Phase-controlled atom–photon entanglement in a three-level Λ–type closed-loop atomic system

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We study the entanglement of dressed atom and its spontaneous emission in a three-level Λ-type closed-loop atomic system in a multi-photon resonance condition and beyond it. It is shown that the von Neumann entropy in such a system is phase-dependent, and it can be controlled by either the intensity or relative phase of applied fields. It is demonstrated that for the special case of the Rabi frequency of applied fields, the system is disentangled. In addition, we take into account the effect of Doppler broadening on the entanglement and it is found that a suitable choice of laser propagation direction allows us to obtain the steady state degree of entanglement (DEM) even in the presence of the Doppler effect.

Keywords: entanglement, quantum entropy, closed-loop system

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1. Introduction

Entanglement is one of the most intriguing features of quantum mechanics, which emanates from quantum correlation between different parts of a system. The concept of entanglement and the name go back to the famous EPR article and Schrödinger's rejoinder[1–4] in 1935 on the foundations of quantum measurement. Generally, a system consisting of two sub-systems is said to be entangled if its quantum state cannot be described by a simple product of the quantum states of the two components.[5] One consequence of this form of the wave function is that a measurement on one of the sub-systems yields information about the other sub-system.

Recently, it has become clear that the importance of the phenomenon extends well beyond fundamental questions about quantum theory; indeed, entanglement lies at the heart of most of the quantum-information processing tasks such as quantum computing,[6] quantum teleportation,[7,8] quantum cryptography,[9,10] and dense coding.[11]

Entanglement can occur as a result of interaction between different parts of a system. One of the fundamental interactions in physics is the interaction between light and matter. In recent years, a theoretical description of entanglement evolution between atoms and quantizeds field in the Jaynes–Cummings model has been proposed.[12–18] Moreover, entanglement of an atom and its spontaneous emission was investigated.[19] The effect of spontaneously generated coherence (SGC) on the atom–photon entanglement was also studied.[20] It was also shown that the induced steady state entanglement between an atom and its spontaneous emission can be controlled by the intensity and detuning of the applied field.[21] In another study, it was shown that the atom–photon entanglement depends on the relative phase of applied fields.[22]

Generally, atom–photon entanglement can be used in new interesting quantum concepts. "Many new concepts, for example, the quantum repeater[23] or quantum networks for distributed quantum computing thus require the faithful mapping of quantum information between photonic quantum channels and matter-based quantum memories and processors. Entanglement between matter and light is crucial for achieving this task."[24,25]

It has been demonstrated that the optical properties of a quantum system containing the closed-loop configuration in a multi-photon resonance condition are phase-dependent.[26–28] In this paper, we investigate the entanglement of a dressed atom and its spontaneous emission in a three-level Λ-type closed-loop atomic system. It is shown that the entanglement can be controlled by either the intensity or relative phase of applied fields. In addition, we find that under the special condition the atom and photons become disentangled. Note that the phase-dependent behavior, here, arises from the closed-loop configuration, but in our previous work[22] it has been applied by the spontaneously generated coherence of spontaneous emission.[29]

2. Model and solution

Consider a Λ-type three-level atomic system including two ground states $|1\rangle$ and $|2\rangle$ and an excited state $|3\rangle$ which is coupled by two coherent optical fields $\boldsymbol{E}_{\rm R}$ and $\boldsymbol{E}_{\rm L}$ and a microwave driving field $\boldsymbol{E}_{\rm m}$ as shown in Fig. 1. The left (right) field $\boldsymbol{E}_{\rm L} = E_{\rm L}\exp[-{\rm i}(\omega_{\rm L}t+\varphi_{\rm L})]+{\rm h.c}$ ($\boldsymbol{E}_{\rm R} = E_{\rm R}\exp[-{\rm i}(\omega_{\rm R}t+\varphi_{\rm R})]+{\rm h.c}$) with frequency $\omega_{\rm L}(\omega_{\rm R})$ and initial phase $\varphi_{\rm L}(\varphi_{\rm R})$ drives the transition $|1\rangle \leftrightarrow |3\rangle$ ($|2\rangle \leftrightarrow |3\rangle$) with the Rabi frequency $\Omega_{\rm L} = \boldsymbol{E}_{\rm L}.\boldsymbol{d}_{13}/\hbar$ ($\Omega_{\rm R} = \boldsymbol{E}_{\rm R}.\boldsymbol{d}_{23}/\hbar$),

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while the transition $|1\rangle \leftrightarrow |2\rangle$ is excited by a microwave driving field $\boldsymbol{E}_{\rm m} = E_{\rm m}\exp[-{\rm i}(\omega_{\rm m}t+\varphi_{\rm m})]+{\rm h.c}$ with frequency $\omega_{\rm m}$, initial phase $\varphi_{\rm m}$, and Rabi frequency $\Omega_{\rm m} = \boldsymbol{E}_{\rm m}.\boldsymbol{d}_{12}/\hbar$. It is well known that the existence of microwave driving field turns this Λ-type system into a closed-loop system. Here $E_{\rm L}(E_{\rm R})$ and $E_{\rm m}$ are the amplitudes of the left (right) and microwave driving fields, respectively. $\boldsymbol{d}_{12}$, $\boldsymbol{d}_{13}$, and $\boldsymbol{d}_{23}$ display the corresponding dipole moments of transitions. The spontaneous decay rates from $|3\rangle$ to level $|1\rangle$ ($|2\rangle$) is denoted by $2\gamma_{31}$ ($2\gamma_{32}$) while the spontaneous decay from level $|2\rangle$ to level $|1\rangle$ has been ignored.

