Tunneling effect in cavity-resonator-coupled arrays

Ma Hua(马 华)[a)†], Qu Shao-Bo(屈绍波)[b)], Liang Chang-Hong(梁昌红)[c)], Zhang Jie-Qiu(张介秋)[b)], Xu Zhuo(徐 卓)[a)], and Wang Jia-Fu(王甲富)[b)]

[a)] *Electronic Materials Research Laboratory, Key Laboratory of Ministry of Education, Xi'an Jiaotong University, Xi'an* 710049, *China*
[b)] *The College of Science, Air Force University of Engineering, Xi'an* 710051, *China*
[c)] *The College of Electronic Engineering, Xidian University, Xi'an* 710126, *China*

(Received 21 April 2012; revised manuscript received 15 November 2012)

The quantum tunneling effect (QTE) in a cavity-resonator-coupled (CRC) array was analytically and numerically investigated. The underlying mechanism was interpreted by treating electromagnetic waves as photons, and then was generalized to acoustic waves and matter waves. It is indicated that for the three kinds of waves, the QTE can be excited by cavity resonance in a CRC array, resulting in sub-wavelength transparency through the narrow splits between cavities. This opens up opportunities for designing new types of crystals based on CRC arrays, which may find potential applications such as quantum devices, micro-optic transmission, and acoustic manipulation.

Keywords: quantum tunneling effect, surface plasmon, cavity-resonator, photonic crystal

PACS: 78.66.Bz, 42.79.Dj, 71.36.+c, 73.20.Mf **DOI:** 10.1088/1674-1056/22/5/057805

1. Introduction

The extraordinary optical transmission (EOT) through metallic films with a periodic array of sub-wavelength holes has attracted considerable interest in the last decade since the report by Ebbesen *et al.*[1] Unlike the prediction of standard aperture theory in the case of a single tiny hole,[2] the EOT phenomenon is typically associated with the excitation of resonances, including Fabry-Perot-like resonance inside narrow slits[3–5] and the surface plasmon resonance that enhances light tunneling through sub-wavelength holes.[6] Another mechanism, proposed by Zhou *et al.*,[7] makes electromagnetic (EM) waves tunnel through the classically-opaque flat slab with negative permittivity, showing perfect transparency accompanied by high magnetic fields at interfaces. A similar phenomenon involving a face-centered-cubic array of close-packed tungsten carbide beads in water was explored by Yang *et al.*,[8] in which ultrasonic tunneling in three-dimensional phononic crystals was demonstrated. In fact, the underlying mechanism of such phenomena can be classified as the tunneling effect, which is one of the most interesting subjects in quantum mechanics. Due to its potential applications in sub-wavelength photolithography, near-field microscopy, wavelength tunable filters, optical modulators and flat-panel displays, the tunneling effect has been widely investigated and generalized to classical areas such as EM waves[9–16] and acoustic waves.[17–19]

In this paper, we present a particular investigation into sub-wavelength transparency through a cavity-resonator-coupled (CRC) array. Compared in terms of mechanism with those instances of EOT mentioned above, the sub-wavelength transparency through a CRC array also results from the resonance-based tunneling effect. However, such a tunneling effect is caused by cavity resonance, so, as we shall demonstrate, it is in fact a quantum tunneling effect (QTE), which may result in some special energy-band structures related to the CRC array, and thus can be used to design new types of crystals. In the following, the physical mechanism is first demonstrated in the EM wave case by taking the photon as the field quantum, and then is generalized to the acoustic wave and matter wave cases.

2. Theoretical analysis

For the sake of simplicity, we consider a one-dimensional (1D) CRC array in two-dimensional space. As shown schematically in Fig. 1(a), the CRC array is a series system of N cavity resonators (CRs), and the "sidewall" between two adjacent CRs has a narrow split (NS). The length and the width of the NS are extremely small compared with the size of the cavity, causing the split coupling effect between adjacent cavities. From the viewpoint of classical theory, for EM waves propagating through such a system, the transparency is very low if the wavelength is far bigger than the size of the NS. However, in quantum mechanics, this may be changed since the particles localized in a potential well can traverse the potential barriers via tunneling effects. Therefore, it is useful to establish equivalence between matter waves and EM waves in our discussion by taking the photon as the field quantum. In fact, by comparing the static Schrodinger equation $((\hbar^2/2m)\nabla^2 - (V(r) - E))\phi(r) = 0$ with the time-harmonic EM Helmholtz equation $(\nabla^2 + n_{\text{eff}}^2\omega^2/c^2)\varphi(r) = 0$, where $\varphi(r)$ represents the EM field intensity, we find that these two equations have the same form if we regard the optical index as

