Dependence of effective screening length in granular columns on bead and silo sizes and their ratio*

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Apparent mass measurements at the bottom of silos have been carried out. An important parameter in the Janssen model known as the effective screening length has been investigated for different bead and silo diameters as well as of their ratios. It is found that the effective screening length augments with the grain diameter d in addition to the granular column size. It is also revealed that λ exhibits stronger correlation with the bead diameter than that of the silo. This phenomenon is attributed to the reduced-shielding of the vertical stresses to the horizontal ones.

Keywords: granular systems, granular solids, static sand piles, granular compaction

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1. Introduction

Granular materials are found everywhere in our daily life and have vast applications in various branches of engineering like mechanical, agriculture, civil, electrical and pharmaceutical manufacturing.[1–3] They also play a significant role in many geological phenomena, such as landslides, erosion, and plate tectonics.[4–6] These materials exhibit many complex behaviors. Granular media lack characterization from the fundamental point of view, specifically the dense one. However, certain empiric engineering models exist, which describe the mean behaviors in a reasonable range. Such models have also been found to be helpful in investigating the stresses in granular media. The pressure measurement at the bottom of a silo is an indirect way of investigating the stresses in a silo. In this regard, the Janssen model is employed, which describes the mean pressure at the bottom of confined granular packing.[7] It shows that unlike hydrostatic, the bottom part of a silo does not experience augment in mean pressure as the height of the granular column increases. Rather it exhibits an exponential behavior since the top mass of the granular pile is carried away by the silo wall. The Janssen model has been verified in many experiments.[8–14] Specifically, Vanel and Clement devised a well controlled experiment to investigate the apparent mass and its fluctuation at the bottom of a granular pile descending with a slow velocity.[9] In other experiments, the side walls were pulled upward to fully mobilize the frictional forces of the packing. Those experiments also validated the Janssen model.

Although the Janssen model has been extensively investigated, however, few experimental studies are carried out where the variations of the characteristic length with the bead size in a granular column are explored. In this study, a series of experiments are performed to investigate the dependence of effective screening length λ on the diameters of granules d and cylinder D as well as their ratio D/d.

2. Experimental set-up and measurement procedure

The beads have been poured into a cylinder with the help of a funnel that is placed on the top of the silo; see Ref. [8] for details on the procedure. The base of the silo is formed by a cylindrical piston. The design of the piston does not allow it to touch the cylinder wall during its motion, and also no leakage of grains takes place. The piston is placed at a certain height inside the silo at the beginning of each experimental run. Then mass M beads are filled through the funnel and held above the cylinder. Once the cylinder has been filled with the beads, 30 s to 40 s are allowed so that the elastic energy amalgamated by the pressure sensor, may be relaxed. After that the piston is allowed to descend slowly with speed 0.02 mm/s, and it covers a total distance of 20 mm. The bottom end of the piston is fixed on the pressure sensor, by which the force exerted by the beads on the top horizontal surface of the piston can be measured. Hence the sensor measures the average static pressure of the whole pile, and the data is recorded by a computer throughout the process. The corresponding force experienced by the pressure sensor is known as instantaneous effective mass M_t, as it differs in value from the total filling mass M_f. The aim of allowing the granules to descend slowly is that the friction between the grains and the silo wall may attain the maximum value.

In these experiments, six cylinders and beads of various

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diameters have been used. The granules used in this experiment have diameters (d) of 3.0 mm, 4.0 mm, 6.0 mm, 7.0 mm, 8.0 mm, and 12.0 mm respectively, whereas the silos have internal diameters of 25.4 mm, 29.3 mm, 33.1 mm, 39.4 mm, 46.3 mm, and 53.80 mm respectively. Figure 1 shows the typical plots of instantaneous effective mass M_t with time. These plots have been obtained for the filling mass $M_f = 600$ g for different grain sizes. It can be observed that instantaneous effective mass M_t shows relatively higher fluctuations when the piston starts descending. Then eventually a comparatively stable state is achieved where the fluctuations are smaller. Here the average of 500 data points has been taken as a single value M_e. The inset of Fig. 1 illustrates the evolution of data points of instantaneous effective mass M_t. In order to ensure the reproducibility and the minimization of error, the experiment has been carried out 5 times. After that the values obtained are averaged to determine a single data point of effective mass M_e.[14]

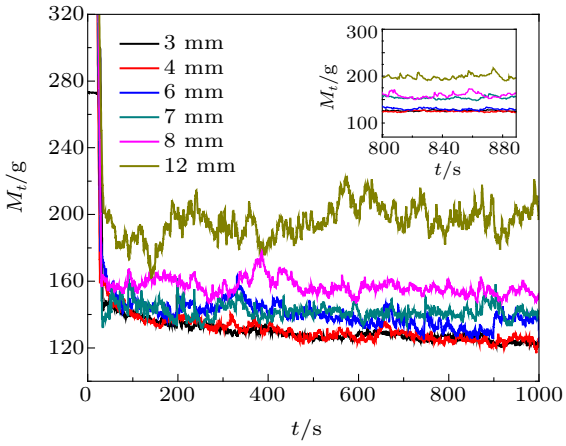


Fig. 1. (color online) Evolution of instantaneous effective mass with time, the inset shows the relatively stable state.

