Totally asymmetric exclusion processes at constrained m-input n-output junction points*

Li Shao-Da(李少达)[a)b)], Liu Ming-Zhe(刘明哲)[a)†], and Pei Xiang-Jun(裴向军)[a)]

[a)] State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, Chengdu 610059, China
[b)] College of Earth Sciences, Chengdu University of Technology, Chengdu 610059, China

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Totally asymmetric exclusion processes at constrained m-input n-output junction points under random update are studied by theoretical calculation and computer simulation in this paper. At the junction points, the hopping rate of particles from m-input parallel lattices to n-output parallel lattices is assumed to be equal to r/n $(0 < r < 1)$. The mean-field approach and Monte Carlo simulations show that the phase diagram can be classified into three regions at any value of r. More interestingly, there is a threshold $r_c = n(1-\sqrt{1-m/n})/m$. In the cases of $r > r_c$ and $r < r_c$, qualitatively different phases exist in the system. With the increase of the value of m/n, the regions of (LD, LD) and (MC, LD) or (HD, LD) decrease, and the (HD, HD) is the only phase that increases in the region (LD stands for low density, HD stands for high density, and MC for maximal current). Stationary current and density profiles are calculated, showing that they are in good agreement with Monte Carlo simulations.

Keywords: exclusion process, mean-field approximation, phase diagram, Monte Carlo simulations

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1. Introduction

In the last few decades, traffic flow theory has been proved to be a rewarding research topic[1,2] since traffic is closely associated with our daily life and can be observed at almost all levels of nature (e.g., from microscopic molecular motors to macroscopic objects like vehicles). Theoretical investigation of traffic flow is normally based on totally asymmetric simple exclusion processes (TASEPs).[3–7] To describe traffic more realistically, it is necessary to study traffic flow on more complex geometries. Two-channel TASEPs with different lane-changing rules between the two channels have been studied, see Refs. [8]–[12] for example. More recently, multiple-channel TASEPs have been investigated.[13–16] These efforts have greatly deepened our understanding of particle traffic on multiple-channel systems.

Junctions may be seen as a geometry extension of the multiple channels connected by junction points. At junction points, cross traffic becomes complex. In this sense, junction points may cause bottlenecks, which normally leads to traffic congestion. In biological physics, the crowding of molecular motors around junctions has been thought of as a source of some human diseases such as neurodegenerative diseases, kidney diseases, and other human diseases.[17,18]

In this paper, the TASEP with a junction connected by m-input parallel channels and n-output parallel channels is studied using a mean-field approach and Monte Carlo simulations ($m < n$ for the case). At the junction, the particles of m-input parallel channels move to one of the n-output parallel channels with equal rate r/n. Here $r < 1$ is a kind of local inhomogeneity,[19,20] which might be related to the cases (i) that drivers need to slow down at the divergence to consider which route to take; (ii) that additional frictions exerted on mobile objects force them to slow down.[21] In the natural and man-made traffic systems, m-input n-output junctions are the commonly used traffic facilities. Therefore, it is necessary to extend the work in Ref. [21] to a general case in order to better understand traffic dynamics and driver behavior at junctions. For simplicity, random sequential update rules are chosen in the extended model. It is found that the phase diagram can be classified into three regions at any value of r. More interestingly, there is a threshold $r_c = n(1-\sqrt{1-m/n})/m$ when $m < n$. In the cases of $r > r_c$ and $r < r_c$, qualitatively different phases can be exhibited in the phase diagram and with the increase of parameter m/n, the regions of (LD, LD) and (MC, LD) or (HD, LD) decrease, and the (HD, HD) is the only phase that increases in the region increases. Here LD stands for low density, HD for high density, and MC for maximal current.

The rest of the present paper is organized as follows. The model and theoretical analysis are conducted in Section 2. In Section 3, the results of theoretical calculation and Monte Carlo simulations are discussed. Finally, some conclusions are given in Section 4.

