An LCOR model for suppressing cascading failure in weighted complex networks*

Chen Shi-Ming(陈世明)[a)b)], Pang Shao-Peng(庞少鹏)[a)†], and Zou Xiao-Qun(邹小群)[a)]

[a)] School of Electrical and Electronic Engineering, East China Jiaotong University, Nanchang 330013, China
[b)] Key Laboratory of Advanced Control & Optimization of Jiangxi Province, Nanchang 330013, China

(Received 3 August 2012; revised manuscript received 6 November 2012)

Based on the relationship between capacity and load, cascading failure on weighted complex networks is investigated, and a load-capacity optimal relationship (LCOR) model is proposed in this paper. Compared with three other kinds of load-capacity linear or non-linear relationship models in model networks as well as a number of real-world weighted networks including the railway network, the airports network and the metro network, the LCOR model is shown to have the best robustness against cascading failure with less cost. Furthermore, theoretical analysis and computational method of its cost threshold are provided to validate the effectiveness of the LCOR model. The results show that the LCOR model is effective for designing real-world networks with high robustness and less cost against cascading failure.

Keywords: cascading failure, load-capacity optimal relationship model, robustness, cost

PACS: 89.75.Fb, 89.75.–k, 05.50.+q **DOI:** 10.1088/1674-1056/22/5/058901

1. Introduction

Cascading failure is a failure in a system of interconnected parts in which the failure of a part can trigger the failure of successive parts, and the entire system may be affected even though intentional attacks and random failures emerge very locally.[1] This issue is crucial for many networked structures such as power grids, computer networks, transportation networks, etc. Evidence has demonstrated that in such a network, when an element fails, its 'load' (e.g., power, information, or traffic flow) is redistributed to the other elements of the network. Some of the increased load may then exceed the capacity of their respective element, leading to further failures and eventually to a cascading breakdown of the entire network.[2–4] Instances of breakdown with severe consequences include power outages,[5] internet congestion,[6] traffic jams,[7] and many other problems of social and economical systems.[8–10] Examples include large blackouts in the US due to failure in the power grid and the rail and road traffic disruption due to snowstorm in southern China.

Many studies have been devoted to the field of controlling cascading failure.[11–15] While these are applicable solutions, they cannot eliminate all failures.[16] How to improve the network robustness against cascading failure and how to design manmade networks with lower cost are two important problems. The study on the relationship between load and capacity is one of the most effective ways to solve the above problems, which has drawn a great deal of attention in the past decades.[17,18] Previous studies on cascading failure in networks assume that the capacity of an element is directly proportional to its initial load. For example, Wang and Chen[19] investigated the cascading failure on weighted complex networks by adopting a local weighted flow redistribution rule (LWFRR), and the strongest robustness against cascading failure at a specific weighting strength was found in this model. The relationship between the edge capacity C_{ij} and the initial load L_{ij} is $C_{ij} = TL_{ij}$, where T is a capacity parameter. There exists a critical threshold T_c (minimum threshold) at which removal of any one edge no longer leads to cascading failure. The critical threshold T_c is the maximum of the critical thresholds of edges, and substantial useless capacity exists in other edges. Although the linear relationship can realize the robustness against cascading failure, it wastes much cost.

The study on the nonlinear relationship between capacity and load started with the work of Kim and Motter.[20] It was shown that real systems tend to have larger unoccupied portions of the capacity on network elements with smaller capacity, which contrasts with key assumptions involved in previous studies. It was then shown that the nonlinear relationship could better improve the network robustness against cascading failure with less cost, and many load-capacity nonlinear models were proposed for controlling cascading failure. For example, a high robustness and low cost model for cascading failure was proposed by Wang and Kim.[21] In this model, the capacity of a node is assigned by a monotonically increasing function of its load. With the use of a simple functional form with two free parameters, the authors proved that it is possible to make networks more robust while spending less cost. Yang and Wang[22] discovered an effective solution to suppress cas-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60804066 and 61165007) and the Scientific and Technological Project of Education Department of Jiangxi Province, China (Grant Nos. GJJ12286 and KJLD12068).

