Adaptive synchronization control of coupled chaotic neurons in an external electrical stimulation*

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In this paper we present a combined algorithm for the synchronization control of two gap junction coupled chaotic FitzHugh–Nagumo (FHN) neurons in an external electrical stimulation. The controller consists of a combination of dynamical sliding mode control and adaptive backstepping control. The combined algorithm yields an adaptive dynamical sliding mode control law which has the advantage over static sliding mode-based controllers of being chattering-free, i.e., a sufficiently smooth control input signal is generated. It is shown that the proposed control scheme can not only compensate for the system uncertainty, but also guarantee the stability of the synchronized error system. In addition, numerical simulations are also performed to demonstrate the effectiveness of the proposed adaptive controller.

Keywords: FitzHugh–Nagumo neuron, synchronization, adaptive control

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1. Introduction

Synchronization processes are ubiquitous in nature and play an important role in many different fields such as biology, ecology, sociology, and technology.[1] Recently, increasing evidence have been provided to show that synchronization is specially relevant to a variety of neuronal phenomena.[2] Classical phenomena such as mutual synchronization, entrainment, and chaotic synchronization have been observed in many biological experiments and numerical simulations.[3–5] Experimental observations demonstrated that synchronous neuronal oscillations correlate with many physiological mechanisms of normal and pathological brain functions. For example, synchronization plays a crucial role in the mechanisms of information processing and information prefacing within different brain areas.[6] Moreover, synchronous oscillations in the sensorimotor cortex provide integration and coordination of the information needed for the motor control.[7] On the other hand, an abnormal synchronization of individual neurons may result in the emergence of some pathological conditions, such as epilepsy, essential tremor, and Parkinson's disease.[8–10]

The investigation of synchronization control in a chaotic system has recently become a subject of increasing interest.[11–15] It has been shown that synchronization in a neuronal system may be influenced by many factors. Without control, identical coupled neurons can eventually synchronize only when the coupling strength exceeds a certain critical value,[16,17] which may be beyond the physiological condition. In biological experiments, the synchronization of two coupled neurons can be realized upon depolarization by an external DC current.[4] Theoretically, many control methods have been developed to achieve the synchronization of neuronal systems, such as adaptive control,[18,19] backstepping control,[20] nonlinear control,[21] and feedback control.[22,23]

In a previous literature,[20] we have studied the synchronization control of two electrically coupled chaotic FitzHugh–Nagumo (FHN) neurons in the external electrical stimulation by backstepping method. The FHN model is a simplified version of the Hodgkin–Huxley model of neuronal dynamics, yet captures the main dynamical features of biological neurons.[24] Using the external electrical stimulation, an individual FHN neuron may exhibit chaotic behavior.[25] The backstepping scheme needs only one controller to synchronize the coupled neurons and can be applied to a variety of chaotic neuronal systems, no matter whether they contain the external excitation. However, it does not consider the uncertain factors in practice. In fact, there are many uncertainty sources in complex neural systems. The most important one is the coupling strength between neurons, which is usually unknown in real neural systems, but has a significant influence on their synchronization behaviors.[17]

In this paper, an adaptive backstepping dynamical sliding mode controller, which combines the merits of both the dynamical sliding mode control and the adaptive backstepping control, is used to achieve the chaos synchronization of two coupled FHN neurons with an uncertain coupling strength. We first derive a simple static adaptive sliding model controller via backstepping. However, the control input signal exhibits an obvious bang–bang behavior which may be deemed as a counter effect in a neuronal system. Then, an adaptive dynamical sliding mode controller will be proposed as a means of ob-

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taining a smoother controlled response. In this control scheme, all chattering is completely relegated to the state space of the dynamical feedback controller. As a consequence, a sufficiently smooth control input signal is generated.[26] Moreover, the backstepping technique provides a sufficiently simple way to obtain a more direct sliding surface adaption procedure, while avoiding the excessively restrictive stabilizing features of the adaptive sliding surface.[26–28] Based on the Lyapunov stability theory, the proposed adaptive dynamical sliding mode control scheme can not only compensate for the system uncertainty, but also guarantee the stability of the synchronized error system.

