Controlling hyperchaotic complex systems with unknown parameters based on adaptive passive method

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The aim of this paper is to study the control of hyperchaotic complex nonlinear systems with unknown parameters using passive control theory. An approach is stated to design the passive controller and estimate the unknown parameters based on the property of the passive system. The feasibility and effectiveness of the proposed approach is demonstrated through its application to the hyperchaotic complex Lü system, as an example. The estimated values of the unknown parameters are calculated. The analytical form of the complex controller is derived and used in the numerical simulation to control the hyperchaotic attractors of this example. Block diagrams of this example using Matlab/Simulink are constructed after and before the control to ensure the validity of the analytical results. Other examples of hyperchaotic complex nonlinear systems can be similarly treated.

Keywords: control, passive control, hyperchaotic, complex

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1. Introduction

Passive systems theory is a widely used tool for the analysis of nonlinear systems.[1–16] Passivity is a characterization of system behavior based on energy.[1,2] Passive systems store and dissipate energy without generating their own.[3–7] Therefore passive systems are Lyapunov-stable and have minimum phase.[8–10] Passivity can be used to assess the stability of a single system, but it is more restrictive than direct Lyapunov stability.[11–13] The real benefit is that when two passive systems are combined in negative feedback, the resulting interconnection is passive and stable. These results provide open-loop conditions to guarantee closed-loop stability.[14–16]

Some results on chaos (or hyperchaos) control and synchronization are derived by using the known (or certain) parameters, and the controller is constructed by those known parameters. However, in practical engineering situations, parameters of systems are probably unknown and may change from time to time. When chaotic (or hyperchaotic) systems have some unknown or fully unknown parameters, it is difficult to achieve control or synchronization.[17–20] Thus, it is a very important problem to realize chaos (or hyperchaos) control for these chaotic (or hyperchaotic) systems with unknown parameters.

Many researchers studied the passive control theory to investigate the control and synchronization of chaotic (or hyperchaotic) systems (with real variables) with unknown parameters.[21–30] There also exist, however, interesting cases of nonlinear dynamical systems, where the main variables are complex.[4,17,31–47] For example, the atomic polarization and electric field amplitudes are complex in a ring laser system of two-level atoms.[31,32] Another example is when chaos synchronization is applied to secure communications, where the complex variable (doubling the number of variables) may be used to increase the content of transmitting information signals and enhance their security.[33,46,47]

Motivated by the above discussion, in this paper, it is important to examine the control of hyperchaotic nonlinear systems (with complex variables) with unknown parameters using the adaptive passive method.

The organization of this paper is as follows. Section 2 presents the description of a hyperchaotic complex nonlinear system. An approach is stated to achieve the control of hyperchaotic complex nonlinear systems with fully unknown parameters in Section 3. In Section 4, we investigate the control of hyperchaotic attractors of the hyperchaotic complex Lü system with unknown parameters as an example. Simulation results are given in Section 5 to confirm the validity of the proposed theoretical results. Finally, conclusions are drawn in Section 6.

2. A hyperchaotic complex nonlinear system

A complex dynamical system is called hyperchaotic if it is deterministic, has long-term aperiodic behavior, and exhibits sensitive dependence on the initial conditions, or has at least two positive Lyapunov exponents. It is even more complicated than chaotic complex systems and has more unstable manifolds. Due to hyperchaotic complex systems having char-

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acteristics of high capacity, high security, and high efficiency, they have a broadly applied potential in nonlinear circuits, secure communications, lasers, neural networks, biological systems, and so on. Therefore researchers are interested in studying hyperchaotic complex nonlinear systems nowadays (see Refs. [17], [34]–[39], [42], and [43]).

Consider the hyperchaotic complex nonlinear system as follows:

$$\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}, \bar{\boldsymbol{z}}), \tag{1}$$

where the state complex vector $\boldsymbol{z} = (z_1, z_2, \ldots, z_n)^{\mathrm{T}} \in \mathbb{C}^n$, $z_l = u_{l\mathrm{r}} + \mathrm{j}u_{l\mathrm{i}}$ $(l = 1, 2, \ldots, n)$, $j = \sqrt{-1}$, $\boldsymbol{f} = (f_1, f_2, \ldots, f_n)^{\mathrm{T}} \in \mathbb{C}^n$ is the vector of complex nonlinear function, $f_l = f_{l\mathrm{r}} + \mathrm{j}f_{l\mathrm{i}}$, T denotes transpose, subscripts r and i stand for the real and imaginary parts of the state complex vector respectively, dot represents derivative with respect to time and an overbar denotes complex conjugate variables.

In this paper we study the control of hyperchaotic complex nonlinear systems with unknown parameters of the form (1) using passive control theory which has not been as actively explored in the literature.

Remark 1 The hyperchaotic complex nonlinear system (1) can be rewritten in the real form as

$$\dot{\boldsymbol{u}} = \boldsymbol{F}(\boldsymbol{u}), \tag{2}$$

where $\boldsymbol{u} = (u_{1\mathrm{r}}, u_{1\mathrm{i}}, u_{2\mathrm{r}}, u_{2\mathrm{i}}, \ldots, u_{n\mathrm{r}}, u_{n\mathrm{i}})^{\mathrm{T}} \in \mathbb{R}^{2n}$ is a state real vector, $u_{l\mathrm{r}}$, $u_{l\mathrm{i}}$ are real functions, $(l = 1, 2, \ldots, n)$ and $\boldsymbol{F} = (f_{1\mathrm{r}}, f_{1\mathrm{i}}, f_{2\mathrm{r}}, f_{2\mathrm{i}}, \ldots, f_{n\mathrm{r}}, f_{n\mathrm{i}})^{\mathrm{T}} \in \mathbb{R}^{2n}$ is a smooth nonlinear vector field.

Remark 2 Most hyperchaotic complex systems can be described by Eq. (1), such as hyperchaotic complex Lorenz, Chen, and Lü systems.[17,34–39,42,43]

In order to show the results of the control of systems of the form (1) with unknown parameters we choose, as an example, the hyperchaotic complex Lü system which has been introduced and studied recently in our work[37]

$$\begin{aligned}
\dot{z}_1 &= \alpha(z_2 - z_1), \\
\dot{z}_2 &= \beta z_2 - z_1 z_3 + (1 + \mathrm{j})z_4, \\
\dot{z}_3 &= \frac{1}{2}(\bar{z}_1 z_2 + \bar{z}_2 z_1) - \gamma z_3, \\
\dot{z}_4 &= \frac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2) - \delta z_4,
\end{aligned} \tag{3}$$

where $\boldsymbol{z} = (z_1, z_2, z_3, z_4)^{\mathrm{T}}$, $z_1 = u_{1\mathrm{r}} + \mathrm{j}u_{1\mathrm{i}}$, $z_2 = u_{2\mathrm{r}} + \mathrm{j}u_{2\mathrm{i}}$, $z_3 = u_{3\mathrm{r}}$, $z_4 = u_{4\mathrm{r}}$.

The hyperchaotic complex Lü system (3) is a 6-dimensional continuous real autonomous system. The block diagram of system (3) (before control) using Matlab/Simulink is illustrated in Fig. 1. For the case $\alpha = 42$, $\beta = 26.1$, $\gamma = 6$, $\delta = 10$, system (3) has hyperchaotic attractor as shown in Fig. 2. This example has several applications in engineering, e.g., in secure communications and nonlinear electronic circuits.