†Corresponding author. E-mail: pang_shao_peng@163.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

cading failure in complex networks. In this model, the relationship between the node capacity C_i and the initial load L_i is $C_i = \alpha + \beta L_i$, where $\alpha \geq 0$ and $\beta \geq 0$ are the capacity parameters. Another load-capacity nonlinear model was proposed by Dou and Zhang,[23] which is $C_{ij} = L_{ij} + \beta L_{ij}^{\alpha}$, with α and β being capacity parameters. In addition, negative exponential relationship was found between α and β under certain conditions.

In order to investigate cascading failure based on the relationship between load and capacity, we assume that: the cascaded effect of failure is only considered in the edges; the load of edges is their initial weights; the process of cascading failure complies with the LWFRR; the network robustness is described as the removal of any one edge does not lead to cascading failure; and, the cost of network is the ratio of total capacity to total load. In such case, the load-capacity optimal relationship (LCOR) model is proposed in this paper. We compare the cost thresholds of four kinds of models: the LCOR model; the load-capacity linear model $C_{ij} = TL_{ij}$; the nonlinear relationship model $C_i = \alpha + \beta L_i$; and the nonlinear relationship model $C_{ij} = L_{ij} + \beta L_{ij}^{\alpha}$. Numerical simulations are performed on a number of model networks such as random networks,[24] small-world networks,[25] and scale-free networks.[26] Furthermore, we consider a number of real-world networks including the Chinese railway network, the US airports network, and the Shanghai metro network. We found that the LCOR model is highly robust against cascading failure with less cost. Through theoretical analysis and computational method of its cost threshold, we prove the effectiveness and universality of the LCOR model.

2. Cascading model

We use the LWFRR to investigate the cascading failure in weighted complex networks, which is introduced in Ref. [19]. In the LWFRR, when an edge is subjected to an attack and removed from the network, the load passing through this edge is redistributed to its neighboring edges. As a consequence, the load of each neighbor edge increases proportionally to its weight. More precisely (see Fig. 1),

$$\Delta L_{mn} = L_{mn} \frac{L_{ij}}{\sum_{a \in \Gamma_m} L_{ma} + \sum_{b \in \Gamma_n} L_{nb}}, \tag{1}$$

where Γ_m and Γ_n are the sets of neighboring nodes of v_m and v_n, respectively (note that Γ_m does not include node v_n, and Γ_n does not include node v_m). The L_{mn} is the load on the edge e_{mn} before being broken, and ΔL_{mn} is the additional load that the e_{ij} receives.

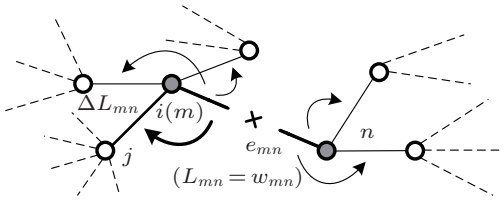


Fig. 1. (color online) Illustration of the LWFRR triggered by an edge-cut-based attack. The initial load L_{mn} of the edge is considered to be its weight. Edge e_{mn} is broken and the load along it is redistributed to the neighboring edges connecting to both ends of v_m and v_n. Among these neighbor edges, the one with higher weight will receive higher shared load from the broken edge.

On one hand, the capacity C_{ij} of edges determines the maximum load that the edge can handle. In the cascading failure process, the load at some edges may exceed their capacity and failure occurs consequently. For example, if

$$C_{ij} < L_{ij} + \Delta L_{mn}, \tag{2}$$

then the edge e_{ij} fails, the process of edges failure and load redistribution is repeated until no edge fails, at which point the cascading failure process can be regarded as being completed.

On the other hand, if we do not consider the cost of networks, cascading failure could never happen by assigning high value to capacity. However, in practice, the capacity is limited severely by cost. For quantitative characterization, we define the cost of a network as the ratio between the total capacity and total load, namely,

$$E = \frac{1}{M} \sum C_{ij}/L_{ij}, \tag{3}$$

where M is the number of edges, and the sums are taken over all edges of the network.

To explore the effect of a small initial attack on four kinds of models, we cut only one edge e_{ij} initially and calculate the avalanche size S_{ij} (the number of broken edges) after the cascading failure process is over. To quantify the robustness of the whole network, we adopt the normalized avalanche size $S_{\mathrm{N}} = \sum_{ij} S_{ij}/M$ obtained via summation over all the avalanche sizes by cutting each edge initially at each time divided by the total number of edges M.

