Dynamics of optical rogue waves in inhomogeneous nonlinear waveguides*

Zhang Jie-Fang(张解放)[a)†], Jin Mei-Zhen(金美贞)[a)], He Ji-Da(何纪达)[b)], Lou Ji-Hui(楼吉辉)[b)], and Dai Chao-Qing(戴朝卿)[c)]

[a)] *Zhejiang University of Media and Communications, Hangzhou* 310018, *China*
[b)] *Institute of Nonlinear Physics, Zhejiang Normal University, Jinhua* 321004, *China*
[c)] *School of Sciences, Zhejiang Agriculture and Forestry University, Lin'an* 311300, *China*

(Received 26 June 2012; revised manuscript received 8 October 2012)

We propose a unified theory to construct exact rogue wave solutions of the (2+1)-dimensional nonlinear Schrödinger equation with varying coefficients. And then the dynamics of the first- and the second-order optical rogues are investigated. Finally, the controllability of the optical rogue propagating in inhomogeneous nonlinear waveguides is discussed. By properly choosing the distributed coefficients, we demonstrate analytically that rogue waves can be restrained or even be annihilated, or emerge periodically and sustain forever. We also figure out the center-of-mass motion of the rogue waves.

Keywords: rogue wave, (2+1)-dimensional nonlinear Schrödinger equation, inhomogeneous nonlinear waveguides

PACS: 42.65.–k, 05.45.Yv, 42.65.Tg **DOI:** 10.1088/1674-1056/22/5/054208

1. Introduction

Rogue waves, also known as freak waves, monster waves, killer waves, giant waves, and extreme waves, are first observed in the sea and are a fascinating destructive phenomenon appearing in nature.[1] Rogue waves are a concept introduced by Draper.[2] Peregrine first presented the basic solution (known as the Peregrine soliton)[3] to describe the rogue wave phenomenon. But rogue waves have not been fully explained so far and were considered as a myth for a long time.[4–6] Recently, the idea of rogue waves has been extended far beyond oceanic expanses. The concept has been applied to pulses emerging in the nonlinear oceanography,[1,7,8] the nonlinear optics,[9–16] the propagation of acoustic waves in superfluids,[17,18] the mean-field theory of the Bose–Einstein condensates,[19–22] optical cavities,[23] ion-acoustic and Alfvén wave propagations in plasmas,[24] the propagation of acoustic-gravity waves in the atmosphere,[25] and even in finance.[26] One possible mathematical model for rogue waves uses the rational solutions of the nonlinear Schrödinger equation (NLSE). In China, relevant studies are still scarce, for example, Yan investigated the financial one- and two-rogon,[26–29] and He *et al.* investigated the rogue wave solutions of the new nonautonomous NLSE and the derivative NLSE.[30–32]

Optical rogue waves, discovered by Solli *et al.*,[9] constitute a fascinating topic in modern nonlinear optics.[12,13] At present, most of the knowledge on the optical rogue waves is from studies of the supercontinuum generation in optical fibers under a variety of operating conditions in the propagation regimes ranging from continuum wave (CW) to femtosecond pulses, and has been shown to share a great similarity with the waves' hydrodynamical counterparts.[4] As far as we know, most of the experimental and theoretical studies are focused on optical pulses propagating in the nonlinear optical material governed by the (1+1)-dimensional nonlinear Schrödinger equation,[33–41] the higher-dimensional cases have been less investigated.[42–45] The goal of this paper is to construct the exact rogue wave solutions of the (2+1)-dimensional NLSE with varying coefficients and investigate the dynamics of the optical rogue waves propagating in inhomogeneous nonlinear waveguides.