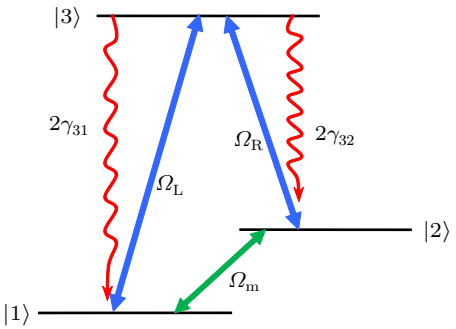


Fig. 1. (color online) A three-level Λ-type closed-loop atomic system driven by two coherent optical fields and a microwave field with corresponding Rabi frequencies $\Omega_{\rm L}$, $\Omega_{\rm R}$, and $\Omega_{\rm m}$, respectively.

Under the dipole and the rotating wave approximations, the total Hamiltonian of this system in the interaction picture reads

$$\begin{aligned} H \;=\; & -\hbar\Omega_{\rm L}\,{\rm e}^{-{\rm i}(\Delta_{\rm L}t+\varphi_{\rm L})}|3\rangle\langle 1| - \hbar\Omega_{\rm R}\,{\rm e}^{-{\rm i}(\Delta_{\rm R}t+\varphi_{\rm R})}|3\rangle\langle 2| \\ & -\hbar\Omega_{\rm m}\,{\rm e}^{-{\rm i}(\Delta_{\rm m}t+\varphi_{\rm m})}|2\rangle\langle 1| + {\rm H.c.}, \end{aligned} \tag{1}$$

where $\Delta_{\rm L} = \omega_{\rm L} - \omega_{31}$, $\Delta_{\rm R} = \omega_{\rm R} - \omega_{32}$, and $\Delta_{\rm m} = \omega_{\rm m} - \omega_{21}$ are the detunings between applied fields and the corresponding atomic transition frequencies. ω_{21}, ω_{31}, and ω_{32} denote corresponding frequencies of transitions $|1\rangle \leftrightarrow |2\rangle$, $|1\rangle \leftrightarrow |3\rangle$, and $|2\rangle \leftrightarrow |3\rangle$, respectively. The so-called three-photon detuning and relative phase are defined as $\Delta = \Delta_{\rm L} - \Delta_{\rm R} - \Delta_{\rm m}$ and $\varphi = \varphi_{\rm L} - \varphi_{\rm R} - \varphi_{\rm m}$, respectively. For simplicity we set $\hbar = 1$ and take the Rabi frequencies of three classical fields to be real frequencies.

The density matrix equations of motion for this system can be written as

$$\begin{aligned} \dot{\rho}_{11} =\; & {\rm i}\Omega_{\rm m}\rho_{21}\,{\rm e}^{-{\rm i}\varphi} + {\rm i}\Omega_{\rm L}^{*}\rho_{31}\,{\rm e}^{{\rm i}\Delta t} - {\rm i}\Omega_{\rm m}\rho_{12}\,{\rm e}^{{\rm i}\varphi} \\ & - {\rm i}\Omega_{\rm L}\rho_{13}\,{\rm e}^{-{\rm i}\Delta t} + 2\gamma_{31}\rho_{33}, \\ \dot{\rho}_{22} =\; & {\rm i}\Omega_{\rm m}\rho_{12}\,{\rm e}^{{\rm i}\varphi} + {\rm i}\Omega_{\rm R}\rho_{32} - {\rm i}\Omega_{\rm m}\rho_{21}\,{\rm e}^{-{\rm i}\varphi} - {\rm i}\Omega_{\rm R}\rho_{23} \\ & + 2\gamma_{32}\rho_{33}, \\ \dot{\rho}_{12} =\; & -{\rm i}(\Delta_{\rm m}\rho_{12} + {\rm i}\Omega_{\rm m}(\rho_{22}-\rho_{11})\,{\rm e}^{-{\rm i}\varphi} + {\rm i}\Omega_{\rm L}^{*}\rho_{32}\,{\rm e}^{{\rm i}\Delta t} \\ & - {\rm i}\Omega_{\rm R}\rho_{13}, \\ \dot{\rho}_{13} =\; & -{\rm i}(\Delta_{\rm L}-\Delta)\rho_{13} + {\rm i}\Omega_{\rm L}^{*}(\rho_{33}-\rho_{11})\,{\rm e}^{{\rm i}\Delta t} \\ & + {\rm i}\Omega_{\rm m}\rho_{23}\,{\rm e}^{-{\rm i}\varphi} - {\rm i}\Omega_{\rm R}\rho_{12} - (\gamma_{31}+\gamma_{32})\rho_{13}, \\ \dot{\rho}_{23} =\; & -{\rm i}(\Delta_{\rm R}\rho_{23} + {\rm i}\Omega_{\rm m}\rho_{13}\,{\rm e}^{{\rm i}\varphi} + {\rm i}\Omega_{\rm R}(\rho_{33}-\rho_{22}) \\ & - {\rm i}\Omega_{\rm L}^{*}\rho_{21}\,{\rm e}^{{\rm i}\Delta t} - (\gamma_{31}+\gamma_{32})\rho_{23}, \\ & \rho_{11}(t)+\rho_{22}(t)+\rho_{33}(t) = 1. \end{aligned} \tag{2}$$