There exists a cost threshold E_{c}. It is a minimum threshold at which removal of any one edge does not lead to cascading failure. More precisely,

$$\begin{aligned} &S_{\mathrm{N}} = 0, && (E > E_{\mathrm{c}}), \\ &0 < S_{\mathrm{N}} \leq M, && (E < E_{\mathrm{c}}), \end{aligned} \tag{4}$$

when $E > E_{\mathrm{c}}$, no cascading failure occurs; while for $E < E_{\mathrm{c}}$, S_{N} increases and the cascading failure emerges. The lower the avalanche size S_{N} is, the more robust the network is against cascading failure.

3. Load-capacity optimal relationship model

The capacity of an element was assumed to be directly proportional to its initial load in the previous studies, and then many nonlinear models were proposed for the study of suppressing cascading failure. As an edge fails, its load shifts to neighboring edges in the network, and some of those neighboring edges might be pushed beyond their capacity, which will get broken in turn and shift their load onto other neighboring edges. Thus, to suppress cascading failure, the following condition should be satisfied in the neighboring edges of e_{mn} (see Fig. 1):

$$C_{ij} \geq L_{ij} + \Delta L_{mn}, \tag{5}$$

where e_{ij} is an arbitrary edge in the set of neighboring edges of e_{mn}. It follows from Eqs. (1) and (3) that

$$E_{ij} = \frac{C_{ij}}{L_{ij}} \geq 1 + \frac{L_{mn}}{\sum\limits_{a\in\Gamma_m} L_{ma} + \sum\limits_{b\in\Gamma_n} L_{nb}}. \tag{6}$$

This indicates two points: first, the cost E_{ij} relates to the initial load of e_{mn} and the sum of load on the neighboring edges of e_{mn}, but not its own load; second, when $C_{ij} = L_{ij} + \Delta L_{mn}$, for arbitrary one edge e_{mn} in the network, the cost thresholds of the neighboring edges of e_{mn} for resisting the load redistribution from e_{mn} are the same.

To avoid the emergence of cascading failure with the least cost, we assume $C_{ij} = L_{ij} + \Delta L_{mn}$. For arbitrary one edge e_{ij} in the network, e_{mn} is arbitrary one of its neighboring edges. The E_{ij} is the cost of the edge e_{ij} to avoid the emergence of cascading failure caused by the break of edge e_{mn}, and different e_{mn} will need different cost E_{ij}. Thus, we choose the maximum one to ensure that the breaking of any one edge in the set of neighboring edges of e_{ij} does not lead to cascading failure. More precisely,

$$C_{ij} = \max(E_{ij}) L_{ij}, \tag{7}$$

where E_{ij} is the cost to avoid the emergence of cascading failure caused by the broken of an edge in the set of neighboring edges of e_{ij}. Based on Eq. (7), we present the LCOR model as follows:

$$C_{ij} = \max\left(1 + \frac{L_{mn}}{\sum\limits_{a\in\Gamma_m} L_{ma} + \sum\limits_{b\in\Gamma_n} L_{nb}}\right) L_{ij}, \tag{8}$$

where the edge $e_{mn} \in \Gamma_{ij}$, Γ_{ij} is the set of neighboring edges of e_{ij}, and four kinds of conditions for e_{mn} are $i = m$, $i = n$, $j = m$, and $j = n$. The condition of $i = m$ is shown in Fig. 1.

4. Network data

In this paper, the cascading failure is investigated in inartificially constructed model networks as well as in a number of real-world networks including the railway network, the airports network, and the metro network.

4.1. Model networks

Numerical simulations are performed on a number of inartificially constructed model networks such as random networks, small-world networks, and scale-free networks. To illustrate the simulation results, we assume the weight of edges in two manners.

(i) Each edge weighted by the multiplication of the degree of the nodes,[11]

$$w_{ij} = a_{ij}(k_i k_j)^{\theta}, \tag{9}$$

where w_{ij} is the weight of edge e_{ij}, k_i and k_j are the degree of nodes v_i and v_j, and θ is a tunable weight parameter. The initial load L_{ij} of the edge is considered to be its weight.

(ii) Each edge weighted by the multiplication of the betweenness centrality of the nodes,[27]

$$w_{ij} = a_{ij}(B_i B_j)^{\theta}, \tag{10}$$

where B_i and B_j are the betweenness centrality of the nodes v_i and v_j, θ is a tunable weight parameter $B_i = \sum\limits_{p\neq i\neq q} [\Gamma_{pq}(i)/\Gamma_{pq}]$, Γ_{pq} is the number of shortest paths from the node v_p to v_q, and $\Gamma_{pq}(i)$ is the number of these shortest paths making use of node v_i.