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†Corresponding author. E-mail: liumz@cdut.edu.cn

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2. Model and theoretical analysis

2.1. Model

An m-input n-output (MINO) lattice system with a junction is illustrated in Fig. 1. The system can be seen as two parts: the left and right subsystems. The former includes m one-dimensional single-channel TASEPs, while the latter consists of n one-dimensional single-channel TASEPs. In a discrete time step, particles are assumed to move from the left to the right. There are L sites in each channel. The behavior of the system can be described by applying the following rules (see Fig. 2). At site 1 (the total number of site 1s is m), if the site is empty, a particle can enter into site 1 with rate α; a particle can leave the left subsystem from site L with rate β_{eff} and enter into the right subsystem from site $L+1$ with rate α_{eff}. For simplicity, the number of particles at site L is denoted as K_L and the number of empty site $L+1$ is represented as N_{L+1} ($N_{L+1} \geq 1$). A particle at site L can hop to site $L+1$ with rate r/N_{L+1} in one time step. Particles finally leave the system from each site $2L$ with rate β. This paper deals with the case of $m < n$.

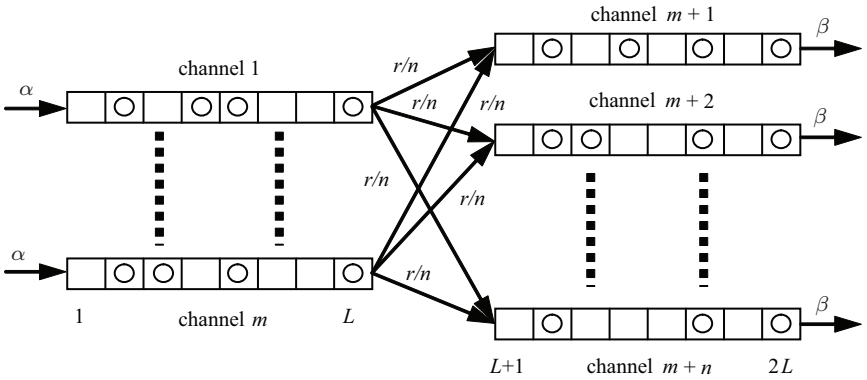


Fig. 1. Schematic diagram of an m-input n-output (MINO) lattice system with a junction. Particles move from the left to the right with hard-core exclusion. There are m inputs and n outputs in the system.

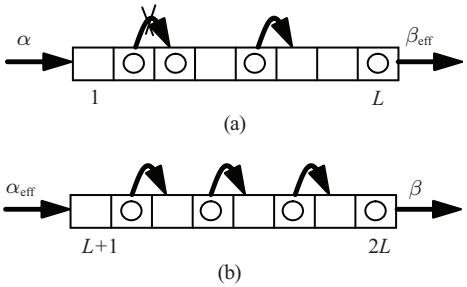


Fig. 2. (a) Entrance probability at site 1 and exit probability at site L are denoted as α and β_{eff}, respectively. (b) Entrance probability at site $L+1$ and exit probability at site $2L$ are denoted as α_{eff} and β, respectively. Solid arrows indicate the allowed hopping.

2.2. Theoretical analysis

In this section, the approximate stationary solutions of the model are presented by using a mean-field approach. The exact results of TASEP on a single lattice with open boundaries and random update is briefly reviewed at first. In this case there are three stationary phases specified by the dynamics at the entrance rate α, at the exit rate β and in the bulk of the system.

For $\alpha < \beta$ and $\alpha < 1/2$, the system is in an LD phase with

$$J = \alpha(1-\alpha), \quad \rho_{\text{bulk}} = \alpha, \quad \rho_1 = \alpha, \quad \rho_N = \frac{\alpha(1-\alpha)}{\beta}, \quad (1)$$

where ρ_1, ρ_N, and ρ_{bulk} are the densities at the entrance, exit, and the bulk of the lattice far from the boundaries, respectively. J denotes the system flux.