2. Rogue wave solutions of (2+1)-dimensional NLSE with varying coefficients

We model the generation of these rogue waves using the following (2+1)-dimensional NLSE in inhomogeneous nonlinear waveguides with distributed coefficients:[34,39,40,42]

$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{\beta(z)}{2}\Delta_{\perp}u+\chi(z)|u|^2u=\mathrm{i}g(z)u, \tag{1}$$

where $u(z,x,y)$ is the complex envelope of the electrical field in the moving frame, z is the normalized distance of propagation, x and y are the normalized coordinates in the transverse plane, and $\Delta_{\perp}$ is the Laplacian (diffraction operator) acting on transverse coordinates (x,y). The function $\beta(z)$ describes the management of diffraction, $g(z)$ denotes the gain ($g>0$) or loss ($g<0$), and $\chi(x)$ represents the focusing ($\chi>0$) or defocusing ($\chi<0$) cubic nonlinearity.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11072219 and 11005092).
†Corresponding author. E-mail: jfzhang2002@yahoo.com.cn
© 2013 Chinese Physical Society and IOP Publishing Ltd

In order to investigate the dynamic properties of the optical rogue wave solution for Eq. (1), we employ the similarity transformation

$$u = \rho(z)U(Z(z),X(z,x,y))\exp[\mathrm{i}\varphi(z,x,y)], \tag{2}$$

where functions $\rho(z)$ and $\varphi(z,x,y)$ represent the amplitude and the phase, respectively, the real function X is the similarity variable, and Z is a real function of z, which denotes the effective propagation distance. The complex function $U(Z,X)$ satisfies the following NLSE with constant coefficients:

$$\mathrm{i}U_Z + \frac{1}{2}U_{XX} + |U|^2U = 0, \tag{3}$$

which is obtained by substituting Eq. (2) into Eq. (1) with the following set of differential equations being satisfied:

$$\rho_z - \frac{1}{2}\beta\rho(\varphi_{xx}+\varphi_{yy}) + g\rho = 0, \tag{4}$$
$$X_z + \beta(X_x\varphi_x + X_y\varphi_y) = 0, \tag{5}$$
$$\varphi_z + \frac{1}{2}\beta(\varphi_x^2+\varphi_y^2) = 0, \tag{6}$$
$$\beta(X_x^2+X_y^2) - Z_z = 0,\ \ \chi\rho^2 - Z_z = 0, \tag{7}$$
$$X_{xx}+X_{yy} = 0. \tag{8}$$

Following Eq. (8), we define the similarity variable as

$$X = k(z)x + l(z)y + p(z), \tag{9}$$

where k, l, and p are three real functions of propagation distance z, which are to be determined. Substituting Eq. (9) into Eq. (5), we obtain

$$\varphi = -\frac{1}{2\beta}\left(\frac{k_z}{k}x^2 + \frac{l_z}{l}y^2\right) + \frac{p_z\beta}{2}\left(\frac{x}{k}+\frac{y}{l}\right) + \varphi_0(z), \tag{10}$$

where $\varphi_0(z)$, $k(z)$, $l(z)$, and $p(z)$ can be determined by inserting Eq. (10) in Eq. (6). After some algebra, we can express the above four real functions as

$$k = k_0\left(c_0 - c_1\int_0^z \beta(s)\,\mathrm{d}s\right)^{-1}, \tag{11}$$
$$l = l_0\left(c_0 - c_1\int_0^z \beta(s)\,\mathrm{d}s\right)^{-1}, \tag{12}$$
$$p = c_2 + \int_0^z \beta(s)\left(c_0 - c_1\int_0^z \beta(s)\,\mathrm{d}s\right)^{-2}\mathrm{d}s, \tag{13}$$
$$\varphi_0 = -\frac{k_0^2+l_0^2}{8k_0^2l_0^2}\int_0^z \beta(s)\left(c_0 - c_1\int_0^z \beta(s)\,\mathrm{d}s\right)^{-2}\mathrm{d}s, \tag{14}$$

where k_0, l_0, c_0, c_1, and c_2 are real constants. We should note that c_1 is related to the initial chirp parameter of the input pulse, $c_0 - c_1\int_0^z \beta(s)\,\mathrm{d}s$ is a positive function of z which describes the change of the pulse's width, and $\int_0^z \beta(s)\,\mathrm{d}s$ stands for the accumulated dispersion.

