The mean first passage time of a three-level atomic optical bistable system subjected to noise*

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The transient properties of a three-level atomic optical bistable system in the presence of multiplicative and additive noises are investigated. The explicit expressions of the mean first-passage time (MFPT) of the transition from the high intracavity intensity state to the low one are obtained by numerical computations. The impacts of the intensities of the multiplicative noise $D_{\rm M}$ and the additive noise $D_{\rm A}$, the intensity of correlation between two noises λ, and the intensity of the incident light y on the MFPT are discussed, respectively. Our results show: (i) for the case of no correlation between two noises ($\lambda = 0.0$), the increase in $D_{\rm M}$ and $D_{\rm A}$ can lead to an increase in the probability of the transition to the low intracavity intensity state, while the increase in y can lead to a retardation of the transition; and (ii) for the case of correlation between two noises ($\lambda \neq 0.0$), the increase in λ can cause an increase in the probability of the transition, and the increase in $D_{\rm A}$ can cause a retardation of the transition firstly and then an increase in the probability of the transition, i.e., the noise-enhanced stability is observed for the case of correlation between two noises.

Keywords: additive noise, multiplicative noise, correlations between two noises, mean first passage time

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1. Introduction

It is well known that the effects of noise on nonlinear dynamical systems have been extensively studied from both theoretical and experimental points of view.[1] In particular, phenomena of such noise-induced transitions, stochastic resonance, and noise-enhanced transport or noise-sustained synchronization have been observed in diverse range of systems in physics, chemistry, biology, and medicine.[2–6] Many of these phenomena have been intensively studied in a two-level atomic optical bistable system, due to its potential applications in all-optical switches, memories, and optical transistors.[7–11] In the case of an optical bistable system, the simplest model which describes amplitude fluctuations in this system is given by a Langevin equation with additive noise,[7,8] and then the stochastic resonance of the same model which was driven by multiplicative noise has studied.[9] Recently, the transient properties and stochastic resonance in the optical bistable system driven by multiplicative and additive noises as well as no correlations in between have been studied.[10–13] A similar model with correlations between multiplicative and additive noises has also been considered likewise in Refs. [14] and [15]. However, there are few experimental investigations of these predicted effects in two-level systems due to the lack of controllability of experimental parameters and, therefore, the accessibility of the appropriate regimes of operation. In contrast, in optical bistability associated with electromagnetically induced transparency in three-level atomic systems inside an optical cavity, both the absorption and dispersion properties of the intracavity medium can be modified and controlled by the additional coupling field. This makes it possible to manipulate the shape of the bistability curve or, equivalently, the width and height of the double-well bistable potential.[16–20] These features make the three-level atomic media ideal candidates for studying noise-induced transitions[21–23] and stochastic resonance[5,6,24–29] in the presence of additive or multiplicative noise. A recent investigation by Wu and co-workers[30] interpreted experimental observations of multiplicative noise-induced transitions in a three-level atomic optical bistable system, they only concentrated on the effect of the multiplicative noise on the system dynamics. However, a system is always disturbed by external environmental random perturbations (i.e., multiplicative noise) and the intrinsic thermal fluctuations (i.e., additive noise) simultaneously, and there are in certain situations where the strong external perturbations can lead to some changes in the internal structure of the system and thus may be correlated with each other as well.[31–34]

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On the level of a Langevin- or Fokker–Planck-type description of a dynamical system, the presence of correlations between two noises can change the dynamics of the system.[35,36] Correlated noise processes have made their presence felt in a wide range of applications, such as the statistical properties of a single mode laser,[37–39] bistable kinetics,[40,41] tumor cell growth,[42,43] barrier crossing dynamics,[44,45] stochastic resonance in linear systems,[46] and steady state entropy production.[47] In the present work, we study the transient properties of the transition between the different stable states in a three-level atomic optical bistable system in the presence of multiplicative and additive noises. Our results show that, in the case of correlation between two noises, the additive noise intensity is different from the case with no correlation.

This paper is organized as follows. In Section 2, the expressions of the mean first-passage time (MFPT) for both the case of no correlation between two noises and the case of correlations between two noises are derived by using the steepest descent method, and then the effects of the multiplicative noise intensity $D_{\rm M}$, the additive noise intensity $D_{\rm A}$, the incident light intensity y, and the intensity λ of correlation between two noises are discussed in detail. Finally, a conclusion is given in Section 3.

