mirror, we choose the dielectric constant around $x = l$ to be

$$\varepsilon(x) = \varepsilon_0[1 + \eta\delta(x-l)], \tag{1}$$

where η is a parameter with the dimension of length which determines the transparency of this plate. The electromagnetic field in the entire cavity including the laser cavity and the environment is governed by the Maxwell wave equation

$$\frac{\partial^2 E}{\partial x^2} - \mu_0\varepsilon_0[1 + \eta\delta(x-l)]\frac{\partial^2 E}{\partial t^2} = 0. \tag{2}$$

We may separate the variables by writing E as

$$E = U(x)\,\mathrm{e}^{-\mathrm{i}\Omega t}. \tag{3}$$

The normal mode of this system can be obtained by solving Maxwell's equations with the proper boundary conditions, the eigenmodes of the entire cavity are

$$U(x) = \begin{cases} M\sin(kx), & 0 \le x \le l, \\ \sin k(l-x), & l \le x \le L. \end{cases} \tag{4}$$

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Thus we obtain

$$M = \frac{\Gamma\Lambda}{[(\omega_j - \Omega)^2 + \Gamma^2]^{1/2}}, \tag{5}$$

where Ω is the frequency of Fox–Li quasimode,[16] ω_j is the universe frequency. $\Gamma = c/\Lambda^2 l$, $\Lambda = \eta/l\tilde{x}$, $\tilde{x}$ is the solution to the equation $\tan(\tilde{x}) = l/\eta\tilde{x}$.

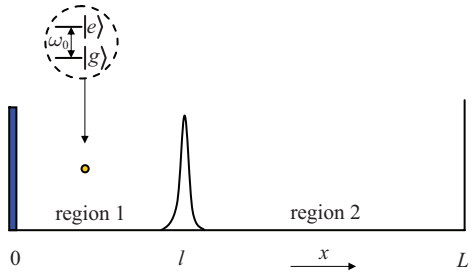


Fig. 1. (color online) Schematic illustration of a setup in which a two-level atom is located inside a laser cavity bounded by a perfect mirror at $x = 0$ and a semitransparent mirror at $x = l$ (region 1). The laser cavity is imbedded in a larger ideal cavity. The larger cavity (region 2) serves as the environment which interacts with the laser cavity and is bounded by a perfect mirror at $x = L$.

In this paper we restrict our investigation to only one quasimode cavity. Here we also assume that different cavity modes are decoupled. Under the electric-dipole approximation, our system is described by the Hamiltonian

$$H = \hbar\omega_0 S_z + \sum_j \hbar\omega_j a_j^\dagger a_j + \sum_j g_j(a_j S_+ - a_j^\dagger S_-), \tag{6}$$

$$g_j = -\mathrm{i}\sqrt{\frac{2\pi\hbar\omega_j}{L}}[\mu \cdot f_j(x)], \tag{7}$$

where ω_0 is the transition frequency of the atom. ω_j are frequencies of the universe modes corresponding to Ω. Operators $a_j^\dagger$ and a_j are creation and annihilation operators of the field modes. S_z, S_+, and S_- are the pseudospin operators of the two-level atom, and μ is its electric-dipole moment. The Casimir–Polder force and relative topics have been discussed under the same model without rotating wave approximation.[10,17] We also assume that the directions of the dipole moment for atom and wave vectors of the cavity fields are perpendicular to the surface of the quantum well. Here $f_j(x)$ is the mode function between $x = 0$ and $x = l$

$$f_j(x) = \frac{\Gamma\Lambda}{[(\omega_j - \Omega)^2 + \Gamma^2]^{1/2}} \sin(k_j x), \tag{8}$$

where Γ is the decay rate of the cavity quasimodes. For those normal modes with frequency ω_j close to a resonant frequency Ω.