The rest of this paper is organized as follows. In Section 2, the dynamic behaviors of two chaotic FHN neurons electrically coupled with gap junctions in the external electrical stimulation are studied. In Section 3, a robust control scheme combining adaptive backstepping method and dynamical sliding mode control technique is designed to synchronize two coupled FHN neurons. The simulation results are also provided to illustrate the effectiveness of the control method. Finally, conclusions are drawn in Section 4.

2. Dynamic behaviors of two FHN neurons electrically coupled with gap junctions

In a neuronal system, the coupling between neurons may occur via two different types of synapses, i.e., the electrical and chemical ones. In the former case, the coupling occurs through gap junctions and its strength depends linearly on the difference between their membrane potentials.[29,30] In our previous study,[16] the model of neurons coupled with gap junction was given, and the influence of gap junction on the synchronization of two coupled FHN neurons was investigated. The circuit diagram of the model is shown in Fig. 1.

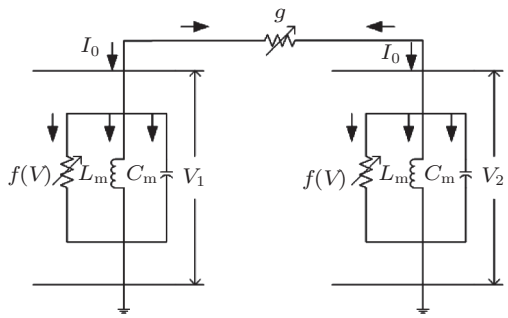


Fig. 1. Circuit diagram of two coupled neurons. V_1 and V_2 denote the membrane potentials for two coupled neurons; $f(V)$, L_m, and C_m are the electrical parameters of neurons; g is the coupling strength between neurons; and I_0 is the external electrical stimulation.

The model of FHN neurons coupled with two gap junctions is described by

$$\begin{aligned}
\dot{X}_1 &= X_1(X_1-1)(1-rX_1)-Y_1-g(X_1-X_2)+I,\\
\dot{Y}_1 &= bX_1,\\
\dot{X}_2 &= X_2(X_2-1)(1-rX_2)-Y_2-g(X_2-X_1)+I,\\
\dot{Y}_2 &= bX_2, \qquad (1)
\end{aligned}$$

where X_i and Y_i $(i=1,2)$ are rescaled membrane voltage and the recovery variable of two FHN neurons, respectively, g is the coupling strength of gap junction, and I is the external electrical stimulation in the form of $I=(A/\omega)\cos(\omega t)$ with A and $\omega=2\pi f$ being the amplitude and frequency, respectively. Throughout this paper, we fix parameters b, r, A, and f to be 1, 10, 0.1, and 127.1, respectively. Thus, the dynamics of individual FHN neurons without coupling is chaotic.[25] According to Ref. [16], if the individual neurons are chaotic, then the synchronization occurs only when the coupling strength of gap junction satisfies some critical conditions. If the coupling strength of the gap junction $g=0.02<0.5$, the synchronization cannot occur; when $g=1>0.5$, the synchronization occurs, as shown in Figs. 2 and 3, respectively.