2. The model

We investigate the effects of noises in a regime of optical bistability in a three-level system, where both absorption and dispersion coexist and are important. Two mechanisms can lead to optical bistability in the system under consideration. The first involves nonlinear absorption and the second involves nonlinear refraction due to Kerr nonlinearity. The deterministic equation of motion for intracavity intensity can be written as[30]

$$\dot{x} = y - x(c_1 + c_2 x + c_3 x^2) = -\frac{{\rm d}V(x)}{{\rm d}x}, \tag{1}$$

where y is the incident light intensity, x is the intracavity light intensity, c_1, c_2, and c_3 are the constant parameters. The potential $V(x)$ corresponding to Eq. (1) is

$$V(x) = -yx + x^2\left(\frac{c_1}{2} + \frac{c_2}{3}x + \frac{c_3}{4}x^2\right), \tag{2}$$

which has two steady stable states $x_1 = [c_2 + \sqrt{A}(\cos(\theta/3) + \sqrt{3}\sin(\theta/3))]/(-3c_3)$, $x_2 = [c_2 - 2\sqrt{A}\cos(\theta/3)]/(-3c_3)$, and one unstable steady state $x_u = [c_2 + \sqrt{A}(\cos(\theta/3) - \sqrt{3}\sin(\theta/3))]/(-3c_3)$, where $A = c_2^2 - 3c_1c_3$, $B = c_1c_2 + 9c_3y$, $T = [3c_3B - 2c_2A]/(2A^{3/2})$, and $\theta = \arccos T$. A bistable potential of $V(x)$ is plotted in Fig. 1. Next, we consider the effects of multiplicative and additive noises in a three-level atomic optical bistable system.

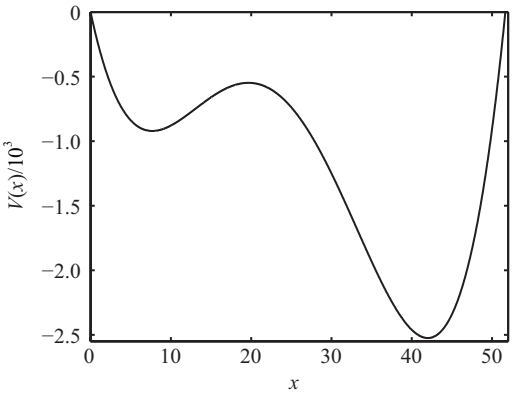


Fig. 1. The bistable potential of Eq. (2). The parameter values are $y = 292$, $c_1 = 59.79$, $c_2 = -3.19$, and $c_3 = 0.046$.

2.1. The case of no correlations between two noises

In the presence of no correlation between multiplicative and additive noises this deterministic equation (1) is replaced by the Langevin equation[30]

$$\dot{x} = y - x(c_1 + c_2 x + c_3 x^2) + x\xi(t) + \eta(t), \tag{3}$$

where $\xi(t)$ and $\eta(t)$ are Gaussian white noise and their statistical properties are given by

$$\begin{aligned} &\langle \xi(t) \rangle = 0, \ \ \langle \xi(t)\xi(t') \rangle = 2D_{\rm M}\delta(t-t'), \\ &\langle \eta(t) \rangle = 0, \ \ \langle \eta(t)\eta(t') \rangle = 2D_{\rm A}\delta(t-t'), \end{aligned} \tag{4}$$

in which $D_{\rm M}$ is the intensity of the multiplicative noise due to the external environmental random perturbations, and $D_{\rm A}$ is the intensity of the additive noise due to the intrinsic spontaneous emission fluctuations. To investigate effects of these noises on the three-level atomic optical bistable system, in this section we consider that the noises $\xi(t)$ and $\eta(t)$ in Eq. (3) are independent of each other, that is,

$$\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = 0. \tag{5}$$

Thus, the approximate Fokker–Planck equation corresponding to Eq. (3) with Eqs. (4) and (5) is derived by using the Novikov theorem[48,49] and Fox approach.[50] It is given as follows:[40]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t), \tag{6}$$

where

$$\begin{aligned} A(x) &= y - x(c_1 + c_2 x + c_3 x^2) + D_{\rm M}x, \\ B(x) &= D_{\rm M}x^2 + D_{\rm A}. \end{aligned} \tag{7}$$