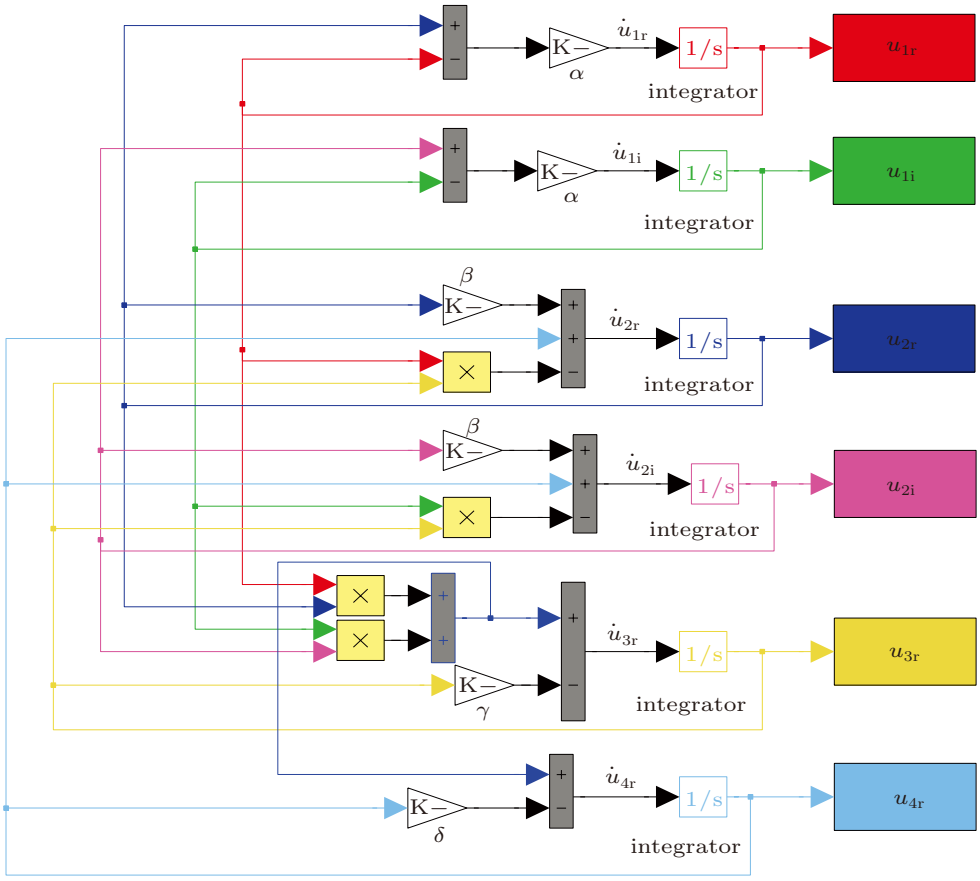


Fig. 1. (color online) Block diagram of system (3) (before control) using Matlab–Simulink.

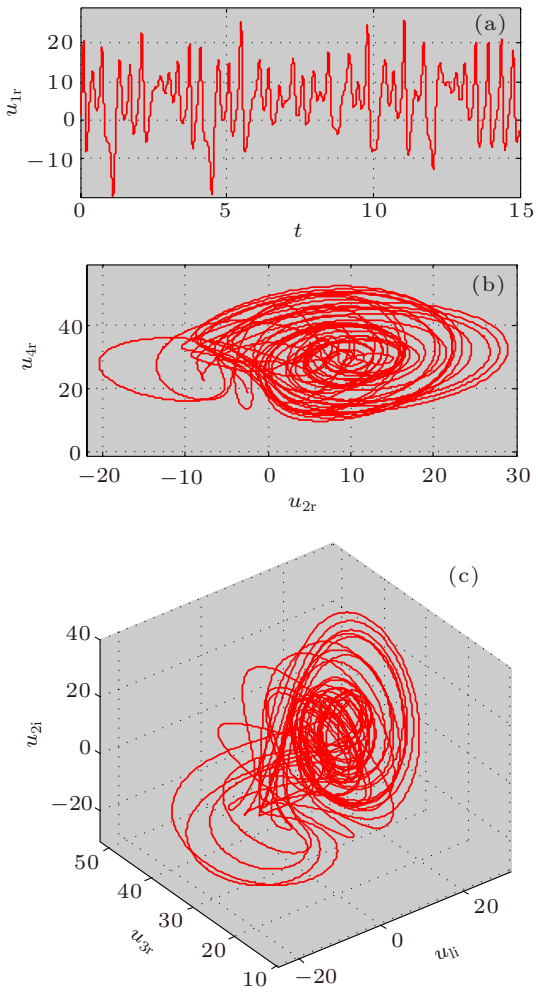


Fig. 2. (color online) Simulation observations of system (3) for $\alpha = 42$, $\beta = 26.1$, $\gamma = 6$, $\delta = 10$ with initial conditions $t_0 = 0$, $u_{1r}(0) = 1$, $u_{1i}(0) = 2$, $u_{2r}(0) = 3$, $u_{2i}(0) = 4$, $u_{3r}(0) = 5$, $u_{4r}(0) = 6$. (a) A state space in (t, u_{1r}) plane. (b) A chaotic attractor in (u_{2r}, u_{4r}) plane. (c) A chaotic attractor in (u_{1i}, u_{2i}, u_{3r}) space.

It is interesting to note that the two positive Lyapunov exponents for the hyperchaotic complex Lü system are $\lambda_1 = 2.60$, $\lambda_2 = 0.83$.[37] When $\alpha = 42$, $\beta = 26.1$, $\gamma = 6$, $\delta = 10$, system (3) has a trivial equilibrium point $E_0 = (0, 0, 0, 0, 0, 0)$, as well as two non-trivial equilibrium points

$$E_1 = (-8.55381, -8.55381, -8.55381, -8.55381, 24.3892, 14.6335),$$

$$E_2 = (9.15381, 9.15381, 9.15381, 9.15381, 27.9308, 16.7585).$$

For more dynamical properties of our system (3), see Refs. [32] and [37].

3. The passivity approach for controlling system (1) with unknown parameters

We consider the hyperchaotic complex nonlinear systems, with unknown parameters, with the controller in the general form as follows:

$$\begin{cases} \dot{z} = \boldsymbol{f}(z, \bar{z}, \boldsymbol{\xi}) + \boldsymbol{g}(z, \bar{z}, \boldsymbol{\xi})\boldsymbol{\Theta}, \\ \boldsymbol{y} = \boldsymbol{h}(z, \bar{z}, \boldsymbol{\xi}), \end{cases} \tag{4}$$

where $\boldsymbol{\Theta} = (\theta_1, \theta_2, \ldots, \theta_m)^{\mathrm{T}} \in \mathbb{C}^m$ is the input (or controller), $\theta_s = \theta_{sr} + \mathrm{j}\theta_{si}$ $(s = 1, 2, \ldots, m)$, $n > m$, $\boldsymbol{y} = (\eta_1, \eta_2, \ldots, \eta_m)^{\mathrm{T}} \in \mathbb{C}^m$ is the output, $\eta_s = \eta_{sr} + \mathrm{j}\eta_{si}$, $\boldsymbol{g} \in \mathbb{C}^{n\times m}$ is complex matrix, $\boldsymbol{f}$ and $\boldsymbol{g}$ are smooth vector fields, $\boldsymbol{h} \in \mathbb{C}^m$ is a smooth mapping and the unknown parameter vector $\boldsymbol{\xi} \in \mathbb{R}^p$.

The real form of system (4) can be written as

$$\begin{cases} \dot{\boldsymbol{u}} = \boldsymbol{F}(\boldsymbol{u}, \boldsymbol{\xi}) + \boldsymbol{G}(\boldsymbol{u}, \boldsymbol{\xi})\boldsymbol{\theta}, \\ \boldsymbol{\eta} = \boldsymbol{H}(\boldsymbol{u}, \boldsymbol{\xi}), \end{cases} \tag{5}$$

where the state variable $\boldsymbol{u} \in \mathbb{R}^{2n}$, the external input $\boldsymbol{\theta} = (\theta_{1r}, \theta_{1i}, \theta_{2r}, \theta_{2i}, \ldots, \theta_{mr}, \theta_{mi})^{\mathrm{T}} \in \mathbb{R}^{2m}$, the output $\boldsymbol{\eta} = (\eta_{1r}, \eta_{1i}, \eta_{2r}, \eta_{2i}, \ldots, \eta_{mr}, \eta_{mi})^{\mathrm{T}} \in \mathbb{R}^{2m}$, $\boldsymbol{G} \in \mathbb{R}^{2n\times 2m}$ is the real form of the matrix $\boldsymbol{g}$ and $\boldsymbol{H} \in \mathbb{R}^{2m}$ is the real vector of $\boldsymbol{h}$. Assume that these functions are locally Lipschitz with $\boldsymbol{F}(\boldsymbol{0}) = \boldsymbol{0}$ and $\boldsymbol{H}(\boldsymbol{0}) = \boldsymbol{0}$.[48] We said that the function $\boldsymbol{N}(\boldsymbol{u})$ is locally Lipschitz if

$$\|\boldsymbol{N}(\boldsymbol{u}) - \boldsymbol{N}(\breve{\boldsymbol{u}})\| \leq k_{\boldsymbol{N}} \|\boldsymbol{u} - \breve{\boldsymbol{u}}\|, \quad \forall \boldsymbol{u}, \breve{\boldsymbol{u}} \in \mathbb{R}^{2n}, \tag{6}$$

$k_{\boldsymbol{N}}$ is appropriate positive constant, in other words $\boldsymbol{N}(\boldsymbol{u})$ have continuous and bounded first partial derivatives with respect to the components of $\boldsymbol{u}$.[48] Also, we assume that the vector field $\boldsymbol{F}$ has at least one equilibrium; thus, without loss of generality, after possible coordinates shift.[10]

Definition 1[4,10] The system of the form (5) is said to have relative degree $[1, 1, \ldots, 1]$ at $\boldsymbol{u} = \boldsymbol{0}$ if the matrix $\boldsymbol{L}_{\boldsymbol{G}}\boldsymbol{H}(\boldsymbol{0}) = [\partial \boldsymbol{H}/(\partial \boldsymbol{u})]\boldsymbol{G}(\boldsymbol{u}, \boldsymbol{\xi})$ (the directional derivative of $\boldsymbol{H}$ in the direction of $\boldsymbol{G}$) is nonsingular.

