Formula for average energy required to produce a secondary electron in an insulator*

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Based on a simple classical model specifying that the primary electrons interact with the electrons of a lattice through the Coulomb force and a conclusion that the lattice scattering can be ignored, the formula for the average energy required to produce a secondary electron (ε) is obtained. On the basis of the energy band of an insulator and the formula for ε, the formula for the average energy required to produce a secondary electron in an insulator (ε_i) is deduced as a function of the width of the forbidden band (E_g) and electron affinity χ. Experimental values and the ε_i values calculated with the formula are compared, and the results validate the theory that explains the relationships among E_g, χ, and ε_i and suggest that the formula for ε_i is universal on the condition that the primary electrons at any energy hit the insulator.

Keywords: width of forbidden band, electron affinity, average energy, insulator

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1. Introduction

The secondary electron emission is an important research topic.[1–10] Computational approaches based on the Monte–Carlo method allow us to calculate the secondary electron emission.[11–15] Modern technologies such as secondary electron dopant mapping[16,17] and critical dimension scanning electron microscopy[14,18] allow us to research the secondary electron emission. The average energy required to produce a secondary electron in an insulator (ε_i) is an important research area of secondary electron emission. Thus many authors have measured ε_i and obtained the empirical relationship between ε_i and the width of the forbidden band (E_g).[19–31] Many authors also have explained the empirical relationship using a free-particle approximation and the conservations of energy and momentum.[32,33]

Based on a simple classical model specifying that the primary electrons interact with the electrons of a lattice through the Coulomb force and a conclusion that the lattice scattering can be ignored, the energy of the secondary electrons (E_1) is deduced. Moreover, the formula for the average energy required to produce a secondary electron (ε) is obtained in this work. According to the energy band of an insulator, the conservation of energy, and the formula for ε, the formula expressing ε_i as a function of E_g and electron affinity χ is deduced. Experimental values and the ε_i values calculated with the formula are compared, and the results suggest that the theory explaining the relationships among E_g, χ, and ε_i is correct and that the formula for ε_i is universal for the estimation of ε_i on the condition that the primary electrons at any energy hit the insulator.

2. Energy of secondary electrons

The primary electrons in the energy range from 10 keV to 1000 keV enter the emitter perpendicularly and travel along a straight line. The minimum distance between an electron in the lattice and a primary electron is s, the time t is measured from the instant of minimum distance, as shown in Fig. 1, and the distance r between the electron of the lattice and the primary electron can be expressed as[34]

$$r = (s^2 + v^2 t^2)^{1/2}, \tag{1}$$

where v is the velocity of the primary electron.

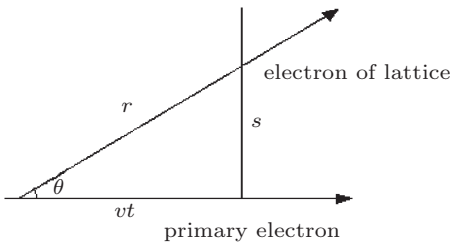


Fig. 1. Distance between an electron in the lattice and a primary electron.

The collisions of greatest interest are those in which the fast primary electron transfers a small amount of its energy to an electron of the lattice. In calculating the momentum transfer for such a collision, one may consider that the primary electron moves with a constant speed along a straight line and interacts with an electron of the lattice whose motion during the collision is negligible. In this approximation, the momentum

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transferred is perpendicular to the primary path and is given by the integral[34]

$$\Delta p = \int_{-\infty}^{\infty} \frac{e^2 \sin\theta}{r^2} \mathrm{d}t = \int_{-\infty}^{\infty} \frac{e^2 s}{(s^2+v^2t^2)^{3/2}} \mathrm{d}t = \frac{2e^2}{vs}, \quad (2)$$

where e is the electronic charge, and θ is the included angle shown in Fig. 1. Based on Eq. (2), E_1 can be written as[34]

$$E_1 = \frac{(\Delta p)^2}{2m} = \frac{e^4}{E_P s^2}, \quad (3)$$

where $E_P = mv^2/2$ is the primary energy, and m is the electron mass.

3. Number of secondary electrons per unit path length

We draw a cylindrical shell in the direction of the velocity of the primary electron, the cylindrical shell is in the radius range from s to $s+\Delta s$, the number of secondary electrons produced in the cylindrical shell of unit path length can be written as

$$\Delta n = (2\pi s \Delta s)N = \pi N \Delta(s^2), \quad (4)$$

where N is the number of secondary electrons per unit volume. An expression for Δn is produced from Eqs. (3) and (4) as follows:

$$\Delta n = \frac{\pi N e^4}{E_P} \Delta\left(\frac{1}{E_1}\right) = \frac{\pi N e^4}{E_P} \frac{\Delta E_1}{E_1^2}. \quad (5)$$

Based on Eq. (1), the number of secondary electrons produced in the cylinder of radius s and unit path length can be written as

$$n = \sum \Delta n = \frac{\pi N e^4}{E_P} \int \frac{\mathrm{d}E_1}{E_1^2}. \quad (6)$$

4. The average energy

Based on Eqs. (3), (5), and (6), the total energy of secondary electrons produced in the cylinder of unit path length can be written as

$$E_{per} = \sum \Delta n E_1 = \frac{\pi N e^4}{E_P} \int \frac{\mathrm{d}E_1}{E_1}. \quad (7)$$