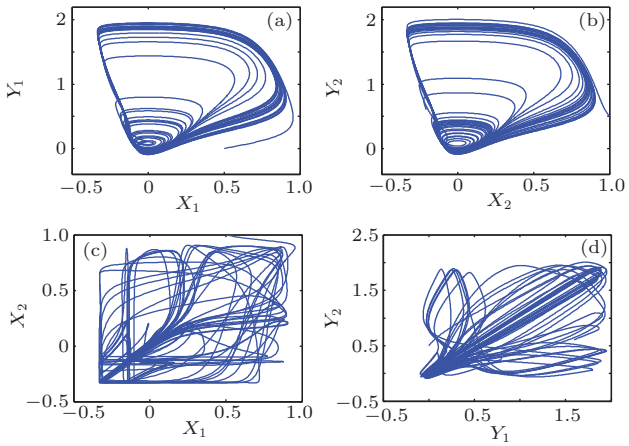


Fig. 2. (color online) Dynamics of two electrically coupled FHN neurons at $f=127.1$ Hz and $g=0.02<0.5$. Phase plane diagrams: (a) X_1-Y_1, (b) X_2-Y_2, (c) X_1-X_2, (d) Y_1-Y_2.

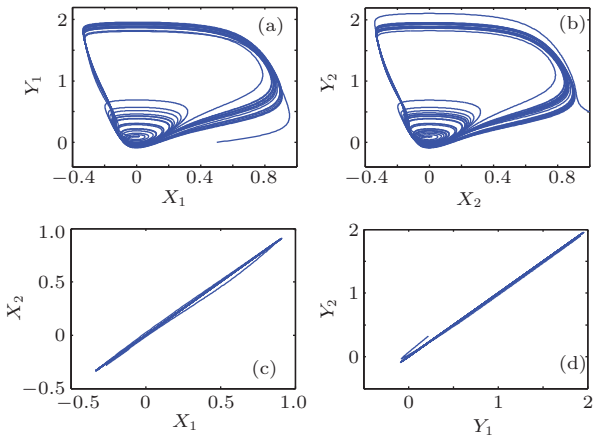


Fig. 3. (color online) Dynamics of two electrically coupled FHN neurons at $f=127.1$ Hz and $g=1>0.5$. Phase plane diagrams: (a) X_1-Y_1, (b) X_2-Y_2, (c) X_1-X_2, (d) Y_1-Y_2.

3. Chaos synchronization via adaptive control

3.1. Description of the coupled FHN neuron system under control

For clarity and without loss of generality, the gap-junction-coupled FHN neuron system under control can be ex-

pressed as

$$\begin{aligned}
\dot{X}_1 &= X_1(X_1-1)(1-rX_1)-Y_1-g(X_1-X_2)+I,\\
\dot{Y}_1 &= bX_1,\\
\dot{X}_2 &= X_2(X_2-1)(1-rX_2)-Y_2-g(X_2-X_1)+I+u,\\
\dot{Y}_2 &= bX_2,
\end{aligned} \tag{2}$$

where g is assumed to be an unknown constant, and u is the added control force such that the dynamical behaviors of two coupled FHN neurons are synchronous.

Let $e_X = X_2 - X_1$ and $e_Y = Y_2 - Y_1$, the error dynamic system of coupled neurons can be written as follows:

$$\begin{aligned}
\dot{e}_X &= [X_2(X_2-1)(1-rX_2)-Y_2-g(X_2-X_1)]\\
&\quad -[X_1(X_1-1)(1-rX_1)-Y_1-g(X_1-X_2)]+u,\\
\dot{e}_Y &= b(X_2-X_1).
\end{aligned} \tag{3}$$

We can rewrite the error system (3) as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}(F(\boldsymbol{x})+u), \tag{4}$$

where

$$\boldsymbol{e} = \begin{bmatrix} e_X \\ e_Y \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} -2g & -1 \\ b & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

and $F(\boldsymbol{x}) = X_2(X_2-1)(1-rX_2) - X_1(X_1-1)(1-rX_1)$ is the nonlinear part of the error dynamics with $\boldsymbol{x} = [X_1, X_2]^{\mathrm{T}}$.

In this way, the problem of synchronization between coupled neurons can be converted into a problem of how to realize the asymptotical stabilization of system (4) at the origin. Our goal is to design an adaptive backstepping dynamical sliding mode controller for the variable X_2 of system (2) to track the reference trajectory X_1 asymptotically, such that $\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = 0$.