Definition 2[10,23,25] System (5) is said to be C^{r}-passive if there exists a C^{r} positive real-valued function $V(\boldsymbol{u})$, $V(\boldsymbol{0}) = 0$, called storage function, such that $\forall t \geq 0$, the following dissipation inequality holds

$$V(\boldsymbol{u}) - V(\boldsymbol{u}(\boldsymbol{0})) \leq \int_0^t \boldsymbol{\theta}^{\mathrm{T}}(\tau)\boldsymbol{\eta}(\tau)\,\mathrm{d}\tau. \tag{7}$$

Definition 3[10,23,25] System (5) is said to be strictly C^{r}-passive if there exists a positive-definite function S such that:

$$V(\boldsymbol{u}) - V(\boldsymbol{u}(\boldsymbol{0})) \leq \int_0^t \boldsymbol{\theta}^{\mathrm{T}}(\tau)\boldsymbol{\eta}(\tau)d\tau - \int_0^t S(\boldsymbol{u}(\tau))\,\mathrm{d}\tau. \tag{8}$$

Remark 3 Setting $\boldsymbol{\theta} = \boldsymbol{0}$ in inequality (7), we get from Definition 2, that V is decreasing along any trajectory of system (5) and it follows that passive systems having a positive-definite storage function V are Lyapunov-stable. Also, we see that V is decreasing along any trajectory of system (5) if $\boldsymbol{\eta} = 0$.[10]

Lemma 1[10] Suppose system (5) is

(i) passive with a storage function V, which is positive-definite,

(ii) zero-state detectable, then $\boldsymbol{u} = \boldsymbol{0}$ can be asymptotically stabilized with $\boldsymbol{\theta} = -\boldsymbol{\Omega}(\boldsymbol{\eta})$ where $\boldsymbol{\Omega}$ is any locally Lipschitz function such that $\boldsymbol{\Omega}(\boldsymbol{0}) = \boldsymbol{0}$ and $\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\Omega}(\boldsymbol{\eta}) > 0$ for all $\boldsymbol{\eta} \neq \boldsymbol{0}$.

If the system (5) has relative degree $[1, 1, \ldots, 1]$,[23,49] it is possible to find $2(n - m)$ real-valued functions $\zeta_1, \zeta_2, \ldots, \zeta_{2(n-m)}$ which together with the $2m$ components of

the output map $\boldsymbol{\eta} = \boldsymbol{H}(\boldsymbol{u},\boldsymbol{\xi})$, qualify as a new set of local coordinates. In the new coordinates $(\boldsymbol{\zeta},\boldsymbol{\eta})$, the system (5) is represented by equations having the following structure (normal form)

$$\begin{cases} \dot{\boldsymbol{\eta}} = \boldsymbol{\rho}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) + \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\theta}, \\ \dot{\boldsymbol{\zeta}} = \boldsymbol{\Upsilon}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}), \end{cases} \tag{9}$$

where $\boldsymbol{\zeta}\in\mathbb{R}^{2(n-m)}, \boldsymbol{\theta},\boldsymbol{\eta}\in\mathbb{R}^{2m}$. The zero dynamics of a system describes the internal dynamics which is consistent with the external constraint $\boldsymbol{\eta} = \boldsymbol{0}$. For system (9), the zero dynamics is given by

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{\Upsilon}(\boldsymbol{\zeta},\boldsymbol{0},\boldsymbol{\xi}) = \boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi}). \tag{10}$$

It is shown by Seron *et al.*[23] that system (9) can be written as

$$\begin{cases} \dot{\boldsymbol{\zeta}} = \boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi}) + \boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta}, \\ \dot{\boldsymbol{\eta}} = \boldsymbol{\rho}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) + \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\theta}. \end{cases} \tag{11}$$

Also, we assume that the $2m\times 2m$ matrix $\boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})$ is nonsingular and we let $\boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})$ in system (9) be a parameter-independent (i.e. $\boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) = \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta})$), so system (11) becomes

$$\begin{cases} \dot{\boldsymbol{\zeta}} = \boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi}) + \boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta}, \\ \dot{\boldsymbol{\eta}} = \boldsymbol{\rho}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) + \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta})\boldsymbol{\theta}. \end{cases} \tag{12}$$

Definition 4[10,23] Suppose $\boldsymbol{L_G H}(\boldsymbol{0})$ is nonsingular, then system (5) is said to be minimum phase if $\boldsymbol{\zeta} = \boldsymbol{0}$ is an asymptotically stable equilibrium of $\boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi})$. In other words, there exists a function $W(\boldsymbol{\zeta},\boldsymbol{\xi})$ which is positive-definite and differentiable in $\boldsymbol{\zeta}$; $\forall\boldsymbol{\xi}\in\mathbb{R}^p$ such that

$$\frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial \boldsymbol{\zeta}}\boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi}) \le 0, \tag{13}$$

$\forall\boldsymbol{\zeta}$ in a neighborhood of $\boldsymbol{\zeta} = \boldsymbol{0}$.

Definition 5 (Convexity condition)[23] The function $\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta}$ is convex in $\boldsymbol{\xi}$ for all $\boldsymbol{\xi}\in\mathbb{R}^p$, if:

$$\frac{\partial(\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta})}{\partial\boldsymbol{\xi}}(\boldsymbol{\xi}_1-\boldsymbol{\xi}_2) + \boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}_2)\boldsymbol{\eta} - \boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}_1)\boldsymbol{\eta} \le 0, \tag{14}$$

$\forall\boldsymbol{\xi}_1,\boldsymbol{\xi}_2\in\mathbb{R}^p$, where

$$\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) = -\boldsymbol{\rho}^{\mathrm{T}}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) - \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}). \tag{15}$$

Theorem 1 Consider the controller and the parameters estimation law as

$$\begin{cases} \boldsymbol{\theta} = \boldsymbol{\sigma}^{-1}(\boldsymbol{\zeta},\boldsymbol{\eta})[\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}}) - \varepsilon\boldsymbol{\eta} + \boldsymbol{\nu}], \\ \dot{\hat{\boldsymbol{\xi}}}^{\mathrm{T}} = -\Big(\dfrac{\partial(\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}})\boldsymbol{\eta})}{\partial\hat{\boldsymbol{\xi}}} - \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\Big)\boldsymbol{\Lambda}, \end{cases} \tag{16}$$

where $\hat{\boldsymbol{\xi}}$ is the estimation vector of the unknown parameter vector $\boldsymbol{\xi}$, $\tilde{\boldsymbol{\xi}} = \boldsymbol{\xi} - \hat{\boldsymbol{\xi}}$, ε is a positive real value, $\boldsymbol{\nu}$ is an external signal vector that is connected with the reference input and $\boldsymbol{\Lambda}$ is a diagonal matrix of positive constant elements λ_l, $l = 1,2,\ldots,p$. If the system (5) satisfies the Definitions 1, 4 and the Definition 5 holds for all $\boldsymbol{\xi}\in\mathbb{R}^p$, then system (12) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point.