From Eqs. (6) and (7), the stationary probability distribution function (SPDF) $P_{\rm st}(x)$ can be derived

$$P_{\rm st}(x) = \frac{N}{\sqrt{D_{\rm M}x^2 + D_{\rm A}}} \exp\left[-\frac{U(x)}{D_{\rm M}}\right], \tag{8}$$

where $U(x)$ is the generalized potential given by

$$U(x) = \frac{c_3}{2}x^2 + c_2 x + \frac{c_1 - c_3 R}{2}\ln(x^2 + R)$$

$$-\frac{c_2R+y}{\sqrt{R}}\arctan\left[\frac{x}{\sqrt{R}}\right], \tag{9}$$

N is the normalization constant, and $R = D_A/D_M$ is the ratio of noise intensities. The extrema of the SPDF obey a general equation $A(x) - B'(x) = 0$, and we obtain

$$y = x(c_1 + D_M + c_2x + c_3x^2). \tag{10}$$

In the case of no correlation between multiplicative and additive noises, we have the extrema of the SPDF for only multiplicative noise processes, the additive noise has no effect on the position of the extrema of the SPDF [see Eq. (10)]. The x as a function of y for different values of D_M exhibits the S-shape curves (i.e., the area of optical bistability), as shown in Fig. 2. Here D_M characterizes the intensity of the multiplicative noise due to the external environmental random perturbations. The curve at $y = 250$ shows three x values. However as the value of D_M increases, the curve changes, i.e., the curve at $y = 250$ gives only one x value. Therefore, the intensity of the multiplicative noise can change the state of the system, namely, noise-induced phase transition.

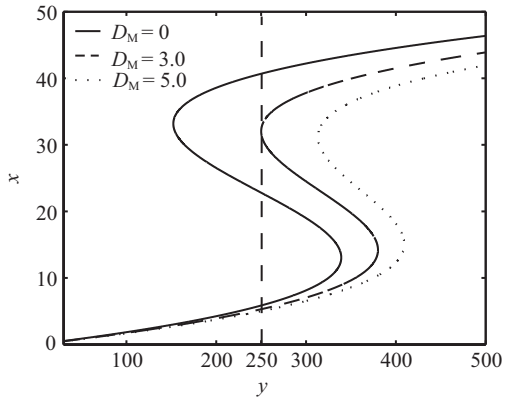


Fig. 2. The intracavity light intensity x as a function of the incident light intensity y for different values of the multiplicative noise intensity D_M due to the external environmental perturbations [according to Eq. (10)]. The other parameters are the same as those in Fig. 1.

Now we are interested in the time from one state to the other state under the influence of environmental random perturbations. Random fluctuations present in this system can induce transitions between those two states. From this point of view, it is interesting to study the mean first passage time (MFPT) of transition from one state (x_2) to the other state (x_1). Using the steepest descent method,[51,52] the explicit expression for the MFPT of the process $x(t)$ to reach the low intracavity intensity state x_1 with the initial condition $x(t = 0) = x_2$ (the high intracavity intensity state). This is given by

$$T = \frac{2\pi}{\sqrt{|V''(x_u)V''(x_2)|}}\exp\left[\frac{U(x_u) - U(x_2)}{D_M}\right]. \tag{11}$$

Note that the above result is valid only when the intensity of two types of noises, measured by D_M and D_A, is small in comparison with the energy barrier height $\triangle U = |\, U(x_u) - U(x_2)\,|$.[53,54] It provides the restriction on the parameters (such as y, c_1, c_2, c_3, D_M, and D_A). The following results in this study are restricted in valid regions. Here, let us stress that a decrease in the MFPT (11) is tantamount to an increase in the probability of the transition to the low intracavity intensity state, while an increase in the MFPT corresponds to a retardation of the transition to the low intracavity intensity state.

In Figs. 3 and 4, the MFPT is presented as a function of the additive noise intensity D_A and the multiplicative noise intensity D_M, respectively. A nearly linear decrease of the MFPT as the function of D_A is observed in Fig. 3, which indicates an increase in the probability of the transition to the low intracavity intensity state. The increase in D_M leads to a decrease in the MFPT, as shown in Fig. 4. This decay is very strong in the case of a large multiplicative noise intensity D_M. The MFPT as a function of y for different values of the multiplicative noise intensity D_M is shown in Fig. 5. An increase in y leads to an increase in the MFPT, which indicates a retardation of the transition to the low intracavity intensity state. But the increase in D_M is related to an increase in the probability of the transition to the low intracavity intensity state.