Now, we seek the corresponding steady state analytical solution for elements of the density matrix for $\gamma_{31} = \gamma_{32} = \gamma = 1$ and $\Omega_{\rm R} = \Omega_{\rm L} = \Omega_0$. The population and coherence terms of the density matrix for $\Delta_{\rm L} = \Delta_{\rm R} = \Delta_{\rm m} = 0.0$ are given by

$$\begin{aligned} \rho_{11} &= \frac{\Omega_{\rm m}^4 + \Omega_0^4 - \Omega_{\rm m}^2(-4+\Omega_0^2) - \Omega_{\rm m}\Omega_0^2(\Omega_{\rm m}\cos[2\varphi] + 2\sin[\varphi])}{2\Omega_{\rm m}^4 + 2\Omega_0^4 - \Omega_{\rm m}^2(-8+\Omega_0^2) - 3\Omega_{\rm m}^2\Omega_0^2\cos[2\varphi]}, \\ \rho_{22} &= \frac{\Omega_{\rm m}^4 + \Omega_0^4 - \Omega_{\rm m}^2(-4+\Omega_0^2) + \Omega_{\rm m}\Omega_0^2(-\Omega_{\rm m}\cos[2\varphi] + 2\sin[\varphi])}{2\Omega_{\rm m}^4 + 2\Omega_0^4 - \Omega_{\rm m}^2(-8+\Omega_0^2) - 3\Omega_{\rm m}^2\Omega_0^2\cos[2\varphi]}, \\ \rho_{12} &= \frac{({\rm e}^{-2{\rm i}\varphi}\Omega_{\rm m}^2(-4-\Omega_{\rm m}^2+\Omega_0^2) - (\Omega_{\rm m}^4 + 2\Omega_0^4 + \Omega_{\rm m}^2(4-3\Omega_0^2)))}{2(2\Omega_{\rm m}^4 + 2\Omega_0^4 - \Omega_{\rm m}^2(-8+\Omega_0^2) - 3\Omega_{\rm m}^2\Omega_0^2\cos[2\varphi])}, \\ \rho_{13} &= \frac{(1-{\rm e}^{-2{\rm i}\varphi})\Omega_{\rm m}\Omega_0(2{\rm i}\Omega_{\rm m} + {\rm e}^{{\rm i}\varphi}(\Omega_{\rm m}^2 - \Omega_0^2))}{-2(2\Omega_{\rm m}^4 + 2\Omega_0^4 - \Omega_{\rm m}^2(-8+\Omega_0^2) - 3\Omega_{\rm m}^2\Omega_0^2\cos[2\varphi])}, \\ \rho_{23} &= \frac{2{\rm i}\sin[\varphi]\Omega_{\rm m}\Omega_0(\Omega_0^2 - \Omega_{\rm m}(\Omega_{\rm m} + 2{\rm i}\cos[\varphi] - 2\sin[\varphi]))}{-2(2\Omega_{\rm m}^4 + 2\Omega_0^4 - \Omega_{\rm m}^2(-8+\Omega_0^2) - 3\Omega_{\rm m}^2\Omega_0^2\cos[2\varphi])}, \\ & \rho_{11}+\rho_{22}+\rho_{33} = 1. \end{aligned} \tag{3}$$

It is clear that all expressions in Eq. (3) are phase-dependent.

3. Evolution of entropy and atom–photon entanglement

Think over a bipartite quantum system described by a density operator ρ_{AF} in a tensor product space $\boldsymbol{C}_A \otimes \boldsymbol{C}_F$. $\boldsymbol{C}_{A(F)}$ is in the Hilbert space of the subsystem $A(F)$. The partial density operator of one part will be obtained by tracing over the other as

$$\rho_{A(F)} = {\rm Tr}_{F(A)}(\rho_{AF}). \tag{4}$$

This bipartite quantum system is called being separable, if its density operator can be written as

$$\rho_{AB} = \rho_A \otimes \rho_B, \tag{5}$$

otherwise it is called an entangled state.[30]

In addition to the generation of entangled states, measurement of the entanglement in a general composite system is a more interesting problem facing researchers at the moment. Here we employ the von Neumann entropy S as a unique measure to estimate the amount of atom–field entanglement in our system. For a system in quantum state ρ, this unique measure is defined as[31]

$$S = -\mathrm{Tr}\,\rho \ln \rho. \tag{6}$$

Araki and Lieb in their fantastic paper[32] showed that for a bipartite quantum system composed of two subsystems A and F (say, the atom and field) at any time t, the system and subsystem's entropies satisfy a remarkable inequality as follows:

$$|S_A(t) - S_F(t)| \leq S_{AF}(t) \leq |S_A(t) + S_F(t)|, \tag{7}$$

where S_{AF} is the total entropy of the composite system and $S_{A(F)}(t) = -\mathrm{Tr}[\rho_{A(F)}(t) \ln \rho_{A(F)}(t)]$ is partial entropy corresponding to a reduced density operator.