Proof Suppose that the storage function takes the form

$$V(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi},\tilde{\boldsymbol{\xi}}) = W(\boldsymbol{\zeta},\boldsymbol{\xi}) + \frac{1}{2}\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta} + \frac{1}{2}\tilde{\boldsymbol{\xi}}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\tilde{\boldsymbol{\xi}}, \tag{17}$$

where $V(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi},\tilde{\boldsymbol{\xi}})$ is the Lyapunov function. Differentiating V with respect to t, then we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi},\tilde{\boldsymbol{\xi}}) = \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\dot{\boldsymbol{\zeta}} + \dot{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{\eta} - \dot{\hat{\boldsymbol{\xi}}}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\tilde{\boldsymbol{\xi}}. \tag{18}$$

Using system (12), we obtain

$$\begin{aligned} \dot{V} &= \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\boldsymbol{\varphi}_0(\boldsymbol{\zeta},\boldsymbol{\xi}) + \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta} \\ &\quad + \big[\boldsymbol{\rho}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) + \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta})\boldsymbol{\theta}\big]^{\mathrm{T}}\boldsymbol{\eta} - \dot{\hat{\boldsymbol{\xi}}}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\tilde{\boldsymbol{\xi}}. \end{aligned} \tag{19}$$

If system (5) satisfies Definitions 1 and 4, then system (5) is a minimum phase system, so equation (19) can be written as

$$\begin{aligned} \dot{V} &\le \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta} + \big[\boldsymbol{\rho}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi}) + \boldsymbol{\sigma}(\boldsymbol{\zeta},\boldsymbol{\eta})\boldsymbol{\theta}\big]^{\mathrm{T}}\boldsymbol{\eta} \\ &\quad - \dot{\hat{\boldsymbol{\xi}}}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\tilde{\boldsymbol{\xi}}. \end{aligned} \tag{20}$$

Substituting Eq. (16) into the inequality (20), we get

$$\begin{aligned} \dot{V} &\le \frac{\partial W(\boldsymbol{\zeta},\boldsymbol{\xi})}{\partial\boldsymbol{\zeta}}\boldsymbol{\psi}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta} + \boldsymbol{\rho}^{\mathrm{T}}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta} + \boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}})\boldsymbol{\eta} \\ &\quad - \varepsilon\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta} + \boldsymbol{\nu}^{\mathrm{T}}\boldsymbol{\eta} + \frac{\partial(\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}})\boldsymbol{\eta})}{\partial\hat{\boldsymbol{\xi}}}(\boldsymbol{\xi}-\hat{\boldsymbol{\xi}}) - \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\tilde{\boldsymbol{\xi}}. \end{aligned} \tag{21}$$

Using Eq. (15), we have

$$\begin{aligned} \dot{V} &\le -\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi})\boldsymbol{\eta} + \boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}})\boldsymbol{\eta} + \frac{\partial[\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}}\boldsymbol{\eta})]}{\partial\hat{\boldsymbol{\xi}}}(\boldsymbol{\xi}-\hat{\boldsymbol{\xi}}) \\ &\quad - \varepsilon\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta} + \boldsymbol{\nu}^{\mathrm{T}}\boldsymbol{\eta} - \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\tilde{\boldsymbol{\xi}}. \end{aligned} \tag{22}$$

When system (5) satisfies Definition 5, for $\boldsymbol{Q}(\boldsymbol{\zeta},\boldsymbol{\eta},\hat{\boldsymbol{\xi}})\boldsymbol{\eta}$ with $\boldsymbol{\xi}_2 = \hat{\boldsymbol{\xi}}$ and $\boldsymbol{\xi}_1 = \boldsymbol{\xi}$, we obtain

$$\dot{V} \le \boldsymbol{\nu}^{\mathrm{T}}\boldsymbol{\eta} - (\varepsilon\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta} + \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\tilde{\boldsymbol{\xi}}). \tag{23}$$

Integrating over both sides of inequality (23) yields

$$\begin{aligned} &V(\boldsymbol{\zeta},\boldsymbol{\eta},\boldsymbol{\xi},\tilde{\boldsymbol{\xi}}) - V(\boldsymbol{\zeta}(\boldsymbol{0}),\boldsymbol{\eta}(\boldsymbol{0}),\tilde{\boldsymbol{\xi}}(\boldsymbol{0})) \\ &\le \int_0^t \boldsymbol{\nu}^{\mathrm{T}}(\tau)\boldsymbol{\eta}(\tau)\,\mathrm{d}\tau - \varepsilon\int_0^t(\boldsymbol{\eta}^{\mathrm{T}}(\tau)\boldsymbol{\eta}(\tau) + \tilde{\boldsymbol{\xi}}^{\mathrm{T}}(\tau)\tilde{\boldsymbol{\xi}}(\tau))\,\mathrm{d}\tau. \end{aligned} \tag{24}$$

According to the definition of V, it is obvious that $V \ge 0$ and using Definition 3, then system (12) is a passive system. According to Lemma 1 system (12) will be asymptotically stabilized at an equilibrium point, and Theorem 1 is proved.

For illustration of this approach we apply it to control hyperchaotic attractors of the complex Lü system (3) as an example of n-dimensional hyperchaotic complex nonlinear system (1) in Section 4.

4. An example

In this section, as an example, we study the passive control of hyperchaotic complex Lü system (3) with unknown parameters using the approach of Section 3. Other examples can be similarly studied. The hyperchaotic complex Lü system with the controller is given by

$$\dot{z}_1 = \alpha(z_2 - z_1),$$

$$\begin{aligned}\dot{z}_2 &= \beta z_2 - z_1 z_3 + (1+\mathrm{j}) z_4 + \theta_1,\\ \dot{z}_3 &= \tfrac{1}{2}(\bar{z}_1 z_2 + \bar{z}_2 z_1) - \gamma z_3,\\ \dot{z}_4 &= \tfrac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2) - \delta z_4,\end{aligned} \tag{25}$$

where $z_1 = u_{1\mathrm{r}} + \mathrm{j}u_{1\mathrm{i}}, z_2 = u_{2\mathrm{r}} + \mathrm{j}u_{2\mathrm{i}}, z_3 = u_{3\mathrm{r}}$, $z_4 = u_{4\mathrm{r}}$, $\boldsymbol{\xi} = [\,\alpha\ \beta\ \gamma\ \delta\,]^{\mathrm{T}}$ is the vector of unknown parameters, $\theta_1 = \theta_{1\mathrm{r}} + j\theta_{1\mathrm{i}}$. Separating the real and imaginary parts, we have

$$\begin{aligned}\dot{u}_{1\mathrm{r}} &= \alpha(u_{2\mathrm{r}} - u_{1\mathrm{r}}),\\ \dot{u}_{1\mathrm{i}} &= \alpha(u_{2\mathrm{i}} - u_{1\mathrm{i}}),\\ \dot{u}_{2\mathrm{r}} &= -u_{1\mathrm{r}}u_{3\mathrm{r}} + \beta u_{2\mathrm{r}} + u_{4\mathrm{r}} + \theta_{1\mathrm{r}},\\ \dot{u}_{2\mathrm{i}} &= -u_{1\mathrm{i}}u_{3\mathrm{r}} + \beta u_{2\mathrm{i}} + u_{4\mathrm{r}} + \theta_{1\mathrm{i}},\\ \dot{u}_{3\mathrm{r}} &= u_{1\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{2\mathrm{i}} - \gamma u_{3\mathrm{r}},\\ \dot{u}_{4\mathrm{r}} &= u_{1\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{2\mathrm{i}} - \delta u_{4\mathrm{r}}.\end{aligned} \tag{26}$$

Theorem 2 The hyperchaotic system (26) is a minimum phase system.