Further, we can get the amplitude and the effective propagation distance as

$$\rho = \rho_0\sqrt{kl}\exp\left[\int_0^z g(s)\,\mathrm{d}s\right], \tag{15}$$
$$Z = \int_0^z \beta(s)(k^2+l^2)\,\mathrm{d}s + Z_0. \tag{16}$$

We should note that the reduction of Eq. (1) into Eq. (3) requires the following controllability condition:

$$\chi = \frac{\beta(k^2+l^2)}{\rho_0^2l^2k^2\exp(2\int_0^z g(s)\,\mathrm{d}s)}. \tag{17}$$

This integrability condition was firstly found in Ref. [44], and also was obtained in the latter researches.[35,36,40]

As we all know, equation (3) has been extensively investigated, and many kinds of analytic solutions have been obtained, such as dark soliton, bright soliton, soliton pair, and rational-oscillatory solutions. In this paper, we are interested in the rational-oscillatory solution, which was firstly derived by Peregrine in 1983.[3] Here applying the Galilean transformation to the rogue wave solution,[3,4] then we have

$$U = \left[1 - \frac{4+8\mathrm{i}(Z-Z_\mathrm{c})}{1+4[X-v(Z-Z_\mathrm{c})]^2+4(Z-Z_\mathrm{c})^2}\right] \times \exp\left[\mathrm{i}\left(1-\frac{v^2}{2}\right)(Z-Z_\mathrm{c}) + \mathrm{i}vX\right], \tag{18}$$

where Z_c and v are two arbitrary constants, $X_\mathrm{c} = v(Z-Z_\mathrm{c})$ is the center of the rogue wave, and $Z = Z_\mathrm{c}$ is the position where the rogue wave emerges and disappears. Therefore, the optical rogue wave solution of Eq. (1) can be easily constructed by substituting Eq. (18) into the similarity transformation (2).

3. Propagation of optical rogue waves in inhomogeneous nonlinear waveguides

Next, we will investigate how to control the optical rogue waves that propagate in inhomogeneous nonlinear waveguides. As far as we know, there are some studies to control the dynamic of rational solitons,[35] such as the latest work of Tian *et al.* who used the relation between the transformed propagation distance Z and the original propagation distance z in the context of the lens-type transformation to control the rational soliton. Inspired by the idea reported in Refs. [35] and [37], we investigate the controllability of the optical rogue waves propagating in inhomogeneous nonlinear waveguides through the relation between the effective propagation distance Z (Eq. (16)) and the normalized propagation distance z. Here, we consider the periodical distributed system with the coefficient

$$\beta = \beta_0 cos(\omega z). \tag{19}$$

According to Eq. (17), the cubic nonlinear coefficient should be expressed as

$$\chi = \frac{\beta_0(k_0^2+l_0^2)}{k_0^2l_0^2}\cos(\omega z)\exp\left[-2\int_0^z g(s)\,\mathrm{d}s\right].$$

For simplicity, we take the parameters $c_2 = 0$ and $Z_0 = 0$. From Eqs. (9) and (16), the similarity variable and the effective propagation distance are respectively given by

$$X = \frac{\omega(k_0 x + l_0 y)}{c_0\omega - c_1\beta_0 \sin(\omega z)} + \frac{\omega}{c_1[\omega - c_1\beta_0 \sin(\omega z)]}, \quad (20)$$

$$Z = \frac{(k_0^2 + l_0^2)\omega}{c_1[\omega - c_1\beta_0 \sin(\omega z)]}. \quad (21)$$