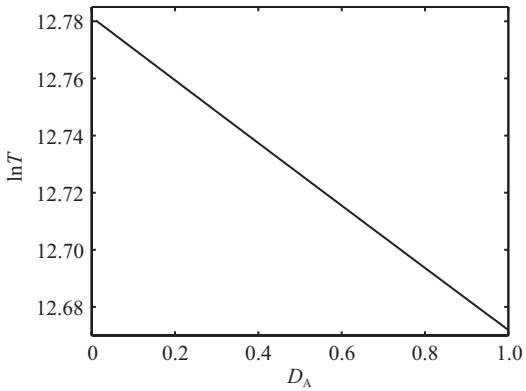


Fig. 3. The MFPT as a function of the additive noise intensity D_A due to the intrinsic spontaneous emission fluctuations [according to Eq. (11)] with $D_M = 0.15$. The other parameters are the same as those in Fig. 1.

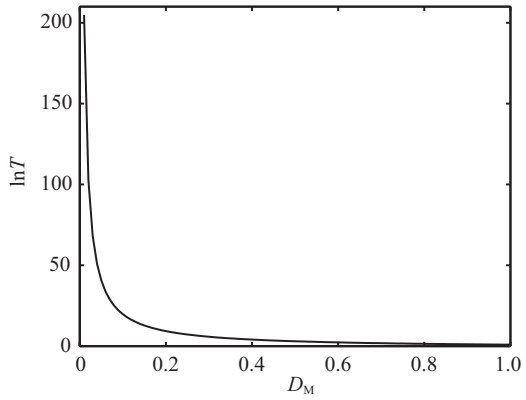


Fig. 4. The MFPT as a function of the multiplicative noise intensity D_M [according to Eq. (11)] with $D_A = 0.15$. The other parameters are the same as those in Fig. 1.

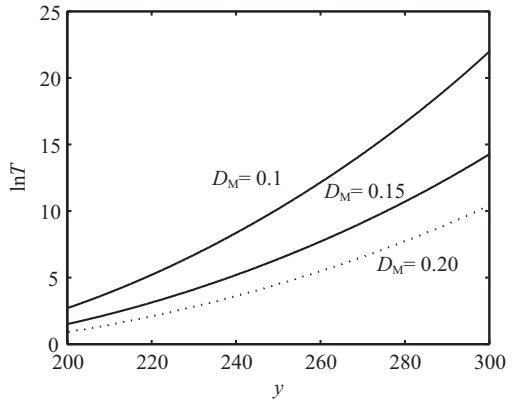


Fig. 5. The MFPT as a function of the incident light intensity y for different values of the multiplicative noise intensity D_M [according to Eq. (11)] with $D_A = 0.15$. The other parameters are the same as those in Fig. 1.

2.2. The case of correlations between two noises

In this section, consider a stochastic three-level atomic optical bistable system with correlation between multiplicative and additive noises, and the correlation form between two noises is assumed as follows:[54–57]

$$\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = 2\lambda\sqrt{D_M D_A}\,\delta(t-t'), \quad (12)$$

where λ is the intensity of correlation between two noises. The Fokker–Planck equation corresponding to Eq. (3) with Eqs. (4) and (12) can be written as

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t), \quad (13)$$

where

$$\begin{aligned} A(x) &= y - x(c_1 + c_2 x + c_3 x^2) + D_M x + \lambda\sqrt{D_M D_A}, \\ B(x) &= D_M x^2 + 2\lambda\sqrt{D_M D_A}\,x + D_A. \end{aligned} \quad (14)$$

