Emission spectrum of a harmonically trapped Λ-type three-level atom

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We theoretically investigate the emission spectrum for a Λ-type three-level atom trapped in the node of a standing wave. We show that the atomic center-of-mass motion not only directly affects the peak number, peak position, and peak height in the atomic emission spectrum, but also influences the effects of the cavity field and the atomic initial state on atomic emission spectrum.

Keywords: emission spectrum, the atomic center-of-mass motion, Λ-type three-level atom

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1. Introduction

The atomic emission spectrum contains a great amount of information about the interaction between light and matter.[1–5] In recent years, there have been significant advances in laser cooling technology.[6–8] Quantum control over the trapped particles' external motion can improve the spectral precision and sensitivity.[9,10] It provides powerful tool for detecting substance structure and controlling the quantum process.[11,12]

The system of a Λ-type three-level atom interacting with the field has been extensively studied due to the potential applications, such as electromagnetically induced transparency,[13,14] high-precision spectroscopy,[15] quantum computing and communicating.[16–18] The study of the emission spectra of a Λ-type three-level atom revealed interesting effects including the vacuum Rabi splitting, quantum beats, and coherent population trapping.[19–22] Three-level atomic systems show a number of interesting physical characteristics due to quantum interference between different atomic transition channels. The interaction of a single atom with the electromagnetic field of a high-quality resonator depends on the atomic center-of-mass motion and the spatial structure of the cavity field mode. We investigated the emission spectrum for a two-level atom trapped in the antinode of the standing wave in Lamb–Dick approximation.[23] We also studied the emission spectrum for a two-level atom driven by a running wave inside an ideal cavity without Lamb–Dick approximation.[24] We showed that the effects of the atomic center-of-mass motion on the emission spectrum for a two-level atom are significantly different in the two cases mentioned above. In this work, we will investigate the emission spectrum for a Λ-type three-level atom harmonically trapped in the node of a standing-wave field in an ideal cavity in the Lamb–Dick regime.

The present paper is organized as follows. In Section 2, we derive the effective Hamiltonian of the system under consideration, and then present an exact expression of emission spectrum for a harmonically trapped Λ-type three-level atom coupled to a single-mode field in cavity. In Section 3, we show the character of atomic emission spectra and discuss the effect of the atomic center-of-mass motion on emission spectra. Finally, we conclude with a brief summary.

2. Model

As depicted in Fig. 1, a Λ-type three-level atom is trapped at a node of a standing wave inside a high-Q cavity. The atomic center of mass oscillates along the axis of the cavity (the x axis). When the atom–field coupling constant g is much larger than the cavity field decay rate and the atomic decay rate, the Hamiltonian of the system is described by[25]

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm I}, \tag{1a}$$

where

$$\hat{H}_0 = \hbar\omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + \hbar\omega_{\rm m}(\hat{b}^+\hat{b} + \frac{1}{2}) + \hbar\sum_{i=1}^{3}\omega_i |i\rangle\langle i|, \tag{1b}$$

$$\hat{H}_{\rm I} = \hbar g\left[\hat{a}\left(\hat{\sigma}_1^+ + \hat{\sigma}_2^+\right) + \hat{a}^+\left(\hat{\sigma}_1 + \hat{\sigma}_2\right)\right]\sin\left[\eta\left(\hat{b} + \hat{b}^+\right)\right]. \tag{1c}$$

Here, $\hat{a}$ ($\hat{a}^+$) is the annihilation (creation) operator for the cavity field of frequency ω, $\hat{b}$ ($\hat{b}^+$) is the annihilation (creation) operator for the atomic center-of-mass motion with

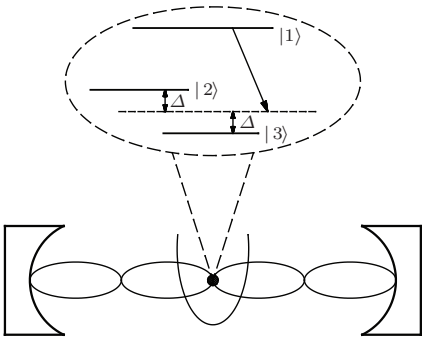


Fig. 1. Schematic diagram of the system.

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frequency $\omega_{\rm m}$, $\hbar\omega_i$ $(i=1,2,3)$ is the energy level of the atomic internal state $|i\rangle$, and $\hat{