The average rate of energy loss per unit path length due to the collisions with atoms and atomic electrons, with which bremsstrahlung quanta are emitted, is the radiative stopping power;[35] the average rate of energy loss per unit path length due to the Coulomb collisions that result in the ionization and excitation of atoms is the collision stopping power;[35] for the primary electrons, the sum of the collision and the radiative stopping powers is the average rate of energy loss per unit path length.[35] For the primary electrons in the energy range from 10 keV to 1000 keV, the collision stopping power is much larger than the radiative stopping power,[36] that is, the energy lost due to the lattice is minor. Thus, it is concluded that the energy lost due to the lattice can be ignored. Therefore, E_{per} is approximately equal to the average rate of energy loss per unit path length, and we obtain

$$-\frac{\mathrm{d}E_P}{\mathrm{d}x} = \sum \Delta n E_1 = \frac{\pi N e^4}{E_P} \int \frac{\mathrm{d}E_1}{E_1}, \quad (8)$$

where x is the distance from the surface of the insulator.

According to Eqs. (6) and (8) and the definition of ε, ε can be written as

$$\varepsilon = \int \frac{\mathrm{d}E_1}{E_1} \Big/ \int \frac{\mathrm{d}E_1}{E_1^2}. \quad (9)$$

5. The average energy in an insulator

When the primary electrons hit an insulator, most secondary electrons come from the valence band. Therefore, based on the energy band of the insulator shown in Fig. 2, the minimum energy required to produce a secondary electron E_{imin} is E_g.

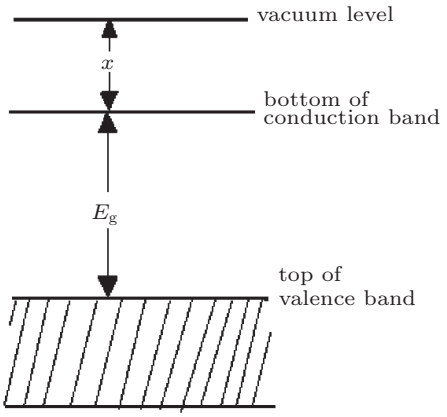


Fig. 2. Energy band of an insulator.

The secondary electrons are conventionally defined as having exit energies $E_s < 50$ eV, while the backscattered electrons have $E_b > 50$ eV.[37] Therefore, based on the energy band of the insulator, the conservation of energy, and the definition of the secondary electrons, the maximum energy required to produce a secondary electron can be written as

$$E_{imax} = \chi + E_g + 50. \quad (10)$$

On the basis of Eq. (9), E_{imin}, and E_{imax}, ε_i can be written as

$$\varepsilon_i = \int_{E_g}^{(\chi+E_g+50)} \frac{\mathrm{d}E_1}{E_1} \Big/ \int_{E_g}^{(\chi+E_g+50)} \frac{\mathrm{d}E_1}{E_1^2} = \frac{E_g(\chi+E_g+50)}{(50+\chi)} \ln\left(\frac{\chi+E_g+50}{E_g}\right). \quad (11)$$

6. Discussion

As can be seen from Table 1, the ε_i calculated with Eq. (11) and the parameters in Table 1 agree well, on the whole, with the experimental values of insulators. Therefore, it is concluded that Eq. (11) and the theory that explains the relationships among E_g, χ, and ε_i are correct.

Table 1. Comparison between experimental and calculated ε_i.

Insulator	E_g/eV	χ/eV	Experimental ε_i/eV	Calculated ε_i/eV
InSb	0.17[19]	4.6[38]	1.2[19]	0.985
PbS	0.41[20]	3.4[38]	2.0[20]	2.015
Ge	0.67[21]	4.1[38]	2.91[21]	2.987
Si	1.12[22]	4.0[38]	3.63[22]	4.454
GaAs	1.42[23]	4.0[38]	4.35[23]	5.340
CdTe	1.52[24]	4.84[39]	4.46[24]	5.644
GaP	2.22[25]	1.96[40]	6.54[25]	7.395
AgBr	2.68[26]	5.81[41]	5.8[26]	8.659
SiC	2.86[27]	3.2[42]	7.5–9.0[27], 6.9[28]	8.968
C	5.47[29]	6.3[38]	16[29], 13.1[30]	14.548
SiO_2	8.4[31]	1.0[38]	18[31]	19.137

For a secondary electron emitter, ε_i is a constant and is not a function of E_P.[43,44] Therefore, equation (11) for ε_i, which is deduced under the condition that the primary electrons in the energy range from 10 keV to 1000 keV hit the insulator, is universal and can be used to estimate ε_i at any energy.

7. Conclusion

Based on 1) a simple classical model specifying that the primary electrons interact with the electrons of a lattice through the Coulomb force, 2) our conclusion that the lattice scattering can be ignored, 3) the definition of ε, and 4) the energy band of the insulator, the formula for ε_i is successfully deduced and shown to be valid by the experimental results in this paper, and the formula for ε_i is shown to be universal for the estimation of ε_i under the condition that the primary electrons at any energy hit the insulator. The theory that explains the relationships among E_g, χ, and ε_i is validated.

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