Specifically, for $c_1 = 0$, they satisfy

$$X = (k_0 x + l_0 y) + \frac{\beta_0 \sin(\omega z)}{\omega}, \quad (22)$$

$$Z = \frac{(k_0^2 + l_0^2)\beta_0 \sin(\omega z)}{\omega}. \quad (23)$$

Next we analyze the case of chirp-free optical rogue wave. From Eqs. (2), (18), and (22), we find the position of the chirp-free optical pulse's center $X_c = \beta_0[v(k_0^2 + l_0^2) - 1]\sin(\omega z)/\omega - vZ_c$, where we have taken $c_0 = 1$, which means that the center of the optical pulse will oscillate periodically when $v(k_0^2 + l_0^2) \neq 1$. Moreover, from Eq. (23), we note that the effective propagation distance Z is a periodic function of the normalized propagation distance z. The continuous real distance z can be set from 0 to infinity, while from Eq. (23), the value of Z is confined as $-(k_0^2 + l_0^2)\beta_0/\omega \leq Z \leq (k_0^2 + l_0^2)\beta_0/\omega$. We know that the evolution of the initial pulse $u(0,x,y)$ in Eq. (1) is equivalent to the evolution of the initial pulse $U(0,X)$ in Eq. (3) under the similarity transformation (2).[35,37] Therefore, if we let $Z < Z_c$, then the rogue wave will not be fully excited (can be restrained), since the rogue wave reaches its maximum amplitude at $Z = Z_c$ in the framework of Eq. (3). While if the maximum value of Z (Eq. (22)) is larger than Z_c, the optical rogue wave will emerge repeatedly along the propagation distance, as shown in Fig. 2. We should note that the word 'repeatedly' here does not mean propagating periodically as shown in Fig. 1(b). Specially, when $(k_0^2 + l_0^2)\beta_0/\omega = Z_c$, the rogue wave could be excited periodically along the propagation distance with period $2\pi/\omega$ (Fig. 2(a)). Finally, if $Z(z \to \infty) = Z_c$, the completely excited optical rogue wave will sustain itself for a long time.

From Figs. 1 and 2, we note that the position where the chirp-free optical rogue waves emerge can be predicted with the similarity transformation employed in this paper. Specifically, the optical rogue wave emerges at $z = \arcsin[Z_c\omega/(k_0^2 + l_0^2)]/\omega + 2n\pi/\omega$, where $n = 0, 1, 2\ldots$ is a non-negative integral number. In Fig. 3. the rogue wave is effectively restrained. And we know that the rogue wave can be restrained to the desire extent according to the similarity transformation. The insets of Figs. 1(a), 2(a), and 3(a) depict the effective propagation distance Z varying with the normalized distance z. From the insets, we can find the principle for controlling the optical rogue wave to emerge repeatedly, periodically, or be restrained.

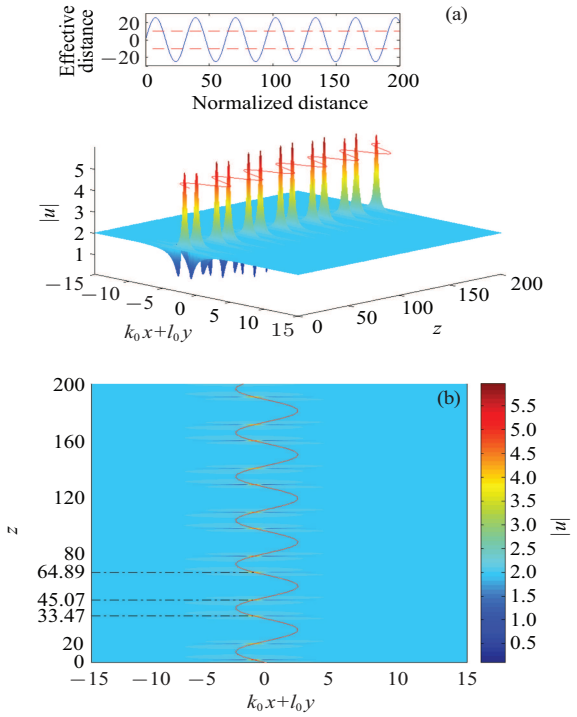


Fig. 1. (color online) (a) Repeated emergence of the optical rogue wave with parameters $g(z) = 0$, $k_0 = 1$, $l_0 = 2$, $Z_c = 10$, $v = 0.1$, $\omega = 0.2$, and the other parameters are specified in the text. In the inset, the solid blue curve depicts the effective propagation distance Z varying with the normalized distance z, and the dashed red lines stand for $Z = \pm Z_c$. (b) The corresponding contour plot. The red solid line denotes the center of the optical pulse.