Proof

First we can compute the matrix $\boldsymbol{L_G H}(0)$ as[4,50]

$$\boldsymbol{L_G H}(0) = \begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}, \tag{27}$$

where

$$\boldsymbol{H} = \begin{bmatrix} u_{2\mathrm{r}}\\ u_{2\mathrm{i}}\end{bmatrix}$$

and

$$\boldsymbol{G} = \begin{bmatrix} 0&0&1&0&0&0\\ 0&0&0&1&0&0\end{bmatrix}^{\mathrm{T}}$$

is computed by comparing system (26) with system (5). Since the matrix $\boldsymbol{L_G H}(0)$ is nonsingular and according to the Definition 1, then system (26) has relative degree $[1,1,\ldots,1]$ at $\boldsymbol{u} = \boldsymbol{0}$, so we can let

$$\begin{aligned}\boldsymbol{\zeta} &= [\,\zeta_1\ \zeta_2\ \zeta_3\ \zeta_4\,]^{\mathrm{T}} = [\,u_{1\mathrm{r}}\ u_{1\mathrm{i}}\ u_{3\mathrm{r}}\ u_{4\mathrm{r}}\,]^{\mathrm{T}},\\ \boldsymbol{\eta} &= [\,\eta_{1\mathrm{r}}\ \eta_{1\mathrm{i}}\,]^{\mathrm{T}} = [\,u_{2\mathrm{r}}\ u_{2\mathrm{i}}\,]^{\mathrm{T}},\end{aligned}$$

thus system (26) has the form

$$\begin{aligned}\dot{\zeta}_1 &= \alpha(\eta_{1\mathrm{r}} - \zeta_1),\\ \dot{\zeta}_2 &= \alpha(\eta_{1\mathrm{i}} - \zeta_2),\\ \dot{\zeta}_3 &= \zeta_1\eta_{1\mathrm{r}} + \zeta_2\eta_{1\mathrm{i}} - \gamma\zeta_3,\\ \dot{\zeta}_4 &= \zeta_1\eta_{1\mathrm{r}} + \zeta_2\eta_{1\mathrm{i}} - \delta\zeta_4,\\ \dot{\eta}_{1\mathrm{r}} &= -\zeta_1\zeta_3 + \beta\eta_{1\mathrm{r}} + \zeta_4 + \theta_{1\mathrm{r}},\\ \dot{\eta}_{1\mathrm{i}} &= -\zeta_2\zeta_3 + \beta\eta_{1\mathrm{i}} + \zeta_4 + \theta_{1\mathrm{i}}.\end{aligned} \tag{28}$$

System (28) can be represented in the form of system (12), so we have

$$\varphi_0(\zeta,\xi) = \begin{bmatrix} -\alpha\zeta_1\\ -\alpha\zeta_2\\ -\gamma\zeta_3\\ -\delta\zeta_4\end{bmatrix}, \quad \psi(\zeta,\eta,\xi) = \begin{bmatrix} \alpha & 0\\ 0 & \alpha\\ \zeta_1 & \zeta_2\\ \zeta_1 & \zeta_2\end{bmatrix},$$

$$\rho(\zeta,\eta,\xi) = \begin{bmatrix} -\zeta_1\zeta_3 + \beta\eta_{1\mathrm{r}} + \zeta_4\\ -\zeta_2\zeta_3 + \beta\eta_{1\mathrm{i}} + \zeta_4\end{bmatrix}, \quad \sigma(\zeta,\eta) = \begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}. \tag{29}$$

Choose a storage function as

$$\begin{aligned}V(\zeta,\eta,\xi,\tilde{\xi}) &= W(\zeta,\xi) + \frac{1}{2}\eta^{\mathrm{T}}\eta + \frac{1}{2}\tilde{\xi}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\tilde{\xi}\\ &= W(\zeta,\xi) + \frac{1}{2}(\eta_{1\mathrm{r}}^2 + \eta_{1\mathrm{i}}^2) + \frac{1}{2}(\lambda_1\tilde{\alpha}^2\\ &\quad + \lambda_2\tilde{\beta}^2 + \lambda_3\tilde{\gamma}^2 + \lambda_4\tilde{\delta}^2),\end{aligned} \tag{30}$$

where $W(\zeta,\xi) = (\zeta_1^2 + \zeta_2^2 + \gamma\zeta_3^2 + \delta\zeta_4^2)/2$, $W(\boldsymbol{0}) = 0$, $\tilde{\alpha} = \alpha - \hat{\alpha}$, $\tilde{\beta} = \beta - \hat{\beta}$, $\tilde{\gamma} = \gamma - \hat{\gamma}$, $\tilde{\delta} = \delta - \hat{\delta}$, $\hat{\alpha},\hat{\beta},\hat{\gamma}$, and $\hat{\delta}$ are the parameters estimation of $\alpha,\beta,\gamma,\delta$ respectively and λ_l $(l = 1,2,3,4)$ are positive constants.

Now we will compute inequality (13) as

$$\begin{aligned}&\frac{\partial W(\zeta,\xi)}{\partial\zeta}\varphi_0(\zeta,\xi)\\ &= [\,\zeta_1\ \zeta_2\ \gamma\zeta_3\ \delta\zeta_4\,][\,-\alpha\zeta_1\ -\alpha\zeta_2\ -\gamma\zeta_3\ -\delta\zeta_4\,]^{\mathrm{T}}\\ &= -(\alpha\zeta_1^2 + \alpha\zeta_2^2 + \gamma^2\zeta_3^2 + \delta^2\zeta_4^2) \le 0.\end{aligned} \tag{31}$$

Since $W(\zeta,\xi) > 0$ and

$$\frac{\partial W(\zeta,\xi)}{\partial\zeta}\varphi_0(\zeta,\xi) < 0,$$

so that according to Definition 4 which means that the controlled system (28) is minimum phase system, and Theorem 2 is proved.

On the other hand, system (26) satisfies Definition 5 for $\xi_2 = \hat{\xi}$, $\xi_1 = \xi$ because

$$\begin{aligned}&\boldsymbol{Q}(\zeta,\eta,\xi)\eta\\ &= (\zeta_1\zeta_3 - \beta\eta_{1\mathrm{r}} - \zeta_4 - \alpha\zeta_1 - \gamma\zeta_1\zeta_3 - \delta\zeta_1\zeta_4)\eta_{1\mathrm{r}}\\ &\quad + (\zeta_2\zeta_3 - \beta\eta_{1\mathrm{i}} - \zeta_4 - \alpha\zeta_2 - \gamma\zeta_2\zeta_3 - \delta\zeta_2\zeta_4)\eta_{1\mathrm{i}},\end{aligned} \tag{32}$$

and

$$\begin{aligned}&\boldsymbol{Q}(\zeta,\eta,\hat{\xi})\eta\\ &= (\zeta_1\zeta_3 - \hat{\beta}\eta_{1\mathrm{r}} - \zeta_4 - \hat{\alpha}\zeta_1 - \hat{\gamma}\zeta_1\zeta_3 - \hat{\delta}\zeta_1\zeta_4)\eta_{1\mathrm{r}}\\ &\quad + (\zeta_2\zeta_3 - \hat{\beta}\eta_{1\mathrm{i}} - \zeta_4 - \hat{\alpha}\zeta_2 - \hat{\gamma}\zeta_2\zeta_3 - \hat{\delta}\zeta_2\zeta_4)\eta_{1\mathrm{i}},\end{aligned} \tag{33}$$

then we have

$$\frac{\partial(\boldsymbol{Q}(\zeta,\eta,\xi)\eta)}{\partial\xi}(\xi - \hat{\xi}) + \boldsymbol{Q}(\zeta,\eta,\hat{\xi})\eta - \boldsymbol{Q}(\zeta,\eta,\xi)\eta = 0. \tag{34}$$

Since system (26) satisfies Definitions 1, 4, and 5, therefore it can be equivalent to a passive system, then we can apply Theorem 1 to system (26) and compute the controllers as

$$\begin{aligned}\theta_{1\mathrm{r}} &= -\hat{\alpha}\zeta_1 - \hat{\beta}\eta_{1\mathrm{r}} - \zeta_4 + (1-\hat{\gamma})\zeta_1\zeta_3 - \hat{\delta}\zeta_1\zeta_4 - \varepsilon\eta_{1\mathrm{r}} + \nu_1,\\ \theta_{1\mathrm{i}} &= -\hat{\alpha}\zeta_2 - \hat{\beta}\eta_{1\mathrm{i}} - \zeta_4 + (1-\hat{\gamma})\zeta_2\zeta_3 - \hat{\delta}\zeta_2\zeta_4 - \varepsilon\eta_{1\mathrm{i}} + \nu_2,\end{aligned} \tag{35}$$

and the adaptive laws of parameters as

$$\begin{aligned}\dot{\hat{\alpha}} &= \lambda_1(\zeta_1\eta_{1\mathrm{r}} + \zeta_2\eta_{1\mathrm{i}} + \tilde{\alpha}),\\ \dot{\hat{\beta}} &= \lambda_2(\eta_{1\mathrm{r}}^2 + \eta_{1\mathrm{i}}^2 + \tilde{\beta}),\\ \dot{\hat{\gamma}} &= \lambda_3(\zeta_1\zeta_3\eta_{1\mathrm{r}} + \zeta_2\zeta_3\eta_{1\mathrm{i}} + \tilde{\gamma}),\\ \dot{\hat{\delta}} &= \lambda_4(\zeta_1\zeta_4\eta_{1\mathrm{r}} + \zeta_2\zeta_4\eta_{1\mathrm{i}} + \tilde{\delta}).\end{aligned} \tag{36}$$