3. The atom dynamical population inversion

We first solve the motion of the operators under the so-called Born–Markov approximation. According to Hamiltonian Eq. (6), we have

$$\frac{\partial S_z}{\partial t} = -\frac{\mathrm{i}}{\hbar}[S_z, H] = -\frac{\mathrm{i}}{\hbar}\sum_j g_j(a_j S_+ - a_j^\dagger S_-). \tag{9}$$

Thus, we need to investigate the time-development of the operators on the right-hand side of Eq. (9), such as

$$\begin{aligned}\frac{\partial(a_j S_+)}{\partial t} &= -\frac{\mathrm{i}}{\hbar}[a_j S_+, H] \\ &= -\frac{\mathrm{i}}{\hbar}\left\{\hbar(\omega_j - \omega_0)a_j S_+ - g_j\left(S_z + \frac{1}{2}\right)\right. \\ &\quad \left. -2\sum_{j'} g_{j'} a_{j'}^\dagger a_{j'} S_z\right\}.\end{aligned} \tag{10}$$

The quantum average of the first term on the vacuum state is zero and we neglect all the terms with two field operators which contribute $o(g_j^2)$ to Eq. (10); this cutoff to the second order of g_j is the Born approximation. We thus obtain the result as follows:

$$\begin{aligned}&\frac{\partial(a_j S_+)}{\partial t} \\ &= -\frac{\mathrm{i}}{\hbar}[a_j S_+, H] \\ &= -\frac{\mathrm{i}}{\hbar}\left\{\hbar(\omega_j - \omega_0)a_j S_+ + g_j\left(S_z + \frac{1}{2}\right)\right\},\end{aligned} \tag{11}$$

Following the same way, we have

$$\frac{\partial(a_j^\dagger S_-)}{\partial t} = \frac{\mathrm{i}}{\hbar}\left\{\hbar(\omega_j - \omega_0)a_j^\dagger S_- + g_j\left(S_z + \frac{1}{2}\right)\right\}. \tag{12}$$

Solving Eqs. (11) and (12) formally, we can obtain

$$\begin{aligned}(a_j S_+)_t &= (a_j S_+)_0 \mathrm{e}^{-\mathrm{i}(\omega_j - \omega_0)t} - \frac{\mathrm{i}}{\hbar} g_j \mathrm{e}^{-\mathrm{i}(\omega_j - \omega_0)t} \\ &\quad \times \int_0^t \mathrm{e}^{\mathrm{i}(\omega_j - \omega_0)t'}\left[S_z(t') + \frac{1}{2}\right]\mathrm{d}t',\end{aligned} \tag{13}$$

$$\begin{aligned}(a_j^\dagger S_-)_t &= (a_j^\dagger S_-)_0 \mathrm{e}^{\mathrm{i}(\omega_j - \omega_0)t} + \frac{\mathrm{i}}{\hbar} g_j \mathrm{e}^{\mathrm{i}(\omega_j - \omega_0)t} \\ &\quad \times \int_0^t \mathrm{e}^{-\mathrm{i}(\omega_j + \omega_0)t'}\left[S_z(t') + \frac{1}{2}\right]\mathrm{d}t'.\end{aligned} \tag{14}$$

Substituting Eqs. (13) and (14) into Eq. (9) and taking the quantum average, we obtain

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\langle S_z(t)\rangle \\ &= -\frac{2}{\hbar^2}\sum_j |g_j|^2 \int_0^{t\to\infty} \cos(\omega_j - \omega_0)(t - \tau) \\ &\quad \times \left(\langle S_z(\tau)\rangle + \frac{1}{2}\right)\mathrm{d}\tau.\end{aligned} \tag{15}$$

Assuming that the decay time of the atom is much larger than ω_0^{-1}, we can replace $S_z(\tau)$ by $S_z(t)$ and the upper limit of the integral can be replaced by ∞, thus, the integration in Eqs. (13) and (14) can be calculated exactly. The above process is the Markov approximation. Under the Born–Markov approximation, we can express Eq. (15) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle S_z(t)\rangle$$

$$= -\frac{2}{\hbar^2}(\langle S_z(t)\rangle + \frac{1}{2})\sum_j |g_j|^2 \times \int_0^{t\to\infty} \cos(\omega_j - \omega_0)(t-\tau)\,\mathrm{d}\tau. \quad (16)$$

By using of the Heitler's definition

$$\pi\delta(x) = \lim_{x\to 0}\int_0^{\tau\to\infty} \cos(\tau x)\,\mathrm{d}\tau \quad (17)$$

and transforming the summation over ω_j into integration, that is $\sum_j \to (L/c\pi)\int \mathrm{d}\,\omega_j$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle S_z(t)\rangle = -\frac{4\mu^2\omega_0\Gamma}{\hbar cl[(\omega_0-\Omega)^2+\Gamma^2]}\left(\langle S_z(t)\rangle + \frac{1}{2}\right) \times \sin^2\left(\frac{\omega_0 x}{c}\right). \quad (18)$$