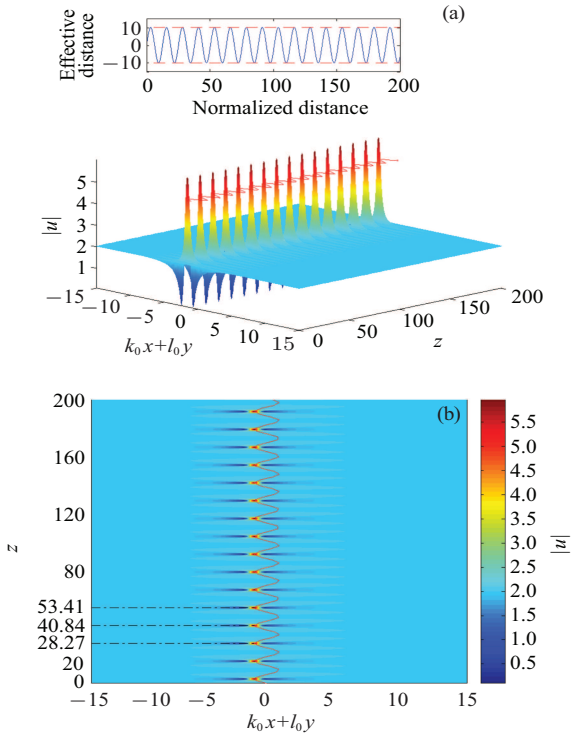


Fig. 2. (color online) (a) Periodic emergence of an optical rogue wave with $\omega = 0.5$, and the other parameters are the same as those in Fig. 1. In the inset, the solid blue curve depicts the effective propagation distance Z varying with the normalized distance z, and the dashed red lines stand for $Z = \pm Z_c$. (b) The corresponding contour plot. The red solid line denotes the center of the optical pulse.

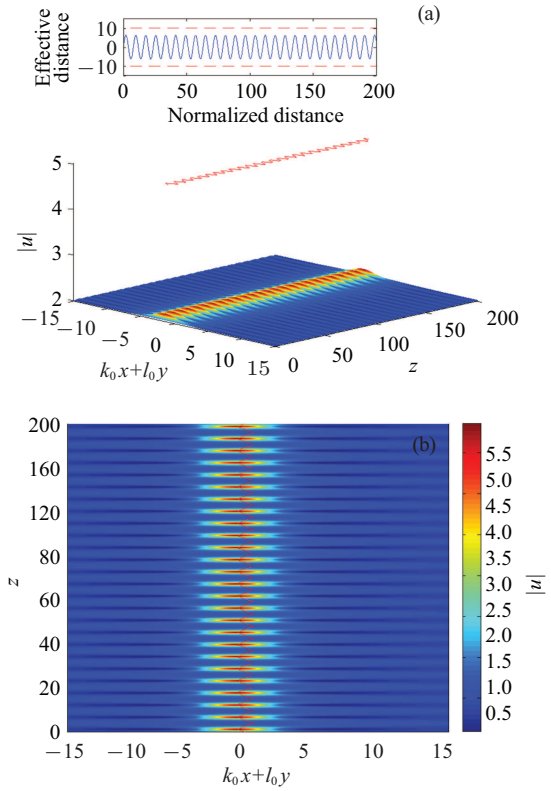


Fig. 3. (color online) (a) Evolution of the restrained optical rogue wave with $\omega = 0.8$, and the other parameters are the same as those in Fig. 1. In the inset, the solid blue curve depicts the effective propagation distance Z varying with the normalized distance z, and the dashed red lines stand for $Z = \pm Z_c$. (b) The corresponding contour plot. The red solid line denotes the center of the optical pulse.