Since

$$\begin{aligned}\zeta &= [\,\zeta_1\ \zeta_2\ \zeta_3\ \zeta_4\,]^{\mathrm{T}} = [\,u_{1\mathrm{r}}\ u_{1\mathrm{i}}\ u_{3\mathrm{r}}\ u_{4\mathrm{r}}\,]^{\mathrm{T}},\\ \eta &= [\,\eta_{1\mathrm{r}}\ \eta_{1\mathrm{i}}\,]^{\mathrm{T}} = [\,u_{2\mathrm{r}}\ u_{2\mathrm{i}}\,]^{\mathrm{T}},\end{aligned}$$

so equation (35) can be written as

$$\begin{aligned}\theta_{1\mathrm{r}} &= -\hat{\alpha}u_{1\mathrm{r}} - \hat{\beta}u_{2\mathrm{r}} - u_{4\mathrm{r}} + (1-\hat{\gamma})u_{1\mathrm{r}}u_{3\mathrm{r}} - \hat{\delta}u_{1\mathrm{r}}u_{4\mathrm{r}} - \varepsilon u_{2\mathrm{r}} + \nu_1,\\ \theta_{1\mathrm{i}} &= -\hat{\alpha}u_{1\mathrm{i}} - \hat{\beta}u_{2\mathrm{i}} - u_{4\mathrm{r}} + (1-\hat{\gamma})u_{1\mathrm{i}}u_{3\mathrm{r}} - \hat{\delta}u_{1\mathrm{i}}u_{4\mathrm{r}} - \varepsilon u_{2\mathrm{i}} + \nu_2,\end{aligned} \tag{37}$$

and equation (36) can be written as

$$\begin{aligned}
\dot{\hat{\alpha}} &= \lambda_1(u_{1\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{2\mathrm{i}} + \tilde{\alpha}),\\
\dot{\hat{\beta}} &= \lambda_2(u_{2\mathrm{r}}^2 + u_{2\mathrm{i}}^2 + \tilde{\beta}),\\
\dot{\hat{\gamma}} &= \lambda_3(u_{1\mathrm{r}}u_{3\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{3\mathrm{r}}u_{2\mathrm{i}} + \tilde{\gamma}),\\
\dot{\hat{\delta}} &= \lambda_4(u_{1\mathrm{r}}u_{4\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{4\mathrm{r}}u_{2\mathrm{i}} + \tilde{\delta}). \qquad (38)
\end{aligned}$$

The analytical formula of the controller θ_1 Eq. (37) and the differential equations for the parameters estimation Eq. (38) are used to control the hyperchaotic complex Lü system as

$$\begin{aligned}
\dot{u}_{1\mathrm{r}} &= \alpha(u_{2\mathrm{r}} - u_{1\mathrm{r}}),\\
\dot{u}_{1\mathrm{i}} &= \alpha(u_{2\mathrm{i}} - u_{1\mathrm{i}}),\\
\dot{u}_{2\mathrm{r}} &= -\hat{\alpha}u_{1\mathrm{r}} + (\tilde{\beta} - \varepsilon)u_{2\mathrm{r}} - \hat{\gamma}u_{1\mathrm{r}}u_{3\mathrm{r}} - \hat{\delta}u_{1\mathrm{r}}u_{4\mathrm{r}} + v_1,\\
\dot{u}_{2\mathrm{i}} &= -\hat{\alpha}u_{1\mathrm{i}} + (\tilde{\beta} - \varepsilon)u_{2\mathrm{i}} - \hat{\gamma}u_{1\mathrm{i}}u_{3\mathrm{r}} - \hat{\delta}u_{1\mathrm{i}}u_{4\mathrm{r}} + v_1,\\
\dot{u}_{3\mathrm{r}} &= u_{1\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{2\mathrm{i}} - \gamma u_{3\mathrm{r}},\\
\dot{u}_{4\mathrm{r}} &= u_{1\mathrm{r}}u_{2\mathrm{r}} + u_{1\mathrm{i}}u_{2\mathrm{i}} - \delta u_{4\mathrm{r}}. \qquad (39)
\end{aligned}$$

Remark 4 If $\varepsilon > 0$ and $v_1 = v_2 = 0$, the hyperchaotic complex Lü system can be controlled to trivial equilibrium point.

Remark 5[9,12] Now we want to steer the state of hyperchaotic complex Lü system to a nonzero equilibrium $E(u_{1\mathrm{r}}^*, u_{1\mathrm{i}}^*, u_{2\mathrm{r}}^*, u_{2\mathrm{i}}^*, u_{3\mathrm{r}}^*, u_{4\mathrm{r}}^*)^{\mathrm{T}}$. Let $\dot{\boldsymbol{u}} = \boldsymbol{0}$ in system (39) we have

$$\begin{aligned}
v_1 &= -(\tilde{\beta} - \varepsilon)u_{2\mathrm{r}}^* + u_{1\mathrm{r}}^*(\hat{\alpha} + \hat{\gamma}u_{3\mathrm{r}}^* + \hat{\delta}u_{4\mathrm{r}}^*),\\
v_2 &= -(\tilde{\beta} - \varepsilon)u_{2\mathrm{i}}^* + u_{1\mathrm{i}}^*(\hat{\alpha} + \hat{\gamma}u_{3\mathrm{r}}^* + \hat{\delta}u_{4\mathrm{r}}^*). \qquad (40)
\end{aligned}$$

So hyperchaotic complex Lü system can be stabilized at E_1 or E_2.

Remark 6 The attractors of system (26) can be converted to periodic (limit cycle) or quasi-periodic solutions when $\varepsilon > 0$ and $v_1 = A_1 \cos\omega_1 t$, $v_2 = A_2 \sin\omega_2 t$, where A_1, A_2 are the amplitudes and ω_1, ω_2 are the frequencies of v_1, v_2 respectively.

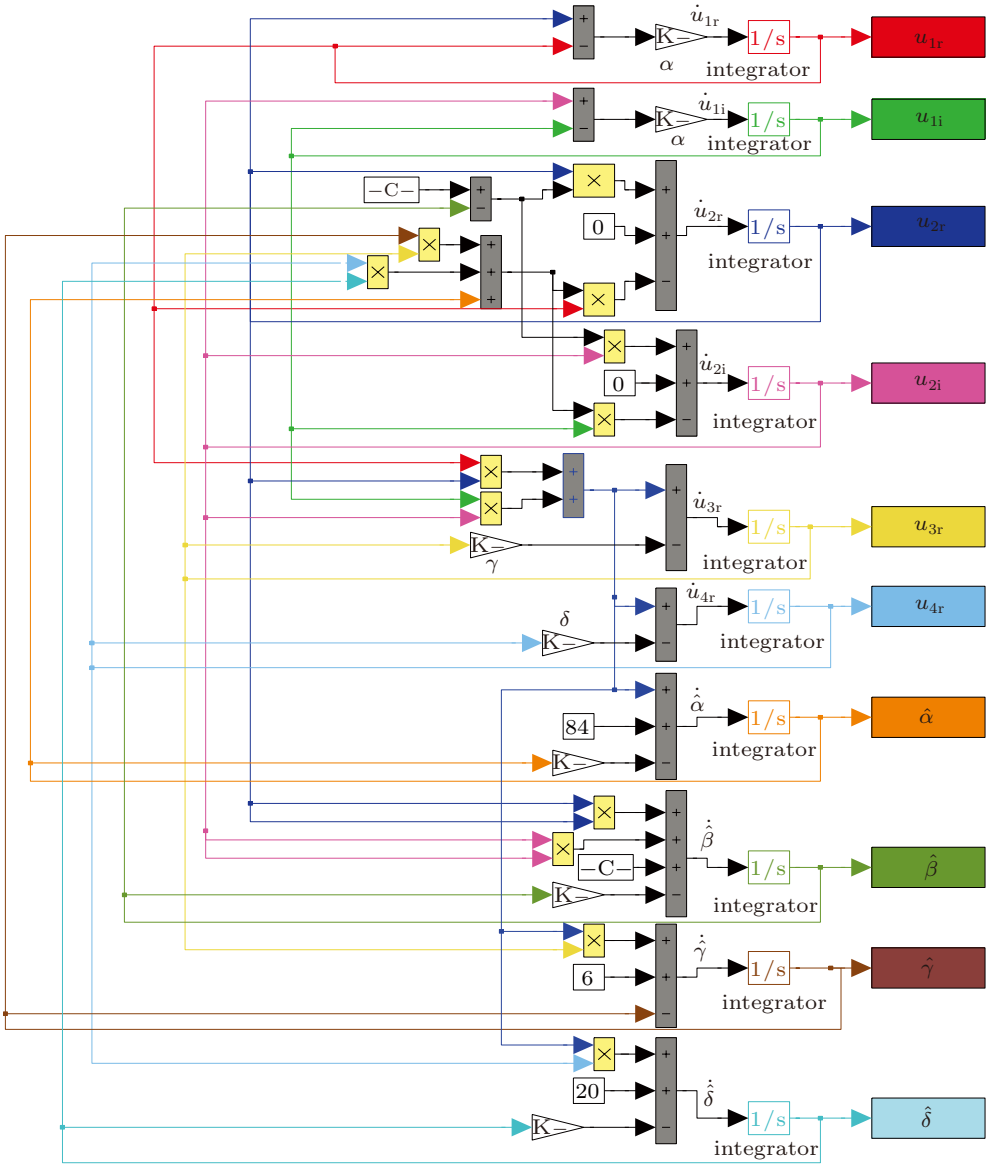


Fig. 3. (color online) Block diagram of system (26) (after control) using Matlab-Simulink.