For $\beta < \alpha$ and $\beta < 1/2$, the system is in a high-density (HD) phase with

$$J = \beta(1-\beta), \quad \rho_{\text{bulk}} = 1-\beta, \quad \rho_1 = 1 - \frac{\beta(1-\beta)}{\alpha}, \quad \rho_N = 1-\beta. \quad (2)$$

For $\alpha > 1/2$ and $\beta > 1/2$, the system is in a maximal-current (MC) phase with

$$J = \frac{1}{4}, \quad \rho_{\text{bulk}} = \frac{1}{2}, \quad \rho_1 = 1 - \frac{1}{4\alpha}, \quad \rho_N = \frac{1}{4\beta}. \quad (3)$$

As shown in Fig. 2, the system can be viewed as two subsystems (i.e., the left subsystem and the right subsystem) and there are m (n) single-channel TASEPs in the left (right) subsystem. Since the total current in the steady state is conserved through the system, we have

$$\begin{aligned} &J_{\text{overall}} = J_1 + J_2 + \cdots + J_m = J_{m+1} + J_{m+2} + \cdots J_{m+n}, \\ &J_1 = J_2 = \cdots = J_m, \quad J_{m+1} = J_{m+2} = \cdots J_{m+n}, \\ &J_{\text{overall}} = mJ_1 = nJ_{m+1}, \end{aligned} \quad (4)$$

where J_i ($i = 1, 2, \ldots, m, m+1, \ldots, m+n$) is the current on the i-th channel, and J_{overall} is the total current of the whole system.

Due to the conservation of the current of the system, the MC phase cannot exist in the right subsystem according to

Eq. (4) since in this paper just the case of $m < n$ is considered. Consequently, there are six possible stationary phases: (LD, LD), (HD, LD), (MC, LD), (LD, HD), (HD, HD), and (MC, HD).

In a similar way to that in Refs. [8], [13], and [21], we adopt effective injection and removal rates for the left (right) subsystem and investigate the possible phase structures of the system in terms of these rates. Thus, the overall current of the system can be calculated by

$$J_{\text{overall}} = \begin{cases} mJ_1 = m\beta_{\text{eff}}\rho_L, \\ nJ_{m+1} = n\alpha_{\text{eff}}(1-\rho_{L+1}). \end{cases} \tag{5}$$

On the other hand, the mean-field expression for the current can be written as

$$J_{\text{overall}} = r\rho_L(1-\rho_{L+1}). \tag{6}$$

Thus, from Eqs. (5) and (6), one can obtain

$$\alpha_{\text{eff}} = \frac{r\rho_L}{n}, \quad \beta_{\text{eff}} = \frac{r(1-\rho_{L+1})}{m}. \tag{7}$$

Now the existence of different stationary phases can be studied. First, we consider the (LD, LD) phase. It is fulfilled when

$$\alpha < \frac{1}{2}, \quad \alpha < \beta_{\text{eff}}; \quad \alpha_{\text{eff}} < \frac{1}{2}, \quad \alpha_{\text{eff}} < \beta. \tag{8}$$

From Eq. (4), we obtain

$$\alpha_{\text{eff}} = \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2} < \frac{1}{2}. \tag{9}$$

According to Eq. (1), the density on the entrance $L+1$ equals the bulk density of right subsystem, so that

$$\rho_{L+1} = \rho_{\text{right,bulk}} = \alpha_{\text{eff}}. \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (7), we have

$$\beta_{\text{eff}} = \frac{r\left(1+\sqrt{1-(4m/n)\alpha(1-\alpha)}\right)}{2m}.$$

Thus, from $\alpha < \beta_{\text{eff}}$ we obtain

$$r > \frac{n\left(1-\sqrt{1-(4m/n)\alpha(1-\alpha)}\right)}{2(1-\alpha)}. \tag{11}$$

Therefore, for the (LD, LD) phase to exist, one has

$$\alpha < \frac{1}{2}, \quad \beta > \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2}, \quad r > \frac{n\left(1-\sqrt{1-(4m/n)\alpha(1-\alpha)}\right)}{2(1-\alpha)}. \tag{12}$$

Now we discuss the (LD, HD) phase, which exists if

$$\alpha < \frac{1}{2}, \quad \alpha < \beta_{\text{eff}}; \quad \beta < \frac{1}{2}, \quad \beta < \alpha_{\text{eff}}. \tag{13}$$