†Corresponding author. E-mail: mahuar@163.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

the static potential.[20,21] For the sake of holding the conceptual coherence between the optical index and the static potential $V(r)$, we adopt $-n_{\rm eff}^2\omega^2/c^2$ as the equivalent of the static potential and thus analogically create the concept of an EM potential well, where $n_{\rm eff}$, ω, and c are the effective refractive index of the media, the angle frequency of the EM wave, and the velocity of light in a vacuum, respectively.

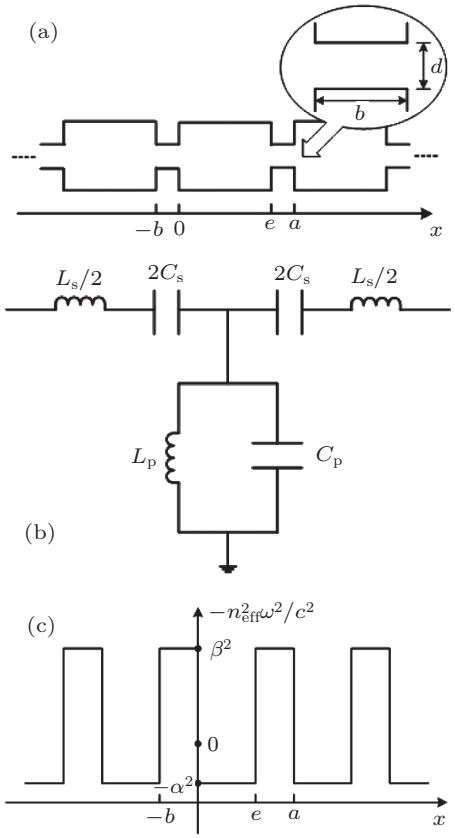


Fig. 1. (a) Sketch of a CRC array and (b) the equivalent circuit for an NS. The CRC array is a series system composed of N CRs with their length being $e = a - b$. Between the two adjacent resonators, there is an NS (shown in the inset) with the length and the width being b and d, respectively. By taking the photon as the field quantum, and introducing $-n_{\rm eff}^2\omega^2/c^2$ as the EM potential, the potential wells ($-n_{\rm eff}^2\omega^2/c^2 = -\alpha^2$) and barriers ($-n_{\rm eff}^2\omega^2/c^2 = \beta^2$) are established in panel (c) for a CRC array, representing the effective behaviors of the NSs.

We adopt the transmission line theory to obtain the effective refractive index of the NS in the 1D case. As shown in Fig. 1(b), the equivalent circuit of the NS element in the CRC array is depicted, and the dispersion relation can be written as[22]

$$\cos\left(n_{\rm eff}\frac{\omega b}{c}\right) = 1 - \frac{\omega^2}{2\omega_0^2}\left(1 - \frac{\omega_s^2}{\omega^2}\right)\left(1 - \frac{\omega_{\rm p}^2}{\omega^2}\right), \qquad (1)$$

where $\omega_0 = 1/\sqrt{L_sC_p}$, $\omega_s = 1/\sqrt{L_sC_s}$, and $\omega_{\rm p} = 1/\sqrt{L_pC_p}$; and L_s, C_s, L_p, and C_p are the equivalent series inductance, series capacitance, shunt inductance, and shunt capacitance, respectively.