Integrating on both sides of Eq. (18), we have

$$\langle S_z(t)\rangle = -\frac{1}{2} + \left(\langle S_z(0)\rangle + \frac{1}{2}\right)\mathrm{e}^{-\gamma t}, \quad (19)$$

where γ is the atom decay rate

$$\gamma = \frac{4\mu^2\omega_0\Gamma}{\hbar cl[(\omega_0-\Omega)^2+\Gamma^2]}\sin^2\left(\frac{\omega_0 x}{c}\right). \quad (20)$$

From Eq. (19), if the atom is in the ground state $|\downarrow\rangle$ initially, $\langle S_z(0)\rangle = -1/2$, $\langle S_z(t)\rangle = -1/2$, it will stay in the ground state. On the contrary, if the atom is prepared in the excited state $|\uparrow\rangle$, $\langle S_z(t)\rangle = -1/2 + \mathrm{e}^{-\gamma t}$.

4. Discussions about the atom decay rate

According to Eq. (20), the atom decay rate versus the quasimode decay rate is the Lorentz type. In particular, when the atom resonates with the Fox–Li quasimode, the atom decay rate is inversely proportional to Γ. As shown in Fig. 2, we plot the atom decay rate at different detunings between Ω and ω_0. The green (solid) line corresponds to resonating case $\omega_0 = \Omega$, the red (dashed) line corresponds to detuning $\Omega - \omega_0 = 10^{14}$, the orange (dashed–dotted) line corresponds to detuning $\Omega - \omega_0 = 2\times 10^{14}$. We find that the larger the detuning is, the smaller the atom decay rate is. When the atom transition frequency ω_0 resonates with Ω, the atom decay rate γ takes maximum value at the same Γ. We find that the atom decay rate is largest when the resonance happens and the three curves coincide with each other as Γ is large enough. Our results show that in the dissipative laser cavity, the atom decay rate is depressed due to the energy exchange between the cavity and outside. The results are equivalent to the fact that: when the two-level system interacts with the multi-modes environment, the atom energy level is broadened and has a finite lifetime. On the other hand, as shown in Eq. (20), the size of the cavity can also affect the atom decay rate, which is inversely proportional to the cavity length. Besides, the decay rate γ is atom-position-dependent, which implies that the decay rate can disappear for some x, that is $x = nc/\omega_0$ with $n = 1, 2, \ldots$.

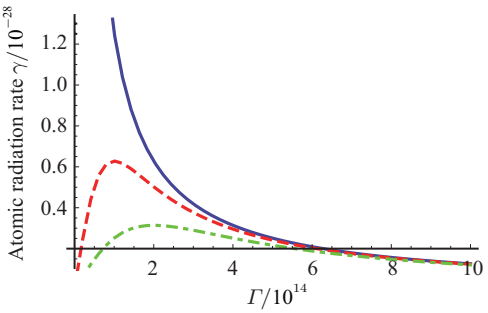


Fig. 2. (color online) The atom decay rate γ as a function of the field mode decay rate for different cases. The parameters are set as $\mu = 8.5\times 10^{-30}$, $\omega_0 = 10^{14}$, $l = 10^{-6}$, and $x = 0.5\times 10^{-6}$. Blue (solid) line, red (dashed) line, and green (dashed–dotted) line represent the resonating cases $\Omega = \omega_0$, $\Omega - \omega_0 = 10^{14}$, and $\Omega - \omega_0 = 2\times 10^{14}$ respectively.

5. Conclusion

In this paper we have explored the atom decay rate in a dissipative laser cavity with the Fox–Li quasimode approach. The time evolutions of the operators are obtained by solving the Heisenberg equation iteratively up to the second order and the atom dynamical decay rate is obtained. We found that when the detuning between the atom and the field mode is different the atom decay rates are different. The larger the detuning is, the smaller the atom decay is. Our result also shows that the size of the cavity can affect the atom decay rate. As the cavity length increases, the atom decay rate decreases.

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