From Eqs. (4), (5), (7), and (13), we can obtain

$$\beta = \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2}, \quad \alpha < \beta_{\text{eff}} = \frac{r(1-\rho_{L+1})}{m}, \quad \beta < \alpha_{\text{eff}} = \frac{\beta(1-\beta)}{1-\rho_{L+1}}. \tag{14}$$

Then, we have

$$\frac{m\alpha}{r} < 1-\rho_{L+1} < 1-\beta, \text{ i.e.} \quad r > \frac{m\alpha}{1-\beta} = \frac{n\left(1-\sqrt{1-(4m/n)\alpha(1-\alpha)}\right)}{2(1-\alpha)}. \tag{15}$$

Thus, the possible conditions for the existence of the (LD, HD) phase are

$$\alpha < \frac{1}{2}, \quad \beta = \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2}, \quad r > \frac{n\left(1-\sqrt{1-(4m/n)\alpha(1-\alpha)}\right)}{2(1-\alpha)}. \tag{16}$$

The (HD, LD) phase is specified by

$$\beta_{\text{eff}} < \frac{1}{2}, \quad \beta_{\text{eff}} < \alpha; \quad \alpha_{\text{eff}} < \frac{1}{2}, \quad \alpha_{\text{eff}} < \beta. \tag{17}$$

The density on the exit site equals the bulk density of the left subsystem in the HD phase while the density on the entrance site equals the bulk density of the right subsystem in the LD phase, so that

$$\rho_L = \rho_{\text{left,bulk}} = 1-\beta_{\text{eff}}, \quad \rho_{L+1} = \rho_{\text{right,bulk}} = \alpha_{\text{eff}}. \tag{18}$$

Substituting Eq. (18) into Eq. (7), we have

$$\alpha_{\text{eff}} = \frac{r(1-\beta_{\text{eff}})}{n}, \quad \beta_{\text{eff}} = \frac{r(1-\alpha_{\text{eff}})}{m}. \tag{19}$$

Therefore,

$$\alpha_{\text{eff}} = \frac{mr-r^2}{mn-r^2}, \quad \beta_{\text{eff}} = \frac{nr-r^2}{mn-r^2}. \tag{20}$$

From Eqs. (17), (18), and (20), we find that the system is in the (HD, LD) phase when

$$\alpha > \frac{nr-r^2}{mn-r^2}, \quad \beta > \frac{mr-r^2}{mn-r^2}, \quad r < n-\sqrt{n(n-m)}. \tag{21}$$

The (HD, HD) phase is defined by the following expression:

$$\beta_{\text{eff}} < \frac{1}{2}, \quad \beta_{\text{eff}} < \alpha; \quad \beta < \frac{1}{2}, \quad \beta < \alpha_{\text{eff}}. \tag{22}$$

Taking into account Eqs. (4) and (7), we obtain

$$\begin{aligned} \beta_{\text{eff}} &= \frac{1-\sqrt{1-(4n/m)\beta(1-\beta)}}{2}, \\ \alpha_{\text{eff}} &= \frac{r\rho_L}{n} = \frac{r(1-\beta_{\text{eff}})}{n} \\ &= \frac{1+\sqrt{1-(4n/m)\beta(1-\beta)}}{2n}. \end{aligned} \tag{23}$$

The expression in the above square root should be positive which leads to

$$\beta < \frac{1-\sqrt{1-m/n}}{2},$$

from Eqs. (22) and (23), the system is in the (HD, HD) phase when

$$\begin{aligned}
&\beta < \frac{1-\sqrt{1-m/n}}{2},\\
&\alpha > \frac{1-\sqrt{1-(4n/m)\beta(1-\beta)}}{2},\\
&r > \frac{m\left(1-\sqrt{1-(4n/m)\beta(1-\beta)}\right)}{2(1-\beta)}.
\end{aligned} \tag{24}$$

The (MC, LD) phase is determined by

$$\alpha > \frac{1}{2},\ \beta_{\rm eff} > \frac{1}{2};\ \alpha_{\rm eff} < \frac{1}{2}, \alpha_{\rm eff} < \beta. \tag{25}$$