3.2. Adaptive dynamical sliding mode controller design

In this section, we aim to design a chattering-free control scheme for the synchronization of coupled chaotic FHN neurons. For comparison, a static adaptive sliding mode controller is designed first to show the bang–bang behavior of the control input signal. Then, we propose an adaptive dynamical sliding model control system to remove the bang–bang nature of control input, i.e., a sufficiently smooth control input signal is obtained.

3.2.1. Static adaptive sliding mode control

Consider the following error dynamic system:

$$\begin{aligned}
\dot{e}_X &= -2ge_X - e_Y + F + u,\\
\dot{e}_Y &= e_X,
\end{aligned} \tag{5}$$

where g is assumed to be an unknown constant. Let $\hat{g}$ denote an estimate of g.

Define the sliding surface s as

$$s = e_X + e_Y, \tag{6}$$

and its time derivative is

$$\dot{s} = \dot{e}_X + \dot{e}_Y = e_X - 2ge_X - e_Y + F + u. \tag{7}$$

Considering the following Lyapunov function:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\phi^2, \tag{8}$$

where $\phi = g - \hat{g}$, then $\dot{V}$ is given by

$$\dot{V} = s\dot{s} + \phi\dot{\phi} = s(e_X - e_Y + F + u) - (2se_X + \hat{g})g + \hat{g}\dot{\hat{g}}. \tag{9}$$

The adaptation law for $\hat{g}$ is designed as

$$\dot{\hat{g}} = -2se_X. \tag{10}$$

Then, we have

$$\dot{V} = s(e_X - e_Y + F + u - \hat{g}2e_X). \tag{11}$$

According to Eq. (11), an adaptive sliding mode control law is proposed to be

$$u = -e_X + e_Y - F + \hat{g}2e_X - k\mathrm{sgn}(s), \tag{12}$$

where $k > 0$. Substituting Eq. (12) into Eq. (11), we obtain

$$\dot{V} = -k|s| \le 0. \tag{13}$$

Therefore, the error dynamic system (5) is negative definite, and the state errors e_X and e_Y will converge to zero, i.e., the static adaptive sliding mode control system is asymptotically stable.

In the following, numerical simulations are carried out to evaluate the effectiveness of the proposed static adaptive sliding mode control system. We choose the control parameter in the simulations to be $k = 0.5$ and the unknown parameter $g = 0.02$. The controller is switched on at time $t = 200$ ms. Dynamical responses of the system are given in Fig. 4. As shown in Fig. 4, two neurons exhibit asynchronous behavior before control. However, as soon as the controller is applied, the dynamic error between two coupled neurons vanishes and the complete synchronization is achieved. The corresponding phase portraits of the system before and after control in the planes of $X_1 - X_2$ and $Y_1 - Y_2$ are shown in Fig. 5. Moreover, figure 6 shows the simulated trajectory for the control input signal $u(t)$. Obviously, in the proceeding static sliding mode-based control scheme, the control input exhibits an obvious bang–bang behavior which may be deemed as counter-effective in neuronal system, as it may damage a biological neuron and change its intrinsic dynamics. In the next section, we will remove the bang–bang nature of the control input by designing an adaptive dynamical sliding mode controller.