Based on Eq. (6), for a closed atom–field system in which both of them start from pure state, the entropies of two interacting subsystems will be exactly equal at all times after the interaction between two subsystems has been switched on. Then Phoenix and Knight[33,34] have shown that under the aforementioned circumstances, information about any of the subsystems is an indication of the entanglement of the whole system, namely a decrease in partial entropy means that each subsystem evolves towards a pure quantum state, whereas a rise in partial entropy means that the two components tend to lose their individualities and become correlated or entangled. The degree of entanglement (DEM) for atom–field entanglement is defined as

$$\mathrm{DEM}(t) = S_A = S_F = -\Big(\sum_{j=1}^{3} \lambda_j \ln \lambda_j\Big), \tag{8}$$

where λ_j is the eigenvalue of the reduced density matrix of the atom.

4. Results and discussion

In this section, we depict our results for the behavior of the system in a three-photon resonance condition and beyond it. All parameters are reduced to dimensionless units through scaling by $\gamma_{31} = \gamma_{32} = \gamma$ and all figures are plotted in the unit of γ. We first discuss the dynamical behavior of the reduced atomic entropy via different values of intensity of applied fields.

In Fig. 2, we plot the dynamical behaviors of DEM in the multi-photon resonance condition. The using parameters are $\Delta_{\mathrm{L}} = \Delta_{\mathrm{R}} = \Delta_{\mathrm{m}} = 0.0\gamma$, $\Omega_{\mathrm{L}} = 0.25\gamma$, $\Omega_{\mathrm{R}} = 0.25\gamma$ (solid), 0.5γ (dashed), and $\Omega_{\mathrm{m}} = 0$ (a), 0.5γ (b). Figure 2(a) shows that in the absence of a microwave field the steady state entropy becomes zero. Figure 2(b) displays that by applying the microwave field to the lower levels, for $\Omega_{\mathrm{L}} \neq \Omega_{\mathrm{R}}$ (dashed) the non-zero steady state entropy is established, while for $\Omega_{\mathrm{L}} = \Omega_{\mathrm{R}}$ (solid) the system becomes disentangled. Thus, the steady state DEM depends on the intensity of applied fields, and it can be controlled by intensity of either the coherent optical or microwave field.

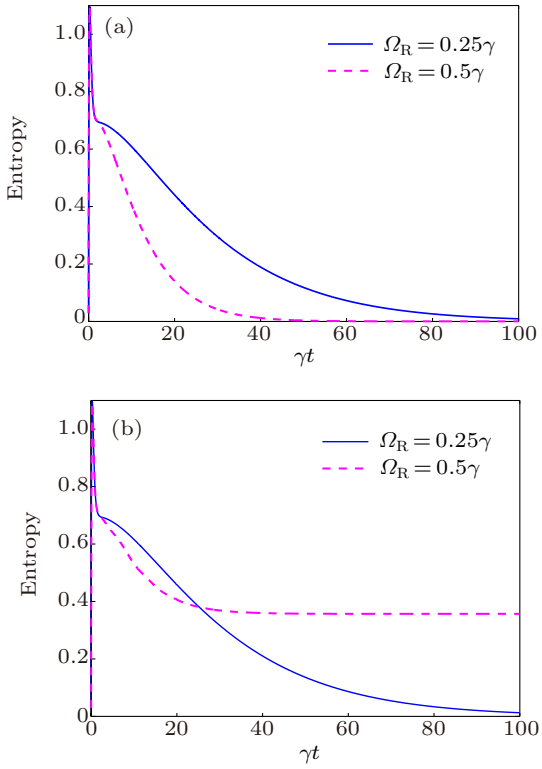


Fig. 2. (color online) Dynamical behaviors of the reduced atomic entropy via different values of the intensity of applied fields in multi-photon resonance condition ($\Delta = 0$). The selected parameters are $\Delta_{\mathrm{L}} = \Delta_{\mathrm{R}} = \Delta_{\mathrm{m}} = 0.0\gamma$, $\Omega_{\mathrm{L}} = 0.25\gamma$, $\Omega_{\mathrm{R}} = 0.25\gamma$ (solid), and 0.5γ (dashed), and $\Omega_{\mathrm{m}} = 0$ (a) and 0.5γ (b).

The relative phase of applied fields is another important parameter to control the DEM in a closed-loop atomic system. In Fig. 3, we display the time evolutions of DEM for different values of relative phase of applied fields in the multi-photon resonance condition. The using parameters are $\Omega_{\mathrm{m}} = 0.5\gamma$, $\Omega_{\mathrm{L}} = 0.25\gamma$, $\Omega_{\mathrm{R}} = 0.25\gamma$ (a), 0.5γ (b), $\Delta_{\mathrm{L}} = \Delta_{\mathrm{R}} = \Delta_{\mathrm{m}} = 0.0\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted). It is shown that the steady state DEM depends on the relative phase of applied fields, and it can be controlled by either intensities or relative phase of applied fields. The interesting result is obtained for $\varphi = 0.0$ (solid) in which for $\Omega_{\mathrm{L}} = \Omega_{\mathrm{R}}$ (Fig. 3(a)) the steady state entropy becomes zero while a nonzero entropy is obtained for $\Omega_{\mathrm{L}} \neq \Omega_{\mathrm{R}}$ (Fig. 3(b)). Note that in the absence of a microwave driving field, the closed-loop configuration is broken, and the DEM does not depend on the relative phase of applied fields.