In the long wavelength limit, equation (1) can be rewritten as

$$n_{\rm eff}^2\frac{\omega^2}{c^2} = \frac{\omega^2}{b^2\omega_0^2}\left(1 - \frac{\omega_s^2}{\omega^2}\right)\left(1 - \frac{\omega_{\rm p}^2}{\omega^2}\right), \qquad (2)$$

which indicates that the NS has a forward pass-band, stop-band, or backward pass-band depending on the frequency. As shown in Fig. 1(b), ω_s and $\omega_{\rm p}$ are in fact the series and shunt resonance frequencies of the equivalent circuit, respectively, and $\omega_s \neq \omega_{\rm p}$ holds true in most cases (we assume $\omega_s < \omega_{\rm p}$). As a result, for $\omega_s < \omega < \omega_{\rm p}$, $n_{\rm eff}^2$ is negative, implying that the NS behaves like an opaque medium when the frequency falls into this band. According to the quantum mechanics, as shown in Fig. 1(c), the EM potential $-n_{\rm eff}^2\omega^2/c^2$ is higher when $n_{\rm eff}^2 < 0$, so each element, composed of one cavity and two NSs, forms a potential well at some specific bands of frequencies where the cavity acts as the well and the NS as the barrier. On this basis, we carry out theoretical analysis on the transparency of the CRC array by solving Maxwell equations.

Suppose that the medium in the cavity regions, and the equivalent in the NSs, are linear, homogeneous and isotropic. The 1D wave equation of EM wave in such media is

$$\left(\frac{\partial^2}{\partial x^2} - n_{\rm eff}^2\frac{\partial^2}{c^2\partial t^2}\right)\varphi(x,t) = 0, \qquad (3)$$

where $\varphi(x,t)$ represents the scalar potential or field intensity. In our solution, the case of EM waves with transverse electric (TE) polarization is considered as an example. Thus, $\varphi(x,t)$ represents specifically the z component E_z of electric fields. With the time dependence $\partial/\partial t = -{\rm j}\omega$, the harmonic form of Eq. (3) can be rewritten as

$$\left(\frac{{\rm d}^2}{{\rm d}x^2} + n_{\rm eff}^2\frac{\omega^2}{c^2}\right)\varphi(x) = 0, \qquad (4)$$

where ω is the angle frequency.

In an N-element CRC array, the transmitted EM wave is of the Bloch form, thus the field variable $\varphi(x)$ can be expressed as $\varphi(x) = \exp({\rm j}kx)u(x)$, where k and $u(x)$ are the Bloch wave vector and the Bloch function, respectively. The differential expression of $\varphi(x)$ is

$$\frac{{\rm d}\varphi}{{\rm d}x} = {\rm e}^{{\rm j}kx}\frac{\rm d}{{\rm d}x}u + {\rm j}k{\rm e}^{{\rm j}kx}u. \qquad (5)$$

Then, equation (4) can be rewritten as

$$\frac{{\rm d}^2}{{\rm d}x^2}u + 2{\rm j}k\frac{\rm d}{{\rm d}x}u + \left(n_{\rm eff}^2\frac{\omega^2}{c^2} - k^2\right)u = 0. \qquad (6)$$

As shown in Fig. 1(c), in the CR region $0 < x < e$, $n_{\rm eff}^2$ is equal to 1. Introducing $\alpha^2 = \omega^2/c^2$ into Eq. (6) yields

$$\frac{{\rm d}^2}{{\rm d}x^2}u + 2{\rm j}k\frac{\rm d}{{\rm d}x}u + (\alpha^2 - k^2)u = 0. \qquad (7)$$

Accordingly, the general solution of Eq. (7) is

$$u(x) = A_0{\rm e}^{{\rm j}(\alpha-k)x} + B_0{\rm e}^{-{\rm j}(\alpha+k)x}. \qquad (8)$$

However, in the NS region $-b < x < 0$, n_{eff}^2 is governed by Eq. (2), and is negative for $\omega_s < \omega < \omega_p$. Let

$$\beta^2 = -n_{\text{eff}}^2 \frac{\omega^2}{c^2} = \frac{\omega^2}{b^2\omega_0^2}\left(\frac{\omega_p^2}{\omega^2} - 1\right)\left(1 - \frac{\omega_s^2}{\omega^2}\right). \quad (9)$$

Substituting Eq. (9) into Eq. (6) yields

$$\frac{d^2}{dx^2}u + 2jk\frac{d}{dx}u - (\beta^2 + k^2)u = 0. \quad (10)$$

The general solution of Eq. (10) is consequently

$$u(x) = C_0 e^{(\beta - jk)x} + D_0 e^{-(\beta + jk)x}. \quad (11)$$

In Eqs. (8) and (11), A_0, B_0, C_0, and D_0 are constants determined by the spatial period of the CRC array, the continuous conditions of $u(x)$, and the differential coefficient $du(x)/dx$. Applying all these physical constraints at positions $-b$, e, and a, respectively, we finally obtain the dispersion relation of Bloch waves in the CRC array