The SPDF $P_{st}(x)$ can be derived as

$$P_{st}(x) = \frac{\widehat{N}}{\sqrt{D_M x^2 + 2\lambda\sqrt{D_M D_A}\,x + D_A}} \exp\left[-\frac{\widehat{U}(x,\lambda)}{D_M}\right], \quad (15)$$

where $\widehat{N}$ is the normalization constant and $\widehat{U}(x,\lambda)$ is

$$\begin{aligned} \widehat{U}(x,\lambda) = {} & \frac{c_3}{2}x^2 + \beta_1 x + \beta_2 \ln(x^2 + 2\lambda\sqrt{R}x + R) \\ & + \frac{\beta_3}{\sqrt{R(1-\lambda^2)}} \arctan\left[\frac{x+\lambda\sqrt{R}}{\sqrt{R(1-\lambda^2)}}\right], \end{aligned} \quad (16)$$

and here

$$\begin{aligned} \beta_1 &= c_2 - 2\lambda c_3\sqrt{R}, \\ \beta_2 &= \frac{c_3}{2}(4\lambda^2 - 1)R - \lambda c_2\sqrt{R} + \frac{c_1}{2}, \\ \beta_3 &= c_2(2\lambda^2 - 1)R - \lambda c_3(4\lambda^2 - 3)\sqrt{R}R - \lambda c_1\sqrt{R} - y, \end{aligned}$$

and $R = D_A/D_M$ is the ratio of noise intensities. Equations (13)–(16) are in agreement with Eqs. (14) and (15) obtained in Ref. [30] by setting $\lambda = 0$, and $D_A = 0$. The extrema of the SPDF obey $A(x) - B'(x) = 0$,

$$y = x(c_1 + D_M + c_2 x + c_3 x^2) + \lambda\sqrt{D_M D_A}. \quad (17)$$

In the presence of correlation between multiplicative and additive noises, we have the extrema of the SPDF in Eq. (17), which shows that the additive noise intensity D_A can also affect the position of the extrema of the SPDF. The S-shape curves (i.e., the area of optical bistability) with D_A is shown in Fig. 6. It is clear that the area of optical bistability moves to left with increase of D_A for the case of strongly negative correlated noises [see $\lambda = -0.9$ in Fig. 6(a)], however for the case of strongly positive correlated noises [see $\lambda = 0.9$ in Fig. 6(b)], the area of optical bistability moves to right with increase of D_A.

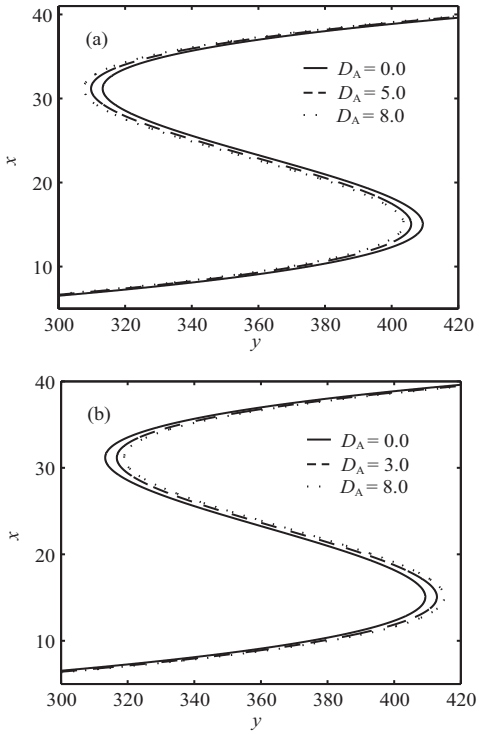


Fig. 6. The intracavity light intensity x as a function of the incident light intensity y for different values of the additive noise intensity D_A [according to Eq. (17)] with $D_M = 5.0$. (a) $\lambda = -0.9$; (b) $\lambda = 0.9$. The other parameters are the same as those in Fig. 1.

Then we can obtain the explicit expression for the MFPT of the process $x(t)$ to reach the low intracavity intensity state x_1 with the initial condition $x(t=0) = x_2$ (the high intracavity intensity state) for the case of correlations between multiplicative and additive noises:

$$T = \frac{2\pi}{\sqrt{|V''(x_u)V''(x_2)|}} \exp\left[\frac{\widehat{U}(x_u,\lambda) - \widehat{U}(x_2,\lambda)}{D_M}\right], \quad (18)$$

in which the intensity of the two types of fluctuations, measured by D_M and D_A, is small in comparison with the energy barrier height $\triangle\widehat{U} = |\,\widehat{U}(x_u,\lambda) - \widehat{U}(x_2,\lambda)\,|$.[53,54] It provides the restriction on the parameters (such as $y, c_1, c_2, c_3, D_M, D_A$, and λ).