To manifest the phase-dependent behavior of entanglement, in Fig. 4 we plot DEM versus the relative phase of applied fields in the steady-state when $\gamma_{31} = \gamma_{32} = \gamma = 1$, $\Omega_{\mathrm{m}} = 0.5\gamma$, $\Omega_{\mathrm{L}} = 0.25\gamma$, $\Omega_{\mathrm{R}} = 0.25\gamma$ (solid), 0.5γ (dashed),

$\Delta_L = \Delta_R = 0$, and $\Delta_m = 0$. One can see that the entanglement shows periodic behavior versus the relative phase of applied fields. The interesting behavior is obtained for $\Omega_R = \Omega_L$, in which the system is disentangled for $\varphi = n\pi$, $(n = 0, \pm 1, \ldots)$.

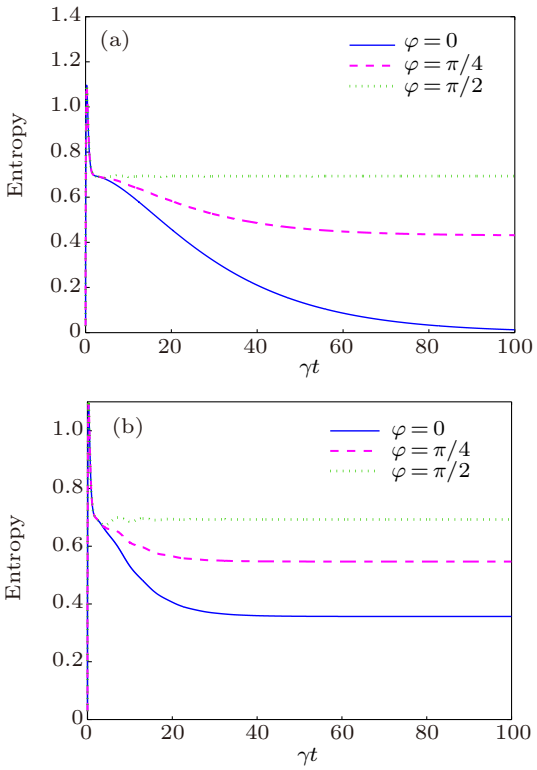


Fig. 3. (color online) Time evolutions of DEM for different relative phases of applied fields in the multi-photon resonance condition. The parameters are $\Omega_m = 0.5\gamma$, $\Omega_L = 0.25\gamma$, $\Omega_R = 0.25\gamma$ (a) and 0.5γ (b), $\Delta_L = \Delta_R = \Delta_m = 0.0\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted).

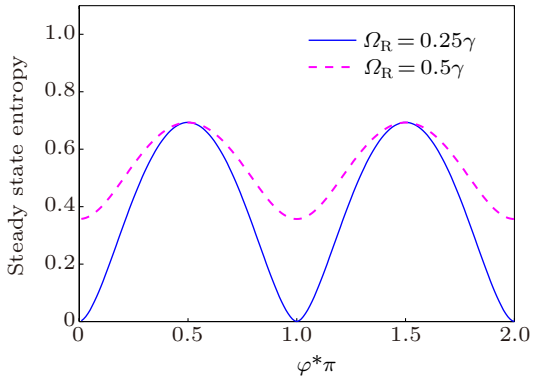


Fig. 4. (color online) Curves of steady state DEM versus relative phase of applied fields for $\gamma_{31} = \gamma_{32} = \gamma = 1$, $\Delta_L = \Delta_R = 0$, $\Delta_m = 0$, $\Omega_m = 0.5\gamma$, $\Omega_L = 0.25\gamma$, and $\Omega_R = 0.25\gamma$ (solid) and 0.5γ (dashed).

In Fig. 5, we display the steady state DEM versus the relative Rabi frequency of optical fields ($\Omega = \Omega_R/\Omega_L$) when other parameters are $\gamma_{31} = \gamma_{32} = \gamma = 1$, $\Delta_L = \Delta_R = \Delta_m = 0.0\gamma$, $\Omega_L = 0.25\gamma$, $\Omega_m = 0.5\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted). It is found out that disentanglement just occurs for equal Rabi frequency of left and right fields in the zero relative phase of applied fields.

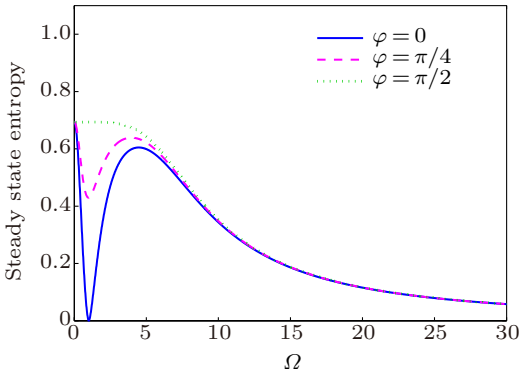


Fig. 5. (color online) Curves of steady state DEM versus the relative Rabi frequency of optical fields ($\Omega = \Omega_R/\Omega_L$).The parameters are $\gamma_{31} = \gamma_{32} = \gamma = 1$, $\Delta_L = \Delta_R = \Delta_m = 0.0\gamma$, $\Omega_L = 0.25\gamma$, $\Omega_m = 0.5\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted).