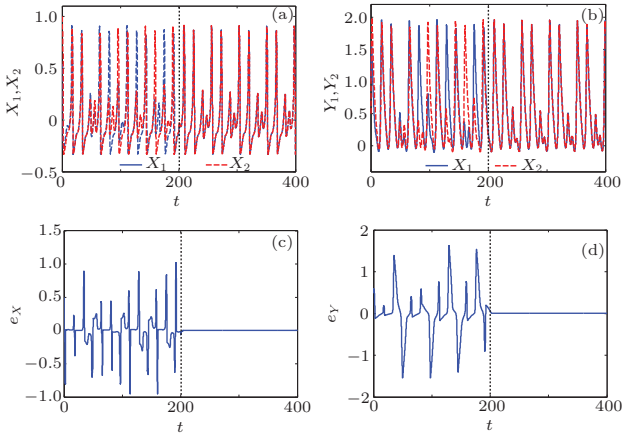


Fig. 4. (color online) Time courses of the coupled FHN neuron system with static adaptive sliding mode control: (a) X_1 and X_2, (b) Y_1 and Y_2. The state errors: (c) $e_X = X_2 - X_1$, (d) $e_Y = Y_2 - Y_1$.

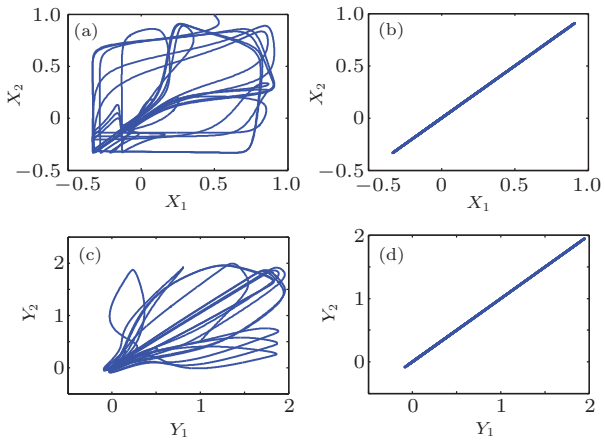


Fig. 5. (color online) Phase portraits of the neural system before control: (a) $X_1 - X_2$, (c) $Y_1 - Y_2$. Phase portraits of the neural system after control: (b) $X_1 - X_2$, (d) $Y_1 - Y_2$.

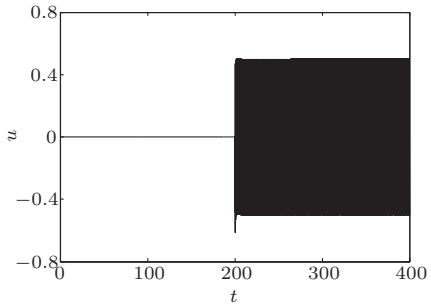


Fig. 6. Static adaptive sliding mode control input signal $u(t)$.

3.3. Adaptive dynamical sliding mode control

In this section, an adaptive dynamical sliding model control scheme is proposed to achieve chaos synchronization of coupled FHN neurons. In order to remove the bang–bang nature of the control input, we resort to the system extension, let the added integration attenuate the discontinuities associated with the input, and relegate them to the first time derivative of the input.

Consider the following extended error dynamic system:

$$\dot{e}_X = -2ge_X - e_Y + F + u,$$

$$\dot{e}_Y = e_X, \quad \dot{u} = v, \tag{14}$$

where v is an auxiliary input representing the time derivative of the original input u, and g is an unknown constant parameter. Let $\hat{g}$ denote an estimate of g. Define the adaptive sliding surface s as

$$s = c_1(e_X + e_Y) + e_X - 2\hat{g}e_X - e_Y + F + u, \tag{15}$$

where c_1 is a positive constant. Define the variable $z = e_X + e_Y$, and its time derivative is

$$\dot{z} = \dot{e}_Y + \dot{e}_X = s - c_1 z - 2(g - \hat{g})e_X, \tag{16}$$

where c_1 is a positive constant.