In Fig. 7, the MFPT is shown depending on the parameter D_A according to Eq. (18). Here D_A characterizes the intensity of the additive noise due to the intrinsic spontaneous emission fluctuations. As a feature there exists a critical additive noise intensity D_A at which the MFPT of the transition to the low intracavity intensity state induced by noise is the largest. The maximum is more pronounced the stronger negative correlated noises is and, simultaneously, it is shifted toward larger values of D_A. This maximum for MFPT as functions of the noise intensity D_A identifies the noise-enhanced stability (NES). The NES was first found numerically by Dayan *et al.*[58] and observed experimentally by Mantegna and Spagnolo,[59] the effect called NES in Ref. [59] was observed in a periodically driven system with a single metastable minimum. In our case, this maximum for MFPT implies that the high intracavity intensity state can be enhanced by the noise, and the mean lifetime of the high intracavity intensity state is longer than the deterministic decay time.

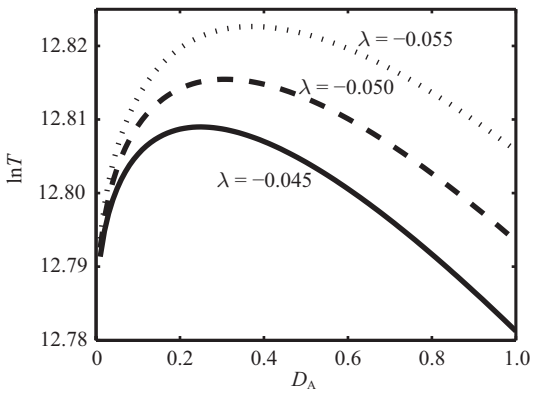


Fig. 7. The MFPT as a function of the additive noise intensity D_A for different values of the cross-correlation intensity λ [according to Eq. (18)] with $D_M = 0.15$ and $y = 292$. The other parameters are the same as those in Fig. 1.

In Fig. 8, the MFPT is presented as a function of the cross-correlation intensity λ for different values of D_M [see Fig. 8(a)] and for different values of D_A [see Fig. 8(b)], respectively. From Fig. 8(a), we can clearly see that the increase in both parameters λ and D_M leads to an increase in the probability of the transition to the low intracavity intensity state. The effects of the additive noise intensity D_A on the MFPT is offered in Fig. 8(b). When D_A approaches to zero, we can obtain no impact of the cross-correlation intensity λ on the MFPT from the inset of Fig. 8(b) (i.e., $D_A = 0.0001$), which is also consistent with Eq. (12). As the value of D_A increases (see Fig. 8(b)), it is clearly seen that there is a critical value of the cross-correlation intensity $\lambda_c \approx 0.1$. For the case of $\lambda < \lambda_c$, the increase in D_A leads to a retardation of the transition to the low intracavity intensity state. However for the case of $\lambda > \lambda_c$, the increase in D_A leads to an increase in the probability of the transition to the low intracavity intensity state.

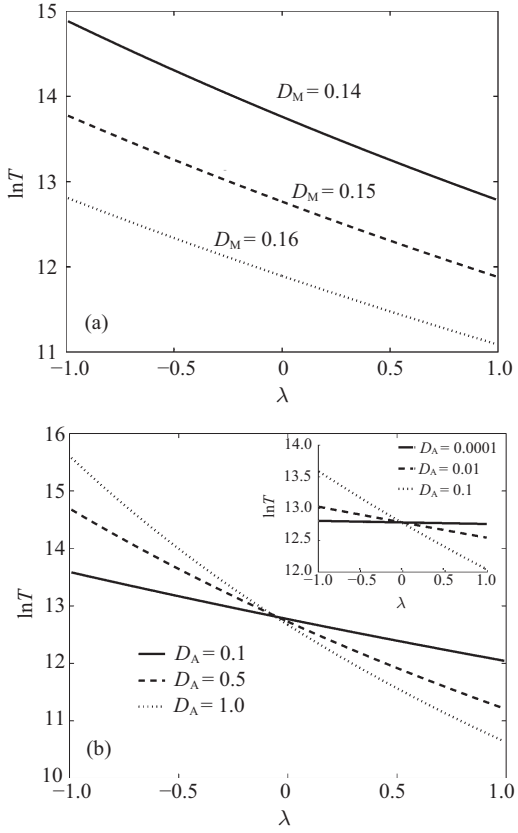


Fig. 8. The MFPT as a function of the cross-correlation intensity λ for different values of the multiplicative noise intensity D_M (a) and for different values of the additive noise intensity D_A (b) [according to Eq. (18)]. (a) $D_A = 0.15$, and $y = 292$; (b) $D_M = 0.15$, and $y = 292$. The other parameters are the same as those in Fig. 1.