Note for Z in the constant coefficient NLSE (3), a rogue wave appears from nowhere and disappears without a trace, while for the continuous real distance z, the rogue wave exhibits a periodic behavior due to the periodic function of the normalized propagation distance z in Eq. (23). That is, when the rogue wave finishes the first excitation (Fig. 2) or restraint (Fig. 3), the next excitation or restraint appears again due to the periodic function of the continuous real distance z, thus the characteristic, i.e., appearing from nowhere and disappearing without a trace, of the autonomous rogue wave is changed, and the periodic excitation or restraint for the nonautonomous rogue wave is exhibited. This kind of behavior for the nonautonomous rogue wave is the breather of a rogue wave on the background of the continuous wave. This new property results from the nonautonomous characteristic of the rogue wave, i.e., the amplitude, the width, the phase, and the effective propagation distance of the solution are influenced by the function $\beta(z)$.

Similarly, we can also investigate the case of chirped optical rogue wave by applying Eqs. (20) and (21) to the solution (Eq. (18)).

It is worth noting that there also exist higher-order rational solutions in Eq. (3),[46,47] thus the dynamics of the higher-order optical rogue wave solutions of Eq. (1) can be investigated. As an example, we construct the second-order optical rogue wave solution of Eq. (1) and discuss the dynamics of the second-order rogue wave. We present the second-order Peregrine soliton solution of Eq. (3) as

$$U = \left(1 - \frac{G + \mathrm{i}H}{D}\right)\exp(\mathrm{i}Z), \tag{24}$$

where

$$\begin{aligned} G &= -\frac{3}{16} + \frac{3}{2}X^2 + X^4 + \frac{9}{2}Z^2 + 6X^2Z^2 + 5Z^4, \\ H &= \left(-\frac{15}{8} - 3X^2 + 2X^4 + Z^2 + 4X^2Z^2 + 2Z^4\right)Z, \\ D &= \frac{3}{64} + \frac{9}{16}X^2 + \frac{X^4}{4} + \frac{X^6}{3} + \frac{33}{16}Z^2 - \frac{3}{2}X^2Z^2 \\ &\quad + X^4Z^2 + \frac{9}{4}Z^4 + X^2Z^4 + \frac{1}{3}Z^6. \end{aligned}$$

Similarly, by applying the Galilean transformation to the second-order Peregrine soliton solution (Eq. (24)), we obtain the second-order optical rogue wave solution of Eq. (1)

$$\begin{aligned} u &= \rho(z)U[X - v(Z - Z_c), Z - Z_c] \\ &\quad \times \exp\left[\mathrm{i}\varphi(z,x,y) + \mathrm{i}\left(1 - \frac{v^2}{2}\right)(Z - Z_c) + \mathrm{i}vX\right], \end{aligned} \tag{25}$$

where ρ, φ, Z, and X are given in Eqs. (9)–(16). To investigate the controllable behaviors and the interaction of the second-order optical rogue waves, we use a diffraction coefficient (Eq. (19)) the same as that discussed in the first-order optical rogue wave. Figures 4(a) and 4(b) present the comparison of the second-order optical rogue waves propagating in bulk optical media with parameters $\omega = 0.33$ and $\omega = 0.4$,