From Eqs. (4) and (7), we obtain

$$\alpha_{\rm eff} = \frac{1-\sqrt{1-m/n}}{2} < \frac{1}{2}, \tag{26}$$

$$\beta_{\rm eff} = \frac{r(1-\rho_{L+1})}{m} = \frac{r(1-\alpha_{\rm eff})}{m} = \frac{r\left(1+\sqrt{1-m/n}\right)}{2m}. \tag{27}$$

Combining Eqs. (25) and (26), the parameters for existence of (MC, LD) phase can be written as

$$\beta > \frac{1-\sqrt{1-m/n}}{2},\ \alpha > \frac{1}{2},\ r > \frac{n\left(1-\sqrt{1-m/n}\right)}{m}. \tag{28}$$

Finally, the (MC, HD) phase exists if

$$\alpha > \frac{1}{2},\ \beta_{\rm eff} > \frac{1}{2};\ \beta < \frac{1}{2},\ \beta < \alpha_{\rm eff}. \tag{29}$$

From Eqs. (4), (5), (7), and (29), we find

$$\begin{aligned}
&\beta = \frac{1-\sqrt{1-m/n}}{2} < \frac{1}{2},\\
&\beta_{\rm eff} = \frac{1}{4\rho_L} > \frac{1}{2},\\
&\alpha_{\rm eff} = \frac{r\rho_L}{n} > \beta.
\end{aligned} \tag{30}$$

Then we have

$$\frac{n\left(1-\sqrt{1-m/n}\right)}{2r} < \rho_L < \frac{1}{2},\ \text{i.e.,}\ r > \frac{n\left(1-\sqrt{1-m/n}\right)}{m}. \tag{31}$$

Hence, the possible conditions for the existence of the (MC, HD) phase are

$$\beta = \frac{1-\sqrt{1-m/n}}{2},\ \alpha > \frac{1}{2},\ r > \frac{n\left(1-\sqrt{1-m/n}\right)}{m}. \tag{32}$$

From the above analyses, we obtain a critical value

$$r_{\rm c} = \frac{n\left(1-\sqrt{1-m/n}\right)}{m}.$$

We note again that the critical value $r_{\rm c}$ is meaningful only when $m < n$. When $m > n$, the value in the square root of $r_{\rm c}$ is not positive, which makes $r_{\rm c}$ nonsensical. For $r > r_{\rm c}$, three phases are as follows:

(LD, LD) phase:

$$\alpha < \frac{1}{2},\ \beta > \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2};$$

(HD, HD) phase:

$$\beta < \frac{1-\sqrt{1-m/n}}{2},\ \alpha > \frac{1-\sqrt{1-(4n/m)\beta(1-\beta)}}{2};$$

(MC, LD) phase:

$$\beta > \frac{1-\sqrt{1-m/n}}{2},\ \alpha > \frac{1}{2}.$$

For $r < r_{\rm c}$, three phases are

(LD, LD) phase:

$$\alpha < \frac{nr-r^2}{mn-r^2},\ \beta > \frac{1-\sqrt{1-(4m/n)\alpha(1-\alpha)}}{2};$$

(HD, HD) phase:

$$\beta < \frac{nr-r^2}{mn-r^2},\ \alpha > \frac{1-\sqrt{1-(4n/m)\beta(1-\beta)}}{2};$$

(HD, LD) phase:

$$\alpha > \frac{nr-r^2}{mn-r^2},\ \beta > \frac{mr-r^2}{mn-r^2}.$$

3. Simulation results and discussion

In this section, Monte Carlo simulation results are presented and compared with the analytical results. In the simulation, the system size is set to $L = 1000$. Stationary phase diagrams are shown in Figs. 3 and 4 which contain three phase regions.