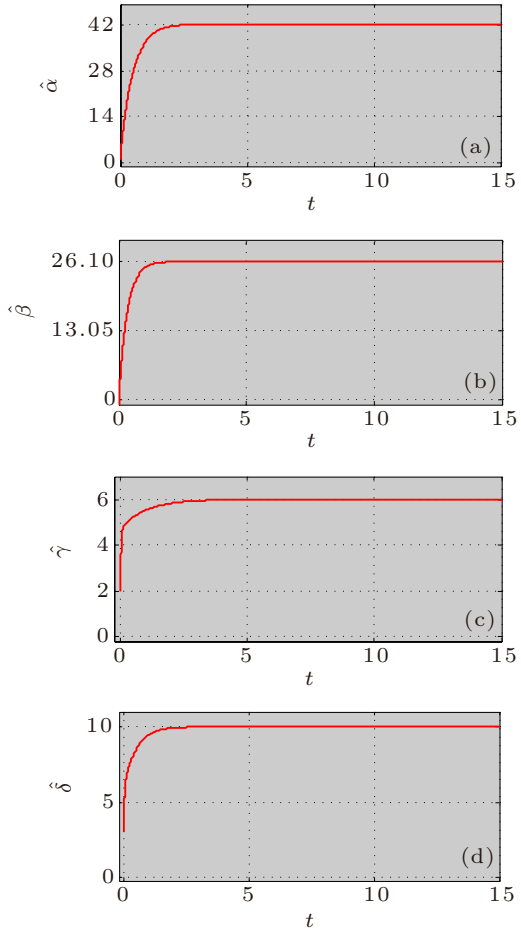


Fig. 4. (color online) Parameters estimation $\hat{\alpha}(t)$, $\hat{\beta}(t)$, $\hat{\gamma}(t)$, $\hat{\delta}(t)$ versus t: (a) $(\hat{\alpha}(t), t)$ diagram; (b) $(\hat{\beta}(t), t)$ diagram; (c) $(\hat{\gamma}(t), t)$ diagram; (d) $(\hat{\delta}(t), t)$ diagram.

5. Numerical simulation results

In this section, the simulation of the hyperchaotic complex Lü system (3) after control is designed by using Matlab/Simulink 7.4 to confirm the validity of the analytical results of the approach of Section 3. Simulink allows block diagram models of dynamic systems to be built quickly. It is widely used in control theory to simulate systems after control.[51] The simulation of the hyperchaotic complex Lü system after control is illustrated in Fig. 3. The block diagram model of system (26) with the complex controller θ_1 Eq. (37) after control (Fig. 3) contains 6 main channels which realize the state variables u_{1r}, u_{1i}, u_{2r}, u_{2i}, u_{3r}, u_{4r}, respectively. There are, also, 4 channels for the uncertain parameters $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, $\hat{\delta}$, respectively. The block diagram model (Fig. 3) is running with the choice $\alpha = 42$, $\beta = 26.1$, $\gamma = 6$, $\delta = 10$ for which hyperchaotic attractor exists,[37] and with initial conditions $t_0 = 0$, $u_{1r}(0) = 1$, $u_{1i}(0) = 2$, $u_{2r}(0) = 3$, $u_{2i}(0) = 4$, $u_{3r}(0) = 5$, $u_{4r}(0) = 6$. The initial values to estimate the unknown parameters are considered as $\hat{\alpha}(0) = 1$, $\hat{\beta}(0) = -1$, $\hat{\gamma}(0) = 2$, $\hat{\delta}(0) = 3$, and $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \mathrm{diag}(2, 3, 1, 2)$. Figure 4 shows the estimated values of the unknown parameters $\hat{\alpha}(t)$, $\hat{\beta}(t)$, $\hat{\gamma}(t)$, $\hat{\delta}(t)$ converge to 42, 26.1, 6, 10 respectively. The simulation observations of hyperchaotic complex Lü system (26) with the controller θ_1 Eq. (37) after control are displayed in Figs. 5–8. Figure 5 shows the state variables of the system (26) approach the trivial fixed point for the choice $\varepsilon = 10$, and $\nu_1 = \nu_2 = 0$ (see Remark 4). While in Fig. 6 they are controlled to the non-trivial equilibrium point E_1 for $\varepsilon = 10$, $\nu_1 = \nu_2 = -8.5538(\hat{\alpha} + \hat{\beta} + 24.389\hat{\gamma} + 14.6335\hat{\delta} - 16.1)$ (see Remark 5). On the other hand, the hyperchaotic attractor of system (3) is converted to periodic attractor for the case $\varepsilon = 10$, $\nu_1 = 10\sin 3t$, $\nu_2 = 3\cos 6t$, as demonstrated in Fig. 7 (see Remark 6). But, if we choose $\nu_1 = 11\sin 5t$, $\nu_2 = 8\cos 7t$, the attractor of system (3) is converted to a quasi-periodic solution as shown in Fig. 8. Figures 4–8 show that the controllers θ_{1r} and θ_{1i} are very effective.

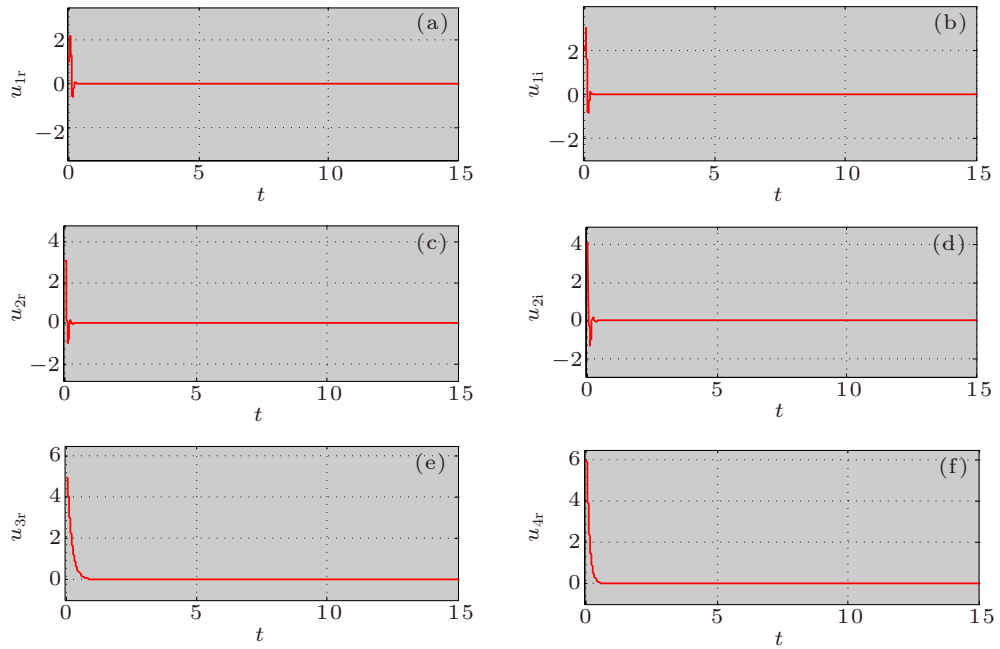


Fig. 5. (color online) Simulation observations of system (26) when $\varepsilon = 10$, $\nu_1 = \nu_2 = 0$ (trivial equilibrium point): (a) (t, u_{1r}) plane; (b) (t, u_{1i}) plane; (c) (t, u_{2r}) plane; (d) (t, u_{2i}) plane; (e) (t, u_{3r}) plane; (f) (t, u_{4r}) plane.