The first Lyapunov function V_1 is chosen to be

$$V_1 = \frac{1}{2}z^2 + \frac{1}{2}(g - \hat{g})^2. \tag{17}$$

Then, $\dot{V}_1$ is given by

$$\dot{V}_1 = z\dot{z} - (g - \hat{g})\dot{\hat{g}} = z[s - c_1 z] - (g - \hat{g})(2ze_X + \dot{\hat{g}}). \tag{18}$$

According to Eqs. (14) and (15), we obtain

$$\begin{aligned} \dot{s} \;=\; & (c_1 + 1 - 2\hat{g})(-2ge_X - e_Y + F + u) \\ & - 2e_X\dot{\hat{g}} + (c_1 - 1)e_X + \dot{F} + v. \end{aligned} \tag{19}$$

We assume the second Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{2}s^2. \tag{20}$$

Thus, $\dot{V}_2$ has the form of

$$\begin{aligned} \dot{V}_2 \;&=\; \dot{V}_1 + s\dot{s} \\ &=\; z[s - c_1 z] - (g - \hat{g})(2ze_X + \dot{\hat{g}}) \\ &\quad + s[v + (c_1 + 1 - 2\hat{g})(-2ge_X - e_Y + F + u) \\ &\quad - 2e_X\dot{\hat{g}} + (c_1 - 1)e_X + \dot{F}]. \end{aligned} \tag{21}$$

An adaptive dynamical sliding mode controller is designed as

$$\begin{aligned} \dot{u} = v \;=\; & -(c_1 + 1 - 2\hat{g})(-2\hat{g}e_X - e_Y + F + u) \\ & + 2e_X\dot{\hat{g}} - (c_1 - 1)e_X - \dot{F} - z - k\,\mathrm{sgn}(s), \end{aligned} \tag{22}$$

where $k > 0$. Substituting Eq. (22) into Eq. (21), we have

$$\begin{aligned} \dot{V}_2 \;=\; & -c_1 z^2 - k|s| - (g - \hat{g})[2ze_X + \dot{\hat{g}} \\ & + (c_1 + 1 - 2\hat{g})2se_X]. \end{aligned} \tag{23}$$

The parameter update law is designed as

$$\dot{\hat{g}} = -2ze_X - (c_1 + 1 - 2\hat{g})2se_X. \tag{24}$$

Then, $\dot{V}_2$ becomes

$$\dot{V}_2 = -z^2 c_1 - k|s| \le 0. \tag{25}$$

Thus, the states z and s will converge to zero. Moreover, $\lim_{t\to\infty} e_X(t) = 0$ and $\lim_{t\to\infty} e_Y(t) = 0$. As a result, the stability of

the proposed adaptive dynamical sliding mode control system (22) can be guaranteed.

In Figs. 7 and 8, numerical simulations are carried out to illustrate the effectiveness of the proposed adaptive dynamical sliding mode control system. The control parameters are set to be $c_1 = 2$ and $k = 0.5$, and the "true" value of the constant parameter is set to be $g = 0.02$. We switch on the controller at time $t = 200$ ms. As one can see, the dynamical errors converge to zero rapidly, and complete synchronization of the neuronal system is obtained. The simulated adaptive dynamical sliding mode control signal $u(t)$ and the parameter estimate trajectory $\hat{g}(t)$ are shown in Fig. 9. Evidently, the control input $u(t)$ no longer exhibits a bang–bang behavior, and a sufficiently smooth control input signal is exactly obtained. Moreover, the trajectory associated with the estimation law for the unknown parameter g also shows an asymptotically stable convergence feature.

In conclusion, we have investigated in detail the synchronization control of coupled neurons from the viewpoint of possible applications in real neuronal systems. In biological experiments, the membrane potential of neurons can be recorded by microelectrodes. The control signal u, obtained from an external controller, should be fed back into the system via the same or another microelectrode in the form of electrical current. As one can see, the amplitude of the control signal is much smaller than the periodic stimulation signal and the membrane potential of coupled neurons, and thus has little influence on the intrinsic properties of individual cells. It is therefore suggested that the proposed control scheme can be applied to a wide class of uncertain neuronal systems.