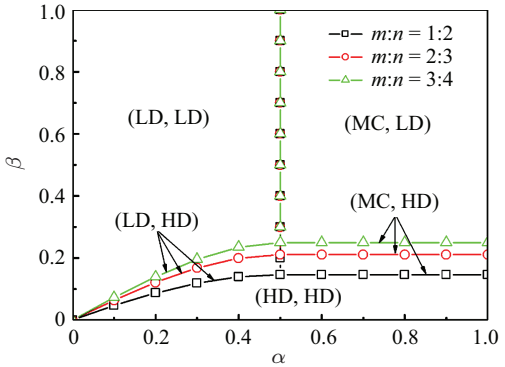


Fig. 3. (color online) Phase diagram of the system, $r > r_{\rm c}$. Symbols represent the Monte Carlo simulation results while lines denote the mean-field calculation results.

In the (LD, LD) phase, all channels are in the LD phase. The typical density profiles with $\alpha = 0.2$, $\beta = 0.8$, and $r = 0.7$ are illustrated in Fig. 5(a). As expected from Eqs. (9), (10), and (12), the analytical results are $\rho_{L,\text{bulk}} = \alpha = 0.2$, $\rho_{R,\text{bulk}} = \alpha_{\text{eff}} = 0.121$. The simulation results are $\rho_{L,\text{bulk}} = 0.203$, $\rho_{R,\text{bulk}} = 0.131$. The comparison of the two results shows that they are in good agreement. The bulk density of each channel is independent of r.

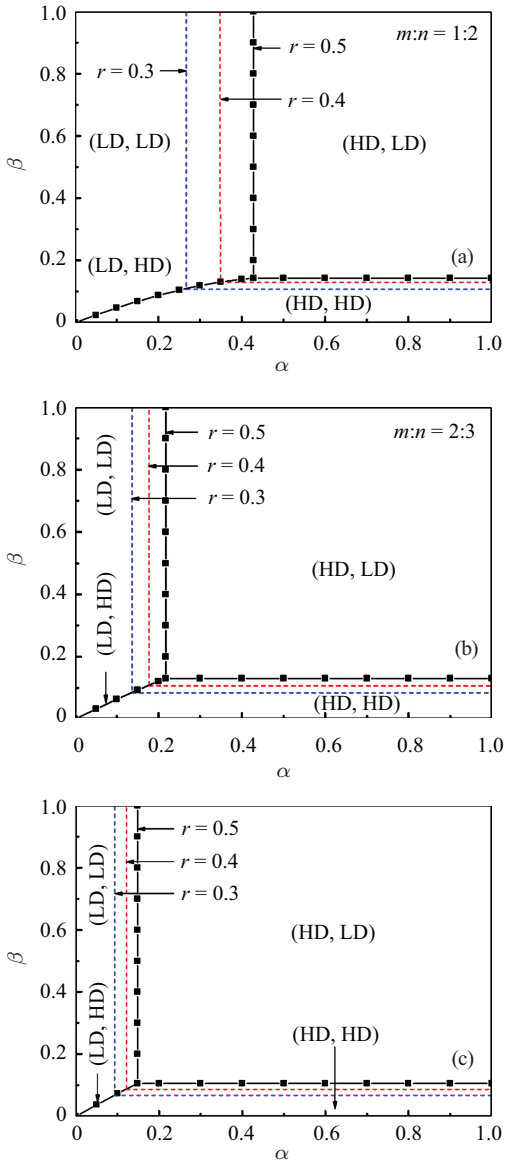


Fig. 4. (color online) Phase diagram of the system with $r = 0:3, 0.4, 0:5 < r_c$, $m : n = 1{:}2$ (a), 2:3 (b), and 3:4 (c). Symbols represent the Monte Carlo simulation results while lines denote the mean-field calculation results.

In the (HD, HD) phase, all channels are in the HD phase. The density profiles with $\alpha = 0.8$, $\beta = 0.05$, and $r = 0.7$ are illustrated in Fig. 5(b). A comparison between the analytical results $\rho_{L,\text{bulk}} = 1 - \beta_{\text{eff}} = 0.923$, $\rho_{R,\text{bulk}} = 1 - \beta = 0.95$, and the simulation results $\rho_{L,\text{bulk}} = 0.926$, $\rho_{R,\text{bulk}} = 0.952$ clearly indicates that the two results are in good agreement.