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta}\sinh(\beta b)\sin(\alpha e) + \cosh(\beta b)\cos(\alpha e) = \cos(ka). \quad (12)$$

Equation (12) is actually the Kronig–Penney equation,[23] which was established in the 1930s and has been successfully used to explain the energy band structure of electronic crystals by virtue of the single electron assumption. Here, we obtain the same result for the 1D CRC array for EM waves, implying that there is a special energy band structure related to the CRC array and that the waves with frequencies in the pass-bands can be perfectly transmitted. As n_{eff}^2 of NSs in frequency regime $\omega_s < \omega < \omega_p$ is negative, an NS is thus equivalent to an opaque medium, or to the potential barrier of quantum mechanics. Consequently, the existence of pass-bands for a CRC array or, in other words, the propagation of the light in a CRC array, must be induced by the excitation of tunneling effects.

3. Numerical simulation

To confirm the above prediction, equation (12) is solved via a numerical method. Let

$$\phi(\omega) = \frac{\beta^2 - \alpha^2}{2\alpha\beta}\sinh(\beta b)\sin(\alpha e) + \cosh(\beta b)\cos(\alpha e), \quad (13)$$

and draw the curve of $\phi(\omega)$ versus ω. We also plot the lines $\phi(\omega) = \pm 1$, which intersect with the $\phi(\omega) \sim \omega$ curve ruled by Eq. (13). As shown in Fig. 2(a), it can be found that $\phi(\omega)$ oscillates with the frequency, indicating that the pass-bands and the stop-bands appear alternately. Namely, in the range of $|\cos(ka)| \leq 1$, k is real, representing the pass-bands; whereas in the range of $|\cos(ka)| > 1$, k is imaginary, representing the stop-bands. Such a result means that the tunneling effect must be excited in the CRC array when the frequency falls into the pass-bands.

In fact, the CRC array, as a whole, can be regarded as a photonic crystal (PC).[24,25] Therefore, the corresponding energy band characteristics can also be observed from the $\phi(\omega)$–ω curve. Since the boundaries of the pass-bands occur at $ka = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$ and the curve of $\phi(\omega)$–ω is a divergent oscillation, the bandwidth decreases with frequency. Note, as shown in Fig. 2(b), that such a bandwidth variation with frequency is not found in regular PCs. This is because n_{eff}^2 of an NS, in this consideration, is frequency dependent in accord with Eq. (2), whereas it is generally assumed to be constant for regular PCs.

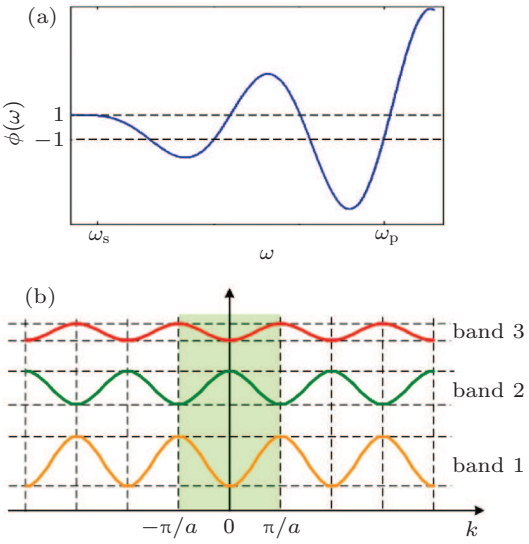


Fig. 2. (color online) In panel (a), the curve of $\phi(\omega)$–ω ruled by Eq. (13) is plotted, which is intersected by lines $\phi(\omega) = \pm 1$, indicating the pass-bands ($|\phi(\omega)| \leq 1$) and the stop-bands ($|\phi(\omega)| > 1$). According to the $\phi(\omega)$–ω curve, the energy band structure of the CRC array is sketched in panel (b), showing a decreasing tendency of the bandwidth with frequency.