4.2. Real-world networks

Although model networks are useful in understanding how real systems behave, they cannot capture many of the structural properties of the real networks. Therefore, we also consider a number of real networks and apply the cascading failure to them.

Chinese railway network (CRN) The railway network contains 2910 nodes and 94720 edges, which is generated by viewing stations as nodes, and an arbitrary pair of stations is considered to be connected by an edge when at least one train stops at both stations, weighted by the train flux. The data comes from the China Railway Passenger Train Timetable for 2011,[28] which have 2910 stations and 4183 trains.

US airport network (UAN) The airlines connection network in the US consists of 332 nodes and 2125 edges. The weights correspond to the number of seats available on the scheduled flights.[29]

Shanghai metro network (SMN) The metro network contains 244 nodes and 3840 edges. In this network, an arbitrary pair of stations is connected by edge when at least one train stops at both stations, weighted by the metro flux. The data comes from the Shanghai metro network map for 2012,[30] which have 244 stations and 11 trains.

5. Simulation results

In order to show the effectiveness and advancement of the LCOR model, four kinds of relationship models are considered in this paper. We calculate their cost threshold E_c meeting the condition that the cascading failure does not occur when any one edge e_{ij} is removed, respectively. Note that we cut only one edge e_{ij} initially to explore the effect of a small initial attack on load-capacity relationship models. The relationship models are listed as follows.

(i) The LCOR model (see Eq. (8)).

(ii) The load-capacity linear model is expressed as

$$C_{ij} = TL_{ij}, \tag{11}$$

where $T > 1$ is the capacity parameter, and the value of cost threshold E_c is the maximum of the critical thresholds of edges.

(iii) A load-capacity nonlinear model proposed by Yang and Wang[22] is expressed as follow:

$$C_{ij} = \alpha + \beta L_{ij}, \tag{12}$$

where $\alpha \geq 0$ and $\beta \geq 0$ are the capacity parameters. For fixed $\beta(\alpha)$, there should exist a critical threshold $\alpha_c(\beta_c)$, below which cascading failure occurs.

(iv) Another load-capacity nonlinear model proposed by Dou and Zhang[23] is expressed as follows:

$$C_{ij} = L_{ij} + \beta L_{ij}^{\alpha}, \tag{13}$$

where α and β are the capacity parameters, and negative exponential relationship is found between α and β under certain conditions.

The simulation results are shown in Fig. 2. Compared with other three kinds of relationship models, the LCOR model could realize robustness against cascading failure with the least cost. In order to show the effectiveness and universal of the LCOR model, we introduce the computational method of cost threshold E_c by theoretical analysis. Two weighting schemes are taken into account in the calculation of cost threshold.

(i) When $w_{ij} = a_{ij}(B_iB_j)^{\theta}$, it follows Eqs. (6) and (10) that

$$E_{ij} = 1 + \frac{(B_mB_n)^{\theta}}{\sum_{a\in\Gamma_m}(B_mB_a)^{\theta} + \sum_{b\in\Gamma_n}(B_nB_b)^{\theta}}. \tag{14}$$

Using Bates' rule,[31] we have

$$\sum_{a\in\Gamma_m}(B_mB_a)^{\theta} = B_m^{\theta}\sum_{B'=B_{\min}}^{B_{\max}} k_m P(B'|B_m)B'^{\theta}, \tag{15}$$

where $P(B'|B_m)$ is the conditional probability that a node with betweenness centrality B' is connected to a node with B_m, $B_{\max}$ and $B_{\min}$ are the maximum and minimum node betweenness centralities, k_m is the degree of node v_m, and Γ_m is the set of neighboring nodes of v_m. We should note that Γ_m includes node v_n.