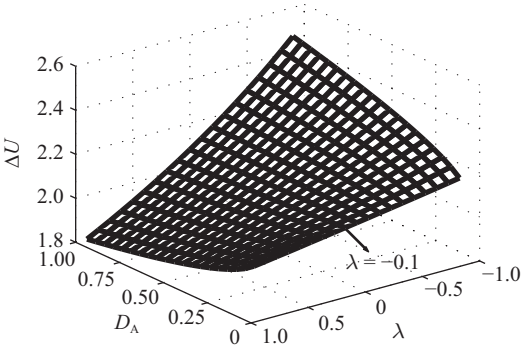


Fig. 9. The three-dimensional curve of the difference between the potential well ΔU as functions of λ and D_A with $D_M = 0.15$ and $y = 292$. The other parameters are the same as those in Fig. 1.

To provide some insight into the above-mentioned phenomenon, we define the difference between the potential well at x_1 and that at x_2 as $\Delta U = |\widehat{U}(x_1) - \widehat{U}(x_2)|$, and plot three-dimensional curve of the difference between the potential well ΔU as functions of λ and D_A in Fig. 9. The greater the difference between the potential well, the longer the transition time from x_2 to x_1, and *vice versa*. The difference between the potential well decreases as the cross-correlation intensity λ increases, thus the MFPT decreases as λ increases (see Fig. 8(b)). When the cross-correlation intensity $\lambda_c \approx 0.1$ (see Fig. 9), the ΔU is unchanged as D_A increases. For $\lambda < \lambda_c$, the

ΔU increases as D_A increases. However for $\lambda > \lambda_c$, the ΔU decreases as D_A increases. Therefore, the point of intersection $\lambda_c \approx 0.1$ is in Fig. 8(b), for $\lambda < \lambda_c$, the MFPT increases with D_A, while for $\lambda > \lambda_c$, the MFPT decreases with D_A.

3. Concluding remarks

In this paper, the transient properties of the transition between the different stable states in a three-level atomic optical bistable system is investigated by using the steepest descent method.[51,52] We have considered both, the case of no correlation between multiplicative and additive noises and that of correlation between two noises. Based on the expressions of the MFPT for both cases, the effects of the multiplicative noise intensity D_M, the additive noise intensity D_A, and the intensity λ of correlations between two noises are discussed. In the case of no correlation between two noises, we have the extrema of the SPDF for only multiplicative noise processes, the additive noise has no effect on the position of the extrema of the SPDF. However, the presence of correlation between two noises changes this picture, not only the multiplicative noise has effect on the position of the extrema of the SPDF, but also the additive noise can also affect it.

In the case of no correlation between two noises, the effects of the multiplicative noise intensity D_M and the additive noise intensity D_A on the MFPT are investigated, respectively. It is found that the increase in both parameters D_M and D_A leads to an increase in the probability of the transition to the low intracavity intensity state. However in the presence of correlations between two noises, the increase in λ can lead to an increase in the probability of the transition to the low intracavity intensity state, and the increase in D_A leads to a retardation of the transition to the low intracavity intensity state firstly and then an increase in the probability of the transition, i.e., a critical additive noise intensity D_A exists at which the MFPT of the transition process induced by noise is the largest. Moreover, in the case of correlation between two noises, the additive noise intensity is different from the case with no correlation. On the other hand, for the case of strongly negative correlated noises, the increase in D_A leads to a retardation of the transition to the low intracavity intensity state, namely, the noise-enhanced stability is observed. However for the case of strongly positive correlated noises, the increase in D_A leads to an increase in the probability of the transition to the low intracavity intensity state. Therefore, we anticipate that our theoretical results will stimulate development of the experimental works to verify the transient properties in three-level atomic optical bistable systems.

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