The excitation of QTE in a CRC array can be attributed to cavity resonances. To demonstrate this, we analyze the field distributions of the EM wave in a CRC array, and verify the theoretical prediction through numerical simulations. We know that EM waves propagating through a periodic structure must be of the Bloch form, which, as a final form, can be expressed as the product of field amplitude and the modulating function $y_h(x,t)$. Adopting the assumptions of $b \ll a$ and $\partial/\partial t = -j\omega$, the spatial modulating function $y_h(x)$ can be factored into two parts: one of them is the resonance term $\sin(m\pi x'/a)$ (where $0 \leq x' \leq a$) in each cavity region for the m-order TE wave, and the other is the Bloch term $\cos(2h\pi x/Na)$. As a result, the spatial modulating function at the first-order resonance is written as

$$y_h(x) = \left|\sin\left(\frac{\pi}{a}x\right)\right| \cos\left(\frac{2h\pi}{Na}x\right), \quad (14)$$

where a is the length of the CR, N the total number of the CRs, and $h = 1, 2, 3, \ldots, N$ is the order of the Bloch wave vectors. In the simulations, we considered a CRC array composed of 18 CRs ($N = 18$). We plotted the curves of $y_h(x)$–x for $h = 1$, 2,

3, 4 and simulated the field distributions for EM waves propagating through the CRC array at the first-order resonance. As shown in Fig. 3, the curves of $y_h(x)$–x are in excellent agreement with the simulated field distributions, indicating that EM waves just at the eigenfrequency of the cavity can exist in the CRC array as the standing modes, and that the number of the Bloch waves at the same eigenfrequency is N, which is in good accord with the general case of crystals. Other simulations were also performed, which indicate that if the frequency is away from the eigenfrequency, there will be no EM waves in the CRC array. This additionally gives evidence of the key role of cavity resonances in exciting the QTE.

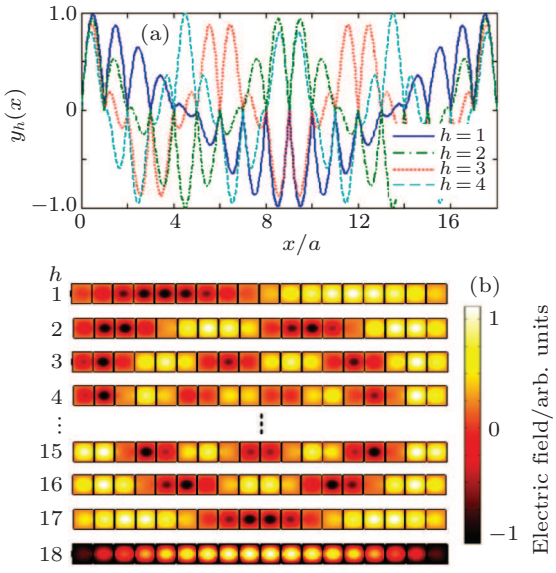


Fig. 3. (color online) A CRC array composed of 18 CRs ($N = 18$) is considered in the simulations. In panel (a), the modulating function $y_h(x)$ versus x ruled by Eq. (13) is plotted, where $h = 1, 2, 3, 4$. In panel (b), the field distributions for TE polarization at the first-order resonance are displayed, where $h = 1, 2, 3, \ldots, 18$.

The above discussion is focused on the EM wave case. However, the conclusion can be generalized to acoustic waves, matter waves, and the like. In principle, such a generalization is based on the same quantum mechanism and on the same wave equation form of these waves. For instance, the 1D acoustic wave equation in ideal liquids has the same form as Eq. (3), and the continuous condition for acoustic waves is similar to that of EM waves. By opening NSs in the walls of some acoustic CRs, and connecting those CRs through split coupling, an acoustic CRC array can be built. We apply the method of dealing with the 1D EM equation to solve the acoustic wave case, and finally arrive at the dispersion relation of acoustic Bloch waves in such an array, which is just Eq. (12). Similarly, if the method described above is adopted to solve the time-independent Schrodinger equation in a 1D quantum CRC array, the same result for matter waves can also be obtained. All these indicate that, for EM waves, acoustic waves and matter waves, respectively, the QTE can be excited by cavity resonance, causing sub-wavelength transparency through the related NS, and that the dispersion relations for these three kinds of waves in the related CRC array can be described by the same expression, namely Eq. (12).