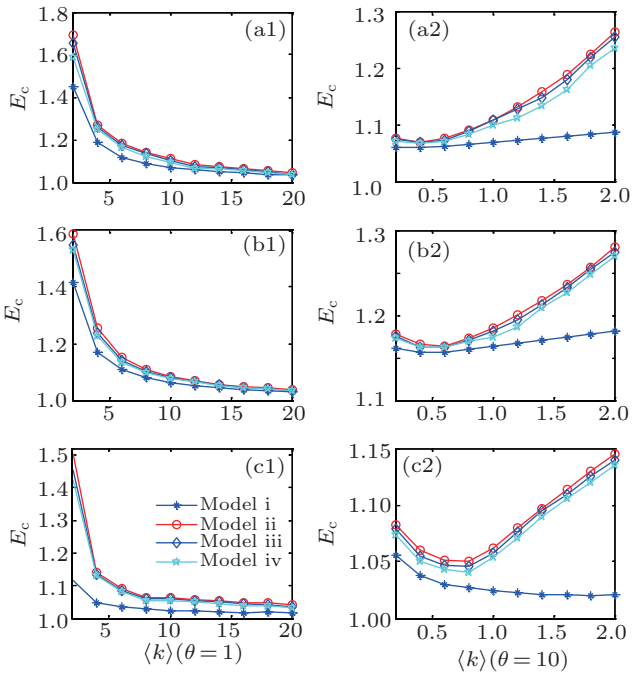


Fig. 2. (color online) The cost threshold E_c as a function of average degree $\langle k\rangle$ and parameter θ for different load-capacity relationship models on (a) ER networks with 1000 nodes, (b) NW networks with 1000 nodes and $P = 0.3$, and (c) BA networks with 1000 nodes. Data show averages over 10 realizations. The weight of edge is set to be $w_{ij} = a_{ij}(B_iB_j)^{\theta}$ in this simulation, and the approximate result appears in the condition of weighting edge by $w_{ij} = a_{ij}(k_ik_j)^{\theta}$.

BA and NW networks have no degree–degree correlation,[32] and these networks do not show any significant betweenness–betweenness correlations.[27] Thus, we have $P(B'|B_m) = P(B')$, and then

$$\sum_{a\in\Gamma_m}(B_mB_a)^{\theta} = B_m^{\theta}k_m\sum_{B'=B_{\min}}^{B_{\max}} P(B')B'^{\theta} = B_m^{\theta}k_m\langle B^{\theta}\rangle. \tag{16}$$

Equation (14) can be simplified into

$$E_{ij} \approx 1 + \frac{(B_mB_n)^{\theta}}{B_m^{\theta}k_m\langle B^{\theta}\rangle + B_n^{\theta}k_n\langle B^{\theta}\rangle}, \tag{17}$$

where $B_m^{\theta}k_m + B_n^{\theta}k_n \geq 2\sqrt{k_mk_n}(B_mB_n)^{\theta/2}$, and equation (17) can be approximated as

$$E_{ij} \approx 1 + \frac{(B_mB_n)^{\theta/2}}{2\sqrt{k_mk_n}\langle B^{\theta}\rangle}. \tag{18}$$

From the above analysis, the maximum cost for arbitrary edge can be chosen by considering the ranges of $\theta < 0$, $\theta = 0$, and $\theta > 0$, respectively. Namely,

$$\max(E_{ij}) \approx \begin{cases} 1 + B_{\max}^{\theta}/2\sqrt{k_mk_n}\langle B^{\theta}\rangle, & (\theta > 0), \\ 1 + 1/2\sqrt{k_mk_n}, & (\theta = 0), \\ 1 + B_{\min}^{\theta}/2\sqrt{k_mk_n}\langle B^{\theta}\rangle, & (\theta < 0), \end{cases} \tag{19}$$

where $B_{\max}$ and $B_{\min}$ are the maximum and minimum node betweenness centralities. Thus, we estimate the cost threshold as

$$E_{\rm c} = \frac{1}{M}\sum \max(E_{ij}) \approx \begin{cases} 1+B^{\theta}_{\max}/2\langle k\rangle\langle B^{\theta}\rangle, & (\theta>0), \\ 1+1/2\langle k\rangle, & (\theta=0), \\ 1+B^{\theta}_{\min}/2\langle k\rangle\langle B^{\theta}\rangle, & (\theta<0). \end{cases} \quad (20)$$

(ii) When $w_{ij} = a_{ij}(k_i k_j)^{\theta}$, we have

$$E_{ij} \approx 1 + \frac{\langle k\rangle (k_m k_n)^{(\theta-1)/2}}{2\langle k^{\theta+1}\rangle}. \quad (21)$$