The physics behind the phenomena can be explained via the population distribution of dressed states. The dressed states of the system in the absence of a microwave field are given by

$$|\varphi\rangle = \frac{1}{\sqrt{\Omega_L^2 + \Omega_R^2}}(\Omega_R|1\rangle - \Omega_L|2\rangle),$$
$$|\psi\rangle = \frac{1}{\sqrt{\Omega_L^2 + \Omega_R^2}}(\Omega_L^*|1\rangle + \Omega_R^*|2\rangle) \quad \text{and} \quad |3\rangle. \tag{9}$$

By applying the microwave driving field to the system, the dressed states for $\varphi = 0$ and $\Omega_L = \Omega_R$ can be written as

$$|NC\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle), \quad |C\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \quad \text{and} \quad |3\rangle, \tag{10}$$

which according to Eq. (9), are same as the dressed states of the system in the absence of microwave field for $\Omega_L = \Omega_R = \Omega_0$.

The corresponding dressed states for $\varphi = \pi/2$ and $\Omega_L = \Omega_R = \Omega_0$ are given by

$$\begin{aligned}
|A\rangle &= \sqrt{\frac{\Omega_m^2}{\Omega_m^2 + 2\Omega_0^2}}\left(-\mathrm{i}\frac{\Omega_0}{\Omega_m}|1\rangle + \frac{\mathrm{i}\Omega_0}{\Omega_m}|2\rangle + |3\rangle\right), \\
|B\rangle &= \sqrt{\frac{\Omega_0^2}{\Omega_m^2 + 2\Omega_0^2}} \times \left\{\left[-\frac{\sqrt{\Omega_m^2 + 2\Omega_0^2}}{\Omega_0}\right.\right. \\
&\quad \left. + \frac{\Omega_m^2 + \Omega_0^2}{\Omega_0(-\mathrm{i}\Omega_m + \sqrt{\Omega_m^2 + 2\Omega_0^2})}\right]|1\rangle \\
&\quad \left. - \frac{\Omega_m^2 + \Omega_0^2}{\Omega_0(-\mathrm{i}\Omega_m + \sqrt{\Omega_m^2 + 2\Omega_0^2})}|2\rangle + |3\rangle\right\}, \\
|C\rangle &= \sqrt{\frac{\Omega_0^2}{\Omega_m^2 + 2\Omega_0^2}} \times \left\{\left[\frac{\sqrt{\Omega_m^2 + 2\Omega_0^2}}{\Omega_0}\right.\right. \\
&\quad \left. - \frac{\Omega_m^2 + \Omega_0^2}{\Omega_0(\mathrm{i}\Omega_m + \sqrt{\Omega_m^2 + 2\Omega_0^2})}\right]|1\rangle \\
&\quad \left. + \frac{\Omega_m^2 + \Omega_0^2}{\Omega(\mathrm{i}\Omega_m + \sqrt{\Omega_m^2 + 2\Omega_0^2})}|2\rangle + |3\rangle\right\}.
\end{aligned} \tag{11}$$

The dynamical behaviors of populations of different dressed states are shown in Fig. 6. The using parameters are $\Omega_{\rm m}=0$ (panels (a) and (b)), 0.5γ (panels (c) and (d)), $\Omega_{\rm L}=0.25\gamma$, $\Omega_{\rm R}=0.25\gamma$, (panels (a), (c), and (d)), 0.5γ (b), and $\varphi=0$ (c), $\pi/2$ (d). Other parameters are same as those in Fig. 2. The populations of $|\phi\rangle$ (solid), $|\psi\rangle$ (dashed), and $|3\rangle$ (dotted) are plotted for $\Omega_{\rm L}=\Omega_{\rm R}$ (a) and $\Omega_{\rm L}\neq\Omega_{\rm R}$ (b). Note that $|\phi\rangle$ is a dark state and during interaction, the population of other two states will be transferred to this state. Thus, the steady state DEM of the system becomes zero (Fig. 2(a)).