3. Results and discussion

In the first experimental trial, using the same diameter of cylinder, i.e. $D = 46.3$ mm, and varying the grain size, i.e., $d = 3.0$ mm, 4.0 mm, 6.0 mm, 7.0 mm, 8.0 mm, 12.0 mm, we investigate the influence on the effective mass. Figure 2 shows the variation in effective mass as a function of the filling mass. The solid lines correspond to fittings for various bead sizes. The dispersed points are the experimental results. It can be observed that when a small mass is poured in the silo, the apparent mass varies linearly with the filled mass. It implies that the hydrostatic behavior is more pronounced in this region. As the filling mass increases, the curves become almost parallel to the x axis, implying that the screening effect is more pronounced and M_e exhibits a saturated value with higher fluctuations. All saturation curves exhibit almost the same trend. However they divide into different curves and augment with the bead sizes. Clearly, this phenomenon needs further investigation, since the variation in the size of granules affects the stress propagation in granular columns.

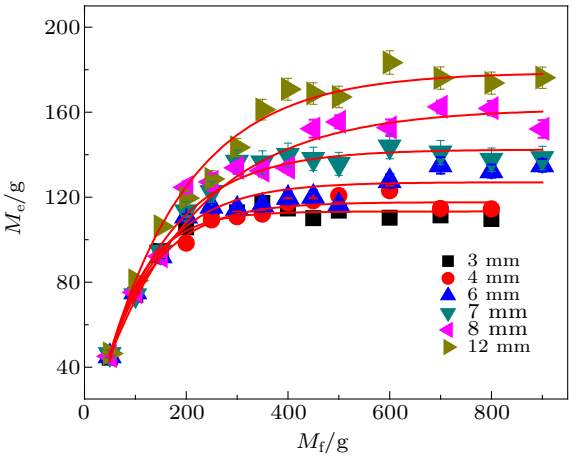


Fig. 2. (color online) Evolution of effective mass with the bead size.

The splitting and the saturation of the stress curves are in agreement with the prediction of the Janssen model. The model assumes that the granular column consists of continuous medium. The second assumption is that a fraction of stresses along the vertical direction are directed to the horizontal ones and are proportional to each other. An important assumption is that the friction between the granules and the cylinder wall has reached its maximum value and directed oppositely to the motion of the imposed velocity. This is somewhat a crude assumption, as it is not possible to describe which contacts between the granular beads and the confining wall have attained the maximum value and are upward.

Considering a horizontal slice of height $\mathrm{d}h$ and radius R, it has been assumed in the Janssen model that the granules' weight in the slice is counterbalanced by the variation of pressure surrounding the granules and the friction force. Hence the equation becomes

$$\frac{\pi R^2 \,\mathrm{d}\sigma_{zz}}{\mathrm{d}h}\,\mathrm{d}h + 2\pi R\mu K\sigma_{zz}\,\mathrm{d}h = \pi R^2 g\rho\,\mathrm{d}h, \tag{1}$$

through simplification, it reduces to

$$\frac{\mathrm{d}\sigma_{zz}}{\mathrm{d}h} + \frac{\sigma_{zz}}{\lambda} = \rho g, \tag{2}$$

while $\lambda = R/(2\mu K)$ is an important parameter in the Janssen model known as the effective screening length. It characterizes the height above which the weight is carried away by the boundary walls, whereas parameter K describes the conversion of vertical stresses into horizontal ones. It is easy to integrate this first-order differential equation, then we get

$$M_e = M_s(1 - \mathrm{e}^{-M_f/M_s}), \tag{3}$$

where the saturation mass can be determined as

$$M_s = \rho\pi(D/2)^3/(2\mu K), \tag{4}$$

here ρ is the material density, D is the internal diameter of the silo, μ is the static friction coefficient between the beads and

the wall surrounding the granular column.

Although in Eq. (3), there is no relation between the bead diameter and the saturation mass, by using Eq. (3), the data point lines can be simulated to determine the values of M_s. Following this procedure, saturation mass M_s for different d is further simulated separately and obtained, so the variation of the saturation mass with the bead diameter can be determined, as depicted in the inset of Fig. 3. It is revealed that M_s augments with the increase of the bead diameter. Using the values of M_s in Eq. (4), we can calculate μK, then determine effective screening length λ. In Fig. 3, the effective screening length is plotted as a function of the bead diameter. It is also revealed that the effective screening length λ increases as the bead diameter is increased. It may be attributed to the fact that the presence of larger diameter beads results in a smaller number of contacts between the silo wall and the beads. On the other hand, the smaller diameter of the beads in the silo gives rise to a larger number of contact points. Consequently, the larger diameter of beads deflects a smaller fraction of vertical stress to the horizontal ones, and a larger fraction of stress is transmitted to the piston of the granular column. Hence the screening effect is reduced in the case of larger diameters of grains and the saturation mass has higher values. This results in the augmentation of characteristic length λ.