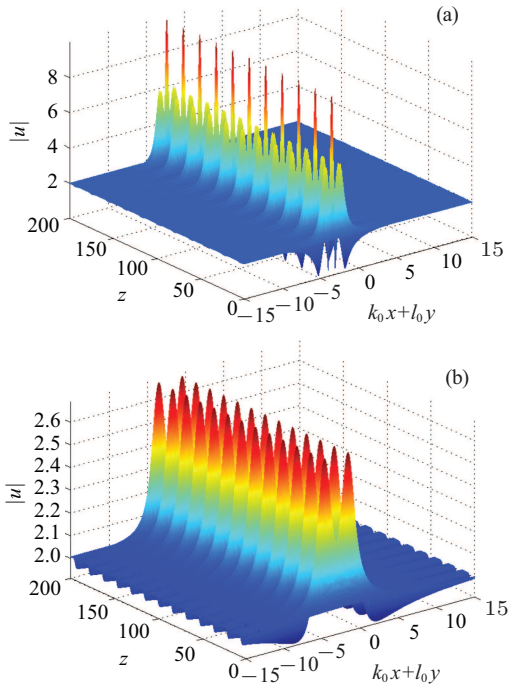


Fig. 4. (color online) (a) Periodical emergence of completed excited second-order optical rogue wave on the continuous-wave background with $\omega = 0.33$. (b) Evolution plot of the restrained rogue wave with $\omega = 0.4$.

respectively. In Fig. 4(a), we can see that the completely excited second-order rogue wave emerges periodically along the propagation distance z. While in Fig. 4(b) we find that the second-order optical rogue wave propagating along the normalized distance z is restricted. The reason for the above phenomena can be clearly explained by the theory discussed in the first-order optical rogue wave. To be specific, when $\omega < [(k_0^2 + l_0^2) - \beta_0 c_1 Z_c]/Z_c$, the rogue will emerge repeatedly along the distance z; when $\omega = [(k_0^2 + l_0^2) - \beta_0 c_1 Z_c]/Z_c$, the rogue will emerge periodically along the distance z; while when $\omega > [(k_0^2 + l_0^2) - \beta_0 c_1 Z_c]/Z_c$, the rogue will be restrained.

4. Conclusion

In summary, we have constructed the exact optical rogue wave solutions of the (2+1)-dimension NLSE with distributed coefficients via a similarity transformation. Moreover, we investigate the dynamics of the first- and the second-order optical rogue waves propagating in a periodical distributed system. As a result, we present the condition for controlling the dynamics of the first- and the second-order optical rogue waves. We believe that the method of controlling the rogue wave shown in this paper would be useful not only in nonlinear optics but also in many other fields of science, such as nonlinear oceanography and Bose–Einstein condensates.