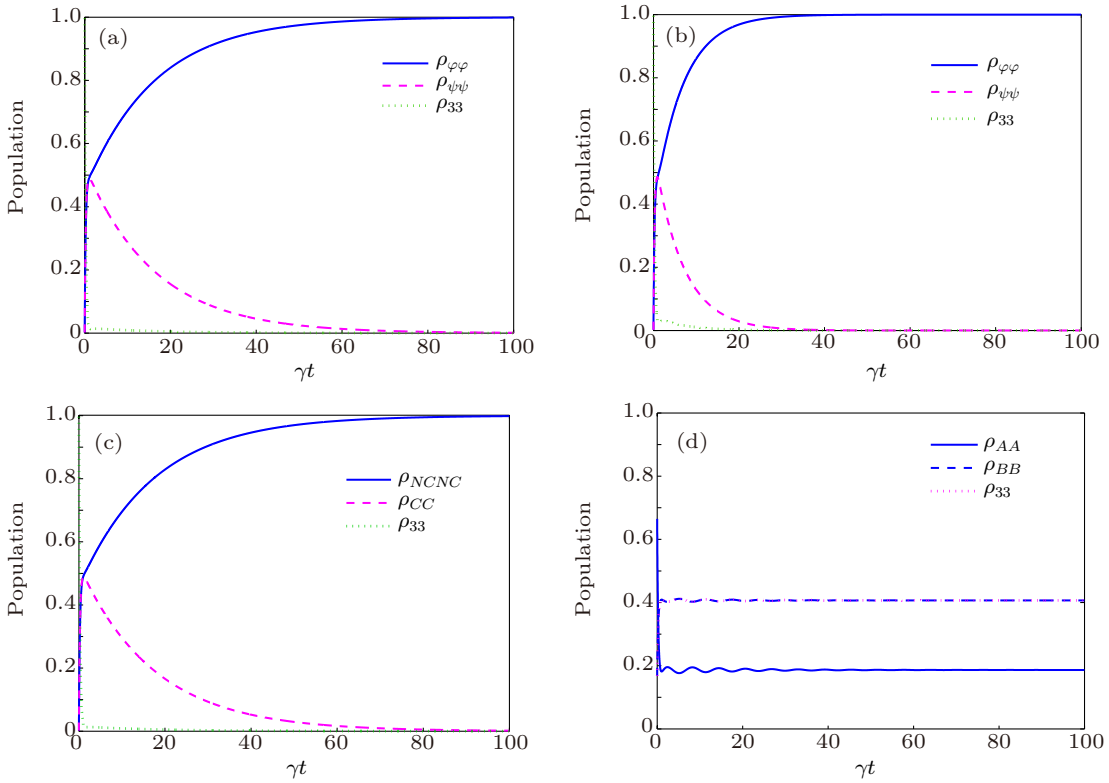


Fig. 6. (color online) Dynamical behaviors of different dressed state populations. The parameters are $\Omega_{\rm m}=0$ (panels (a) and (b)), 0.5γ (panels (c) and (d)), $\Omega_{\rm L}=0.25\gamma$, $\Omega_{\rm R}=0.25\gamma$ (panels (a), (c), and (d)) and 0.5γ (b), and $\varphi=0$ (c) and $\pi/2$ (d). Other parameters are the same as those in Fig. (2). The population of $|\phi\rangle$ (solid), $|\psi\rangle$ (dashed), and $|3\rangle$ (dotted) are plotted for $\Omega_{\rm L}=\Omega_{\rm R}$ (a) and $\Omega_{\rm L}\neq\Omega_{\rm R}$ (b).

For $\Omega_{\rm L}=\Omega_{\rm R}$ all of populations are equally distributed in two lower levels according to the coherent population trapping and the fact that applying the microwave driving field to the lower levels cannot affect the population distribution of the system. Then the same results are obtained for $\Omega_{\rm L}=\Omega_{\rm R}$ and $\varphi=0$ in the absence (Fig. 6(a)) or presence (Fig. 6(c)) of microwave driving field, and all population is transferred to the dark state. The situation is completely different for $\varphi=\pi/2$ in which, all of three dressed states are populated (Fig. 6(d)) and subsequently the non-zero steady state entropy is obtained as shown by dotted line in Fig. 3(a). Thus the atom–photon entanglement just occurs when the population of the system does not trap only in one of the dressed states of the system.

To compare our numerical results with analytical calculation, in Fig. 7, we display the DEM versus relative phase and relative Rabi frequency of optical fields. The using parameters are $\Omega_{\rm L}=0.25\gamma$, $\Omega_{\rm m}=0.5\gamma$, $\Delta_{\rm L}=\Delta_{\rm R}=\Delta_{\rm m}=0.0\gamma$, $\gamma_{31}=\gamma_{32}=\gamma=1$. It is shown that the analytical solutions are in good agreement with the numerical results. The period of DEM variation is $\varphi=\pi$ which can be explained via the obtained analytical results.

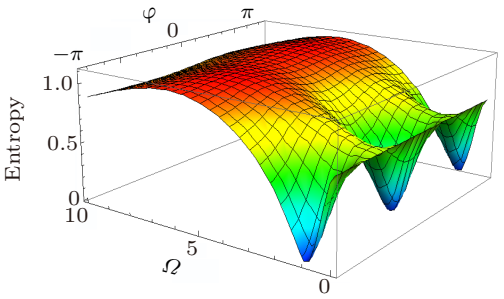


Fig. 7. (color online) Three-dimensional plot of atomic entropy versus relative Rabi frequency and relative phase of applied fields. The selected parameters are $\Delta_{\rm L}=\Delta_{\rm R}=\Delta_{\rm m}=0.0\gamma$, $\gamma_{31}=\gamma_{32}=\gamma=1$, $\Omega_{\rm m}=0.5\gamma$, and $\Omega_{\rm L}=0.25\gamma$.

Now, we are interested in investigating the time-dependent behavior of quantum entropy beyond the multi-photon resonance condition ($\Delta\neq0$). In Fig. 8, we plot the DEMs for different relative phases of applied fields, beyond the multi-photon resonance condition for $\gamma_{31}=\gamma_{32}=\gamma=1$, $\Delta_{\rm L}=0.5$, $\Delta_{\rm R}=\Delta_{\rm m}=0.0\gamma$, $\Delta=0.5\gamma$, $\Omega_{\rm L}=0.25\gamma$, $\Omega_{\rm R}=0.5\gamma$,

$\Omega_m = 0.5\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted). An investigation on Fig. 8 shows that the DEM does not reach a steady state beyond the multi-photon resonance condition.

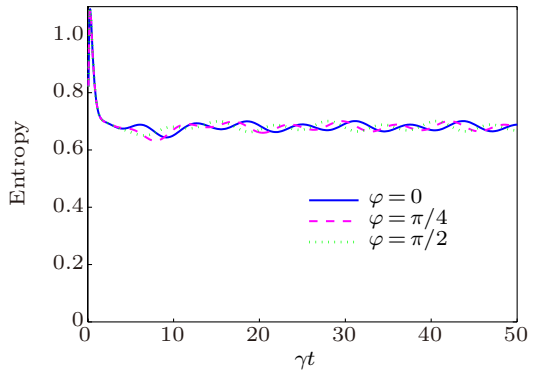


Fig. 8. (color online) DEMs for different relative phases of applied fields, beyond the multi-photon resonance condition for $\gamma_{31} = \gamma_{32} = \gamma = 1$, $\Delta_L = 0.5$, $\Delta_R = \Delta_m = 0.0\gamma$, $\Delta_{=}0.5\gamma$, $\Omega_m = 0.5\gamma$, $\Omega_L = 0.25\gamma$, $\Omega_R = 0.5\gamma$, $\varphi = 0.0$ (solid), $\varphi = \pi/4$ (dashed), and $\varphi = \pi/2$ (dotted).

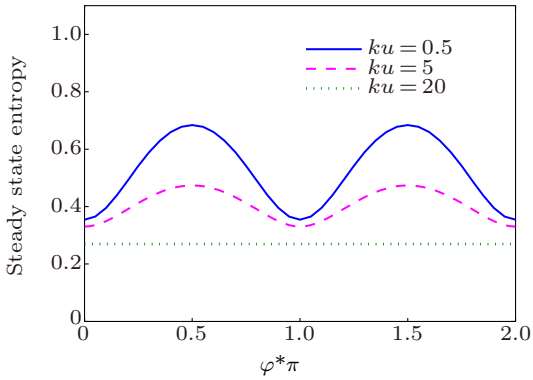


Fig. 9. (color online) Curves of steady state DEM versus relative phase of applied fields in the presence of the Doppler effect for $ku = 0.5\gamma$ (solid), 5γ (dashed), and 20γ (dotted). We assume that $|\boldsymbol{k}_L| = |\boldsymbol{k}_R| = k$ and other parameters are the same as those used for the dashed line in Fig. 4.

Finally we are interested in the influence of the Doppler effect on the obtained results. Doppler broadening due to the atom's thermal velocity $\boldsymbol{v}$ can be applied by replacing the Δ_L, Δ_R, and Δ_m by $\Delta_L - \boldsymbol{k}_L.\boldsymbol{v}$, $\Delta_R - \boldsymbol{k}_R.\boldsymbol{v}$, and $\Delta_m - \boldsymbol{k}_m.\boldsymbol{v}$, respectively. The parameters $\boldsymbol{k}_L$, $\boldsymbol{k}_R$, and $\boldsymbol{k}_m$ are the wave vectors of left, right, and microwave driving fields. To obtain the steady state solutions for the set of equations in expression (2), it is assumed that the multi-photon resonance condition, i.e., $\Delta(\boldsymbol{v}) = \Delta_L - \Delta_R - \Delta_m - \boldsymbol{k}_L.\boldsymbol{v} + \boldsymbol{k}_R.\boldsymbol{v} + \boldsymbol{k}_m.\boldsymbol{v} = 0$ is still satisfied by detunings and propagation directions of applied fields. It is worth noting that a suitable choice of laser propagation direction allows us to obtain the steady state results even in the presence of the Doppler effect. The Maxwellian velocity distribution,

$$f(v) = \frac{1}{\sqrt{2\pi}u} \exp[-v^2/u^2], \tag{12}$$

is used to average our results for populations and quantum entropy. The parameter $u = \sqrt{2k_B T/m}$ shows the Doppler distribution width; m is the mass of a moving atom; T is the temperature of the cell; k_B is the Boltzmann constant. Then the averaged quantum entropy of the system taking the Doppler effect into consideration is given by

$$\bar{S} = \int_{-\infty}^{\infty} f(v)S(v)\,\mathrm{d}v.$$

Figure 9 shows the curves of steady state DEM versus the relative phase of applied fields in the presence of the Doppler effect for $ku = 0.5\gamma$ (solid), 5γ (dashed), 20γ (dotted). We assume $|\boldsymbol{k}_L| = |\boldsymbol{k}_R| = k$ and other using parameters are the same as those used for the dashed line in Fig. 4. It can be seen that increasing the Doppler width of distribution leads to a reduction of the maximum values of averaged quantum entropy and plays a destructive role in atom–photon entanglement.

5. Conclusions

In the present paper, we investigate the atom–photon entanglement in a three-level Λ-type closed-loop atomic system via the von Neumann entropy. The results are obtained in multi-photon resonance condition and beyond it. It is found that the DEM in such a system is phase-dependent and the nonzero steady state entropies are obtained for different values of relative phase of applied fields. We take into consideration the Doppler broadening due to the thermal motion of atoms and find that increasing the Doppler width of distribution reduces the steady state DEM.

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