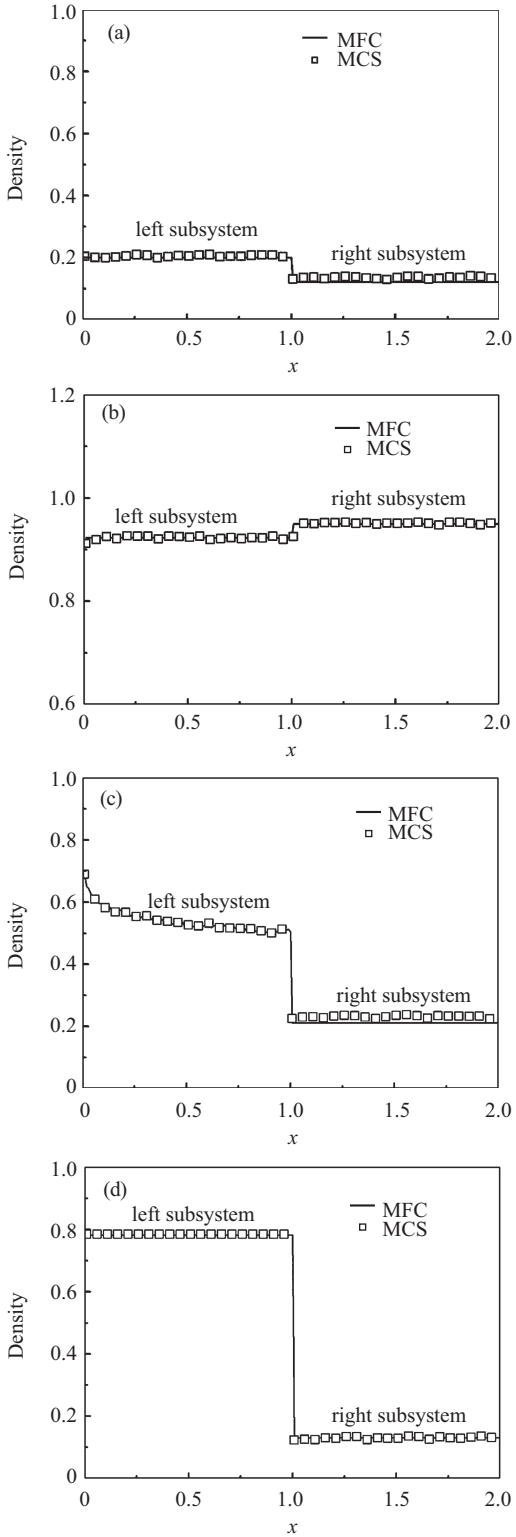


Fig. 5. Density profiles in (a) the (LD, LD) phase; (b) the (HD, HD) phase; (c) the (MC, LD) phase; (d) the (HD, LD) phase. Symbols represent the Monte Carlo simulation results while lines denote our theoretical predictions.

More interestingly, the (MC, LD) phase transferring into the (HD, LD) phase depends on parameter r. According to Eqs. (9), (10), and (12), we can obtain the threshold

$$r_c = \frac{n\left(1 - \sqrt{1 - m/n}\right)}{m} = 0.634.$$

When $r > r_c$, the system is in the (MC, LD) phase. The density profiles corresponding to $\alpha = 0.8$, $\beta = 0.8$, and $r = 0.7$ are shown in Fig. 5(c). The simulation results are in agreement with the analytic ones. Moreover, the region of the (HD, HD) phase expands and the region of the (LD, LD) and (MC, LD) phases shrink with the increase of the parameter m/n, when the parameter r is fixed. When $r < r_c$ the system is in the (HD, LD) phase. The density profiles corresponding to $\alpha = 0.8$, $\beta = 0.8$, and $r = 0.5$ are shown in Fig. 5(d). According to Eqs. (18) and (20), we can obtain the analytic results of density profiles $\rho_{L,\text{bulk}} = 0.783$, $\rho_{R,\text{bulk}} = 0.13$, which accord well with the simulation results $\rho_{L,\text{bulk}} = 0.781$, $\rho_{R,\text{bulk}} = 0.136$. $\rho_{L,\text{bulk}}$ increases but $\rho_{R,\text{bulk}}$ decreases with the decrease of r. Moreover, the region of the (HD, LD) phase expands and the region of the (LD, LD) and (HD, HD) phases shrinks with the decrease of r provided $r < r_c$.