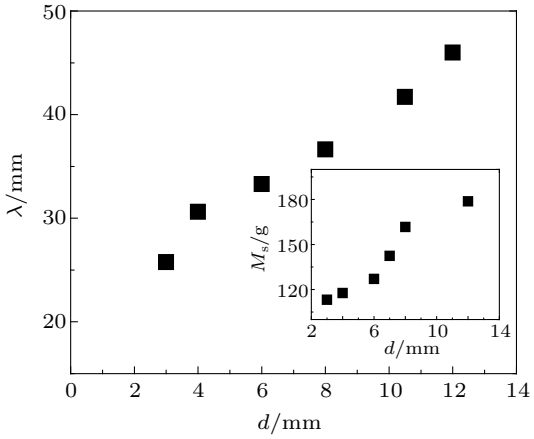


Fig. 3. Characteristic length λ and saturation mass (inset) versus the diameter of granules.

In the next experimental trial, the diameter of the silo is systemically varied while keeping the diameter of granules constant. The variation of the screening length in such a configuration is investigated. In Fig. 4, M_e as a function of M_f has been plotted for different silo diameters, i.e., $D = 25.4$ mm, 29.3 mm, 33.1 mm, 39.4 mm, 46.3 mm, and 53.8 mm. The glass spheres of 3 mm in diameter are used in every case. Each data line shown has an error margin less than 5%. Here the splitting of the data lines is in accordance with Eq. (4). Recently, Liu *et al*. have experimentally verified this relation.[14]

It is observed that the effective length augments in both cases, either the bead or the silo diameter is increased. In Fig. 5, the characteristic lengths for both cases are depicted. The squares represent the case where the silo size is kept constant, i.e., $D = 46.3$ mm, while the bead diameter is varied. The circles illustrate the case of varying the cylinder diameter whereas the bead size is kept constant, i.e., $d = 3$ mm. Figure 5 shows that the characteristic length augments not only with the increase in the size of the silo as expected from Eq. (4) but also with the grain size. It is also revealed in Fig. 5 that the characteristic length λ is more correlated with the variation in the bead diameter than the silo size. This is probably due to the fact that with the increase of the bead diameter, the less stress arches are supported by the silo walls, hence the deflection of vertical stresses towards the horizontal ones is reduced; consequently λ exhibits higher values.

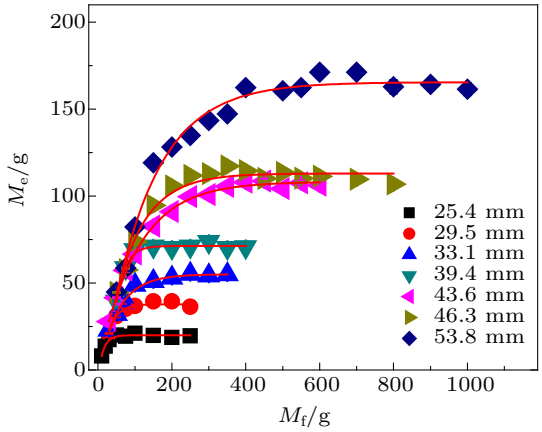


Fig. 4. (color online) Evolution of M_e with M_f for different silos.

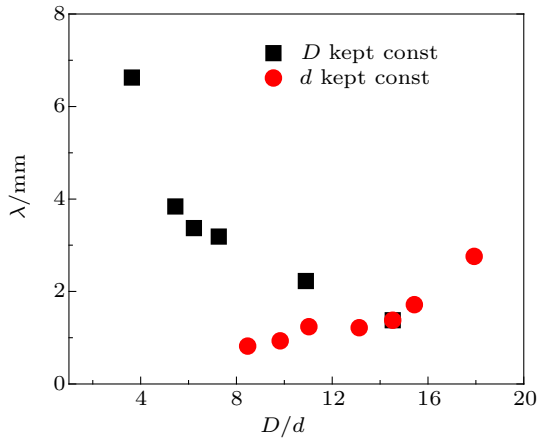


Fig. 5. (color online) Variation of characteristic length with D/d.

4. Conclusion

The dependencies of the effective screening length on bead and cylinder diameters have been investigated. It is found that the effective screening length is an increasing function of the bead diameter. Moreover, the behavior of λ with increasing bead and silo sizes has also been investigated. It is revealed that λ is more correlated and augments more with the former than the latter. Furthermore, it is also evident that the presence

of larger sizes of beads in a silo reduces the deflection of vertical stresses into the horizontal ones and hence increases the characteristic length of the silo. In future work, these results will be complemented by the effects of two important parameters, i.e., the roughness of the silo walls and the roughness of the beads.

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