References

[1] Kharif C, Pelinovsky E and Slunyaev A 2009 *Rogue Waves in the Ocean, Observation, Theories and Modeling* (New York: Springer)
[2] Draper L 1965 *Marine Observer* **35** 193
[3] Peregrine D H 1983 *J. Austral. Math. Soc. Ser. B* **25** 16
[4] Akhmediev N, Ankiewicz A and Taki M 2009 *Phys. Lett. A* **373** 675
[5] Ankiewicz A, Devine N and Akhmediev N 2009 *Phys. Lett. A* **373** 3997
[6] Ruban V, Kodama Y and Ruderman M 2010 *Eur. Phys. J. Spec. Top.* **18** 5
[7] Osborne A R 2009 *Nonlinear Ocean Waves* (New York: Academic Press)
[8] Müller P, Garrett Ch and Osborne A 2005 *Oceanography.* **18** 66
[9] Solli D R, Ropers C, Koonath P and Jalali B 2007 *Nature* **450** 1054
[10] Solli D R, Ropers C and Jalali B 2008 *Phys. Rev. Lett.* **101** 233902
[11] Dudley J M, Genty G, Dias F, Kibler B and Akhmediev N 2009 *Opt. Express* **17** 21497
[12] Dudley J M, Finot C, Millot G, Garnier J, Genty G, Agafontsev D and Dias F 2010 *Eur. Phys. J. Spec. Top.* **185** 125
[13] Jalali B, Solli D R, Goda K, Tsia K and Ropers C 2010 *Eur. Phys. J. Spec. Top.* **185** 145
[14] Kibler B, Fatome J, Finot C, Millot G, Dias F, Genty G, Akhmediev N and Dudley J M 2010 *Nature Phys.* **6** 1
[15] Majus D, Jukna V, Valiulis G, Faccio D and Dubietis1 A 2011 *Phys. Rev. A* **83** 025802
[16] Zaviyalov A, Egorov O, Iliew R and Lederer F 2012 *Phys. Rev. A* **85** 013828
[17] Moslem W M, Sabry R, El-Labany S K and Shukla1 P K 2011 *Phys. Rev. E* **84** 066402
[18] Ganshin A N, Efimov V B, Kolmakov G V, Mezhov-Deglin L P and McClintock P V E 2008 *Phys. Rev. Lett.* **101** 065303
[19] Yang G, Li L and Jia S T 2012 *Phys. Rev. E* **85** 046608
[20] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
[21] Bludov Y V, Konotop V V and Akhmediev N 2009 *Phys. Rev. A* **80** 033610
[22] Ruderman M S 2010 *Eur. Phys. J. Spec. Top.* **185** 57
[23] Montina A, Bortolozzo U, Residori S and Arecchi F T 2009 *Phys. Rev. Lett.* **103** 173901
[24] Vergeles S and Turitsyn S K 2011 *Phys. Rev. A* **83** 061801
[25] Stenflo L and Marklund M 2010 *J. Plasma Phys.* **76** 293
[26] Yan Z Y 2010 *Commun. Theor. Phys.* **54** 947
[27] Yan Z Y, Konotop V V and Akhmediev N 2010 *Phys. Rev. E* **82** 036610
[28] Yan Z Y 2011 *Phys. Lett. A* **375** 4274
[29] Yan Z Y 2011 *J. Math. Anal. Appl.* **380** 689
[30] Wang G Y, He J S and Li Y S 2011 *Commun. Theor. Phys.* **56** 995
[31] Xu S W, He J S and Wang L H 2011 *J. Phys. A: Math. Theor.* **44** 305203
[32] Wang X C, He J S and Li Y S 2011 *Commun. Theor. Phys.* **56** 631
[33] Akhmediev N, Ankiewicz A, and Soto-Crespo J M 2009 *Phys. Rev. E* **80** 026601
[34] Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (San Diego: Academic)
[35] Tian Q, Yang Q, Dai C Q and Zhang J F 2011 *Opt. Commun.* **284** 2222
[36] Hao R Y, Li L, Li Z H, Yang R C and Zhou G S 2005 *Opt. Commun.* **245** 383
[37] Zhang J F, Tian Q, Wang Y Y, Dai C Q and Wu L 2010 *Phys. Rev. A* **81** 023832
[38] Wu L, Li L and Zhang J F 2008 *Phys. Rev. A* **78** 013838
[39] Wu L, Zhang J F, Li L, Tian Q and Porsezian K 2008 *Opt. Express* **16** 6352
[40] Dai C Q, Wang Y Y and Wang X G 2011 *J. Phys. A: Math. Theor.* **44** 155203
[41] Dai C Q, Wang Y Y and Yan C J 2010 *Opt. Commun.* **283** 1489
[42] Wu L, Li L,Zhang J F, Mihalache D, Malomed B A and Liu W M 2010 *Phys. Rev. A* **81** 061805
[43] Dai C Q, Zhang J F and Zhu S Q 2010 *Europhys. Lett.* **92** 24005
[44] Zhong W P, Xie R H, Bellić M, Petrović N, Chen G and Yi L 2008 *Phys. Rev. A* **78** 023821
[45] Zhong W P and Belić M 2009 *Phys. Rev. A* **79** 023804
[46] Akhediev N, Soto-Crespo J M and Ankiweicz A 2009 *Phys. Lett. A* **373** 2137
[47] Ankiewicz A, Clarkson P A and Akhmediev N 2010 *J. Phys. A: Math. Theor.* **43** 122002