4. Summary and conclusion

TASEPs on MINO lattices with a constrained hopping at the junction are investigated by a mean-field approximation and Monte Carlo simulations. The proposed model may be related to cross traffic of molecular motors in biological transport. It is shown that the phase diagram can be classified into three regions according to value of r (r denotes the constrained hopping rate, $0 < r < 1$). Furthermore, a threshold

$$r_c = \frac{n\left(1 - \sqrt{1 - m/n}\right)}{m}$$

is identified when $m < n$. When $r > r_c$, the three regions correspond to the (LD, LD), (HD, HD), and (MC, LD) phases respectively and the phase boundaries are independent of r. In the case of $r > r_c$, the (MC, LD) phase turns into the (HD, LD) phase. Moreover, the phase boundaries become dependent on r. More interestingly, when $r > r_c$ and r is fixed, with the increase of the parameter m/n, the regions of (LD, LD) and (MC, LD) expand, and the (HD, HD) is the only phase that shrinks in the region. When $r < r_c$, the region of the (HD, LD) phase expands and the regions of the (LD, LD) and (HD, HD) phases shrink with the decrease of r. Steady current and density profiles are calculated, and the results show that they are in good agreement with Monte Carlo simulations. It is also interesting to extend this model by considering different hopping rates at the junction.

References

[1] Chowdhury D, Santen L and Schadschneider A 2000 *Phys. Rep.* **329** 199
[2] Schadschneider A 2006 *Physica A* **372** 142
[3] Shi Q H, Jiang R, Hu M B and Wu Q S 2012 *Phys. Rev. E* **85** 041131
[4] Liu M Z, Li S and Wang R 2012 *Chin. Phys. B* **21** 090510
[5] Song X, Liu M Z and Shang J 2012 *Mod. Phys. Lett. B* **26** 1150036
[6] Song X, Liu M Z, Shang J and Wang H 2012 *Chin. Phys. B* **21** 020514
[7] Evans M R and Sugden K E P 2007 *Physica A* **384** 53
[8] Pronina E and Kolomeisky A B 2004 *J. Phys. A: Math. Gen.* **37** 9907
[9] Pronina E and Kolomeisky A B 2006 *Physica A* **372** 12
[10] Reichenbach T, Frey E and Franosch T 2007 *New J. Phys.* **9** 159
[11] Song X, Liu M Z and Cai J 2009 *Phys. Lett. A* **374** 8
[12] Shi Q H, Jiang R, Hu M B and Wu Q S 2011 *J. Stat. Phys.* **142** 616
[13] Wang R, Liu M Z and Jiang R 2008 *Phys. Rev. E* **77** 051108
[14] Jiang R, Wang R, Hu M B, Jia B and Wu Q S 2007 *J. Phys. A: Math. Theor.* **40** 9213
[15] Jiang R, Hu M B, Jia B, Wang R and Wu Q S 2007 *Phys. Rev. E* **76** 036116
[16] Song X, Cai J, Wang R, Liu M and Liu F 2009 *Chin. Phys. B* **18** 5103
[17] Goldstein L S B 2001 *Proc. Natl. Acad. Sci. USA* **98** 6999
[18] Aridor M and Hannan L A 2000 *Traffic* **1** 836
[19] Shaw L B, Kolomeisky A B and Lee K H 2004 *J. Phys. A: Math. Gen.* **37** 2105
[20] Kolomeisky A B 1998 *J. Phys. A: Math. Gen.* **31** 1153
[21] Wang X, Jiang R, Wang R, Hu M B, Nishinari K and Wu Q S 2009 *Int. J. Mod. Phys. C* **20** 1999