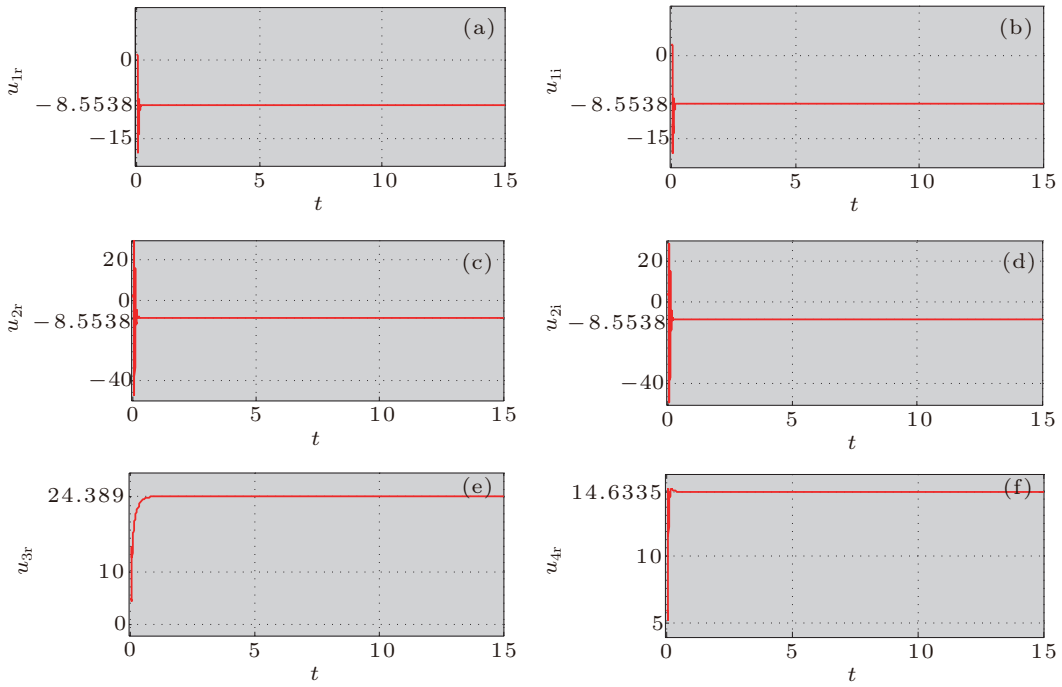


Fig. 6. (color online) Simulation observations of system (26) when $\varepsilon = 10$, $\nu_1 = \nu_2 = -8.5538(\hat{\alpha} + \hat{\beta} + 24.389\hat{\gamma} + 14.6335\hat{\delta} - 16.1)$ (non-trivial equilibrium point): (a) (t, u_{1r}) plane; (b) (t, u_{1i}) plane; (c) (t, u_{2r}) plane; (d) (t, u_{2i}) plane; (e) (t, u_{3r}) plane; (f) (t, u_{4r}) plane.

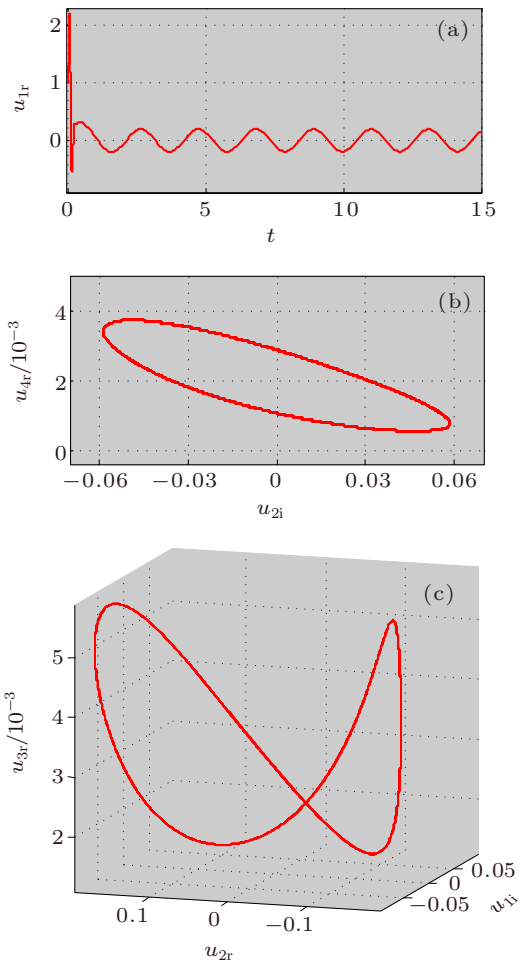


Fig. 7. (color online) Simulation observations of system (26) when $\varepsilon = 10$, $\nu_1 = 10\sin 3t$, $\nu_2 = 3\cos 6t$ (limit cycle): (a) (t, u_{1r}) plane; (b) (u_{2i}, u_{4r}) plane; (c) (u_{1i}, u_{2r}, u_{3r}) space.

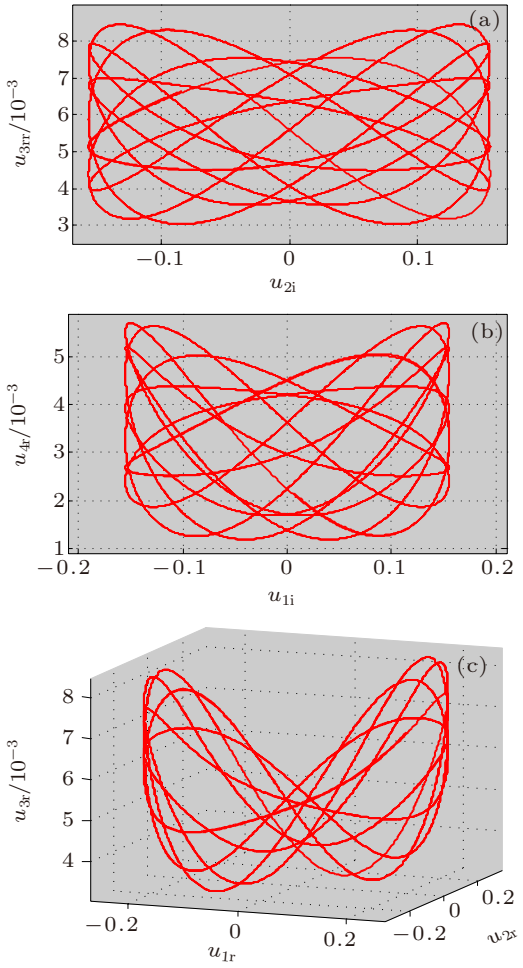


Fig. 8. (color online) Simulation observations of system (26) when $\varepsilon = 10$, $\nu_1 = 11\sin 5t$, $\nu_2 = 8\cos 7t$ (quasi-periodic solution): (a) (u_{2i}, u_{3r}) plane; (b) (u_{1i}, u_{4r}) plane; (c) (u_{1r}, u_{2r}, u_{3r}) space.

6. Conclusion

In this paper, we studied the control of hyperchaotic complex nonlinear systems with unknown parameters (1) using passive control theory which has not been treated in the literature. As an example, the proposed approach of this paper is applied to control hyperchaotic attractors of the hyperchaotic Lü system (3).[37] Using this approach an analytical formula of the complex controller θ_1 Eq. (37) and the differential equations for the parameters estimation Eq. (38) are calculated. Equations (38) are solved, by using block diagram of system (26), and the estimated values of the unknown parameters are displayed in Fig. 4 and it is clear that they approach the exact values. The formula for θ_1 is used to control a hyperchaotic attractor of system (3) to a trivial equilibrium point as shown in Fig. 5, and to a non-trivial one in Fig. 6, while, it is used to convert the hyperchaotic attractor of system (3) to a periodic one in Fig. 7 and to a quasi-periodic one in Fig. 8. It is clear that in our example we used only one complex controller θ_1 to suppress the hyperchaotic behavior of system (26) based on our approach. The approach of this work may be used to study other hyperchaotic complex nonlinear systems, e.g. Lorenz and Chen, which appear in several applications in engineering, physics, and other applied sciences (see, e.g. Refs. [17], [34]–[39], [42], and [43]).

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