4. Conclusion

In conclusion, we have made specific investigations of QTE in the three cases of EM waves, acoustic waves, and matter waves. For EM waves, we obtain the effective refractive index of an NS by using the transmission line theory, and regard such an NS as a frequency-dependent opaque medium. Our research shows that in a CRC array, the QTE can be excited by cavity resonances, causing sub-wavelength transparency through the NS, and that such a phenomenon, as a quantum effect, can be generalized to acoustic waves and matter waves. The most important theoretical result is Eq. (12), which was established by Kronig and Penney,[23] initially to explain the energy band structures of electronic crystals, while in this paper, it is generalized to a CRC array to explain the QTE for EM waves, acoustic waves, and matter waves. The theoretical analyses are in good agreement with the numerical simulations. Our research thus opens up opportunities for designing new types of crystals based on CRC arrays, which may find potential applications in quantum devices, micro-optic transmission, acoustic manipulation, and the like.

References

[1] Ebbesen T W, Lezec H J, Ghaemi H F, Thio T and Wolff P A 1998 *Nature* **391** 667
[2] Bethe H A 1944 *Phys. Rev.* **66** 163
[3] Porto J A, Garcia-Vidal F J and Pendry J B 1999 *Phys. Rev. Lett.* **83** 2845
[4] Takakura Y 2001 *Phys. Rev. Lett.* **86** 5601
[5] Yang F and Sambles J R 2002 *Phys. Rev. Lett.* **89** 063901
[6] Raether H 1988 *Surface Plasmons* (New York: Springer)
[7] Zhou L, Wen W J, Chan C T and Sheng P 2005 *Phys. Rev. Lett.* **94** 243905
[8] Yang S X, Page J H, Liu Z Y, Cowan M L, Chan C T and Sheng P 2002 *Phys. Rev. Lett.* **88** 104301
[9] Xu H, Wang Z Y, Hao J M, Dai J J, Ran L X, Kong J A and Zhou L 2008 *Appl. Phys. Lett.* **92** 041122
[10] Hibbins A P and Sambles J R 2004 *Phys. Rev. Lett.* **92** 143904
[11] Liu W C and Tsai D P 2002 *Phys. Rev. B* **65** 155423
[12] Salomon L, Grillot F, Zayats A V and Fornel F D 2001 *Phys. Rev. Lett.* **86** 1110
[13] Moreno L M, Vidal F J, Lezec H J, Pellerin K M, Thio T, Pendry J B and Ebbesen T W 2001 *Phys. Rev. Lett.* **86** 1114
[14] Yao J and Ye Y H 2012 *Chin. Phys. Lett.* **29** 047802
[15] Li H, Sheng C X and Chen Q 2012 *Chin. Phys. Lett.* **29** 054201
[16] Liu Y S, Lu H F, Xu X L, Gong M G, Liu L and Yang Z 2011 *Chin. Phys. Lett.* **28** 057803
[17] Ke M Z, He Z J, Peng S S, Liu Z Y and Shi J 2007 *Phys. Rev. Lett.* **99** 044301
[18] Lima M M, Kosevich Y A, Santos P V and Cantarero A 2010 *Phys. Rev. Lett.* **104** 165502
[19] Naber W J, Fujisawa T, Liu H W and Wiel W G 2006 *Phys. Rev. Lett.* **96** 136807
[20] Qian B C 1993 *Quantum Mechanics* (California: Electronic Industry Press) pp. 33–37 (in Chinese)
[21] Baudon J, Hamamda M, Grucker J, Boustimi M, Perales F, Dutier G and Ducloy M 2009 *Phys. Rev. Lett.* **102** 140403
[22] Caloz C and Itoh T 2003 *IEEE-MTT Int'l Microwave Symp.* Vol. 1 June, 2003 Philadelphia, PA, pp. 195–198
[23] Kronig R and Penney W G 1931 *Proc. Roy. Soc. A* **130** 499
[24] Yablonovitch E 1987 *Phys. Rev. Lett.* **58** 2059
[25] John S 1987 *Phys. Rev. Lett.* **58** 2486