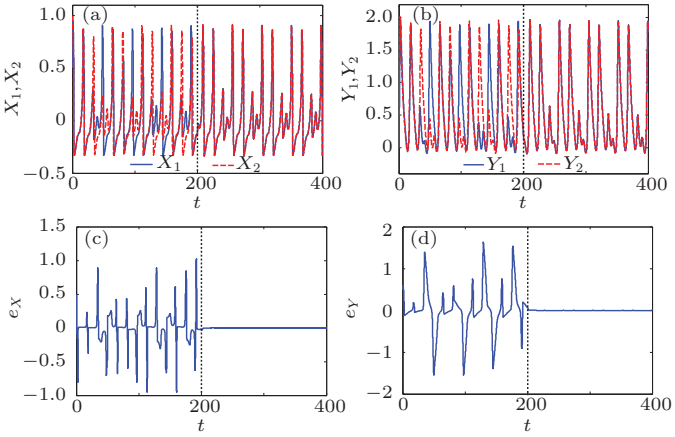


Fig. 7. (color online) Time courses of the coupled FHN neuron system with adaptive dynamical sliding mode control: (a) X_1 and X_2, (b) Y_1 and Y_2. The state errors: (c) $e_X = X_2 - X_1$, (d) $e_Y = Y_2 - Y_1$.

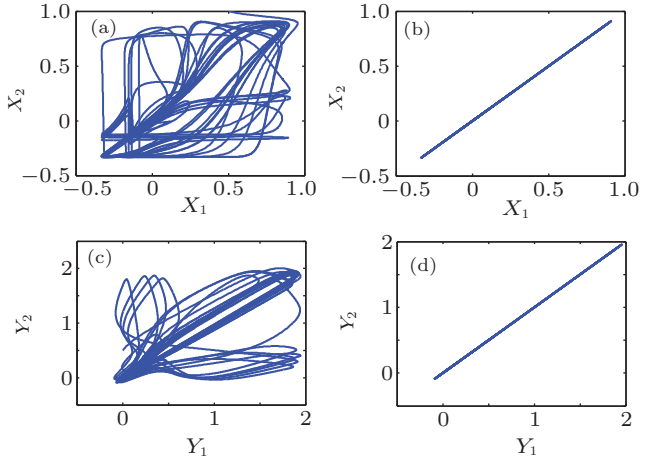


Fig. 8. (color online) Phase portraits of the neural system before control: (a) $X_1 - X_2$, (c) $Y_1 - Y_2$. Phase portraits of the neural system after control: (b) $X_1 - X_2$, (d) $Y_1 - Y_2$.

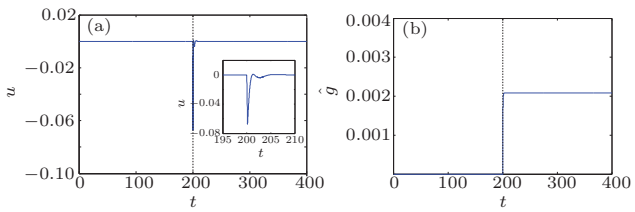


Fig. 9. (color online) (a) Adaptive dynamical sliding mode control signal $u(t)$. (b) The parameter estimate trajectory $\hat{g}(t)$.

4. Conclusions

In this paper, an adaptive backstepping dynamical sliding mode control scheme, which combines the merits of both the adaptive backstepping control and the dynamical sliding mode control, is used to synchronize two electrically coupled chaotic FHN neurons in the external electrical stimulation. A static adaptive sliding mode approach is used first to compensate an uncertainty which occurs in the control system. However, the control input exhibits a bang–bang behavior which may damage a biological neuron and change its intrinsic dynamics. In order to remove the bang–bang nature of the control input, an adaptive dynamical sliding mode control law is designed based on backstepping technique. It is shown that the proposed adaptive dynamical sliding mode control scheme can not only compensate for the system uncertainties, but also guarantee the stability of the synchronized error system. The simulation results demonstrate the effectiveness of the proposed adaptive controller and the smoothed control input signal.

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