The maximum cost for an arbitrary edge can be chosen by considering the ranges of $\theta < 1$, $\theta = 1$, and $\theta > 1$, respectively. We estimate the cost threshold as

$$E_{\rm c} = \frac{1}{M}\sum \max(E_{ij}) \approx \begin{cases} 1+\langle k\rangle k^{\theta-1}_{\max}/2\langle k^{\theta+1}\rangle, & (\theta>1), \\ 1+\langle k\rangle/2\langle k^{2}\rangle, & (\theta=1), \\ 1+\langle k\rangle k^{\theta-1}_{\min}/2\langle k^{\theta+1}\rangle, & (\theta<1). \end{cases} \quad (22)$$

The two results are both based on a mean-field type of approximation. From the result, we can see that the value of the cost threshold $E_{\rm c}$ is influenced by the topology, degree distribution, and weight distribution of network.

We also compare the critical threshold of different relationship models in real-world weighted networks including the Chinese railway network, the US airports network, and the Shanghai metro network. The cost thresholds for each model are shown in Table 1. The LCOR model has the least cost threshold. Particularly in the railway network and the airport network, the result of Model i is much better than other models. The reason is that some individual edges need high capacity in the networks. For the Chinese railway network, the maximum of cost $E_{\max} = 1.8879$ exists in one edge. Thus, the cost threshold of Model ii equals the maximum cost, and vast useless cost exists in other edges. The same phenomenon occurs in the Model iii and Model iv. However, the LCOR model could fully solve this problem based on reasonable capacity allocation. Therefore, it could achieve robustness against cascading failure with the least cost.

Table 1. The cost threshold $E_{\rm c}$ of the Chinese railway network with 2910 nodes and 94720 edges, the US airports network with 332 nodes and 2125 edges, and the Shanghai metro network with 244 nodes and 3840 edges. Data shows averages over 10 realizations.

Network	Model i	Model ii	Model iii	Model iv
Railway	1.0264	1.5010	1.4612	1.4371
Airport	1.0527	1.8879	1.8604	1.8223
Metro	1.0202	1.0556	1.0498	1.0437

In addition, we provide the $S_{\rm N}$ as a function of the cost E for different real-world weighted networks. As shown in Fig. 3, the Model i in the real-world networks could improve the network robustness against cascading failure with the least cost. For example, for the values of the cost as $E = 1.01$ in the Shanghai metro network, the average avalanche size of Model i is $S_{\rm N} = 0.547$, while $S_{\rm N} \geq 0.7$ for other models. Similar patterns are observed for other real-world networks including the Chinese railway network and the US airports network. In any of these examples, the appropriate relationship between capacity and load based on the LCOR model could ensure the robustness of the network against cascading failure with the least cost. The observation suggests that the robustness of a network with any structure against cascading failure can be substantially improved by assigning appropriate capacity for the edges, and one of effective ways is to assign capacity reasonably based on the LCOR model.

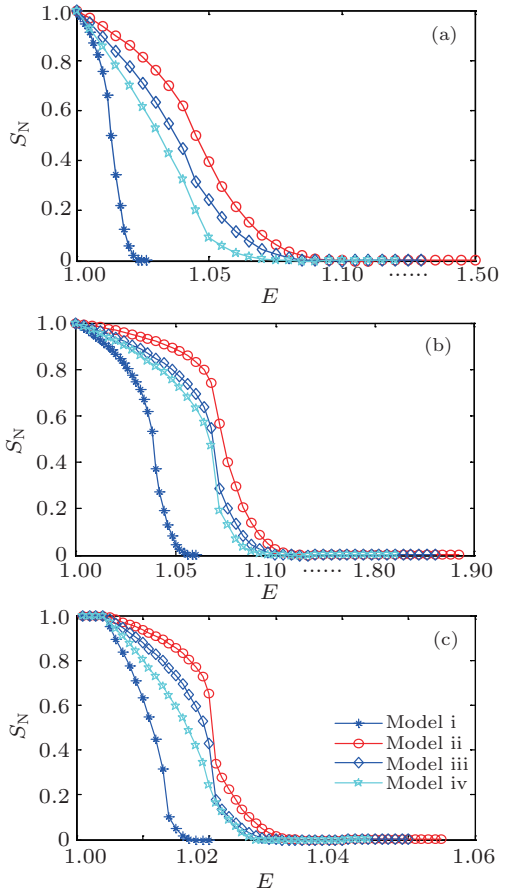


Fig. 3. (color online) $S_{\rm N}$ as a function of cost E for (a) the Chinese railway network, (b) the US airports network, and (c) the Shanghai metro network. Data shows averages over 10 realizations.

6. Conclusion

The increasing complexity of infrastructure networks makes them very sensitive to local failures, and the cascading failure may occur in such networks. In this paper, we investigate the cascading failure on weighted complex networks by focusing on the relation between capacity and load. Then the LCOR model is proposed for against cascading failure with the least cost. Many model networks as well as a number of

real-world weighted networks including the railway network, the airports network and metro network are used in the comparison analysis of four relationship models. The result reveals that the capacity allocation based on the LCOR model could realize robustness against cascading failure with the least cost. Meanwhile, we give the computational method of the cost threshold of the LCOR model.

According to the simulation results and the comparison analysis, we show that the LCOR model is effective and universal for designing networks with high robustness to cascading failure and less cost, and it has immediate applications in designing real-world networks including power grids, computer networks, and transportation networks.

References

[1] Huang L, Lai Y C and Chen G R 2008 *Phys. Rev. E* **78** 036116
[2] Zhao L, Park K and Lai Y C 2004 *Phys. Rev. E* **70** 035101
[3] Xia Y X, Fan J and Hill D 2010 *Physica A* **389** 1281
[4] Babaei M, Ghassemieh H and Jalili M 2011 *IEEE Transactions on Circuits and Systems II* **58** 527
[5] Sachtjen M L, Carreras B A and Lynch V E 2000 *Phys. Rev. E* **61** 4877
[6] Guimerá R, Arenas A, Díaz-Guilera A and Giralt F 2002 *Phys. Rev. E* **66** 026704
[7] Wu J J, Sun H J and Gao Z Y 2007 *Physica A* **386** 407
[8] Motter A E and Lai Y C 2002 *Phys. Rev. E* **66** 065102
[9] Watts D J 2001 *Proceedings of the National Academy of Sciences of the United States of America* (USA: Academy of Sciences) p. 5766
[10] Wang W X, Yang R and Lai Y C 2010 *Phys. Rev. E* **81** 035102
[11] Wu Z H and Fang H J 2008 *Chin. Phys. Lett.* **25** 3822
[12] Hines P, Huaiwei L, Jia D and Talukdar S 2005 *Proceedings of the 2005 IEEE International Conference on Networking Sensing and Control* (Arizona: Tucson) p. 273
[13] Zheng J F, Gao Z Y, Fu B B and Li F 2009 *Chin. Phys. B* **18** 4754
[14] Hu K, Hu T and Tang Y 2010 *Chin. Phys. B* **19** 080206
[15] Wang J W and Rong L L 2008 *Chin. Phys. Lett.* **25** 3826
[16] Talukdar S N, Ilic J A M, Lave L B and Morgan M G 2003 *Electr. J.* **16** 7
[17] Xia Y X, Fan J and Hill D 2010 *Physica A* **389** 1281
[18] Babaei M, Ghassemieh H and Jalili M 2011 *IEEE Transactions on Circuits and Systems II* **58** 527
[19] Wang W X and Chen G R 2008 *Phys. Rev. E* **77** 026101
[20] Kim D H and Motter A E 2008 *J. Phys. A: Math. Theor.* **41** 224019
[21] Wang B and Kim B J 2007 *Europhys. Lett.* **78** 48001
[22] Yang R, Wang W X, Lai Y C and Chen G 2009 *Phys. Rev. E* **79** 026112
[23] Dou B L and Zhang S Y 2011 *J. System Simulation.* **23** 7 (in Chinese)
[24] Erdös P and Rényi A 1960 *Publ. Math. Inst. Hung. Acad. Sci.* **5** 17
[25] Newman M E J and Watts D J 1999 *Phys. Lett. A* **263** 341
[26] Barabasi A L and Albert R 1999 *Science* **286** 509
[27] Mirzasoleiman B, Babaei M, Jalili M and Safari M 2011 *Phys. Rev. E* **84** 046114
[28] China Railway Passenger Train Timetable
[29] USA Airports Timetable
[30] Shanghai Metro Timetable
[31] Howson C and Urbach P 1993 *Scientific Reasoning: The Bayesian Approach* LaSalle p. 267
[32] Huang Z X, Wang X R and Zhu H 2004 *Chin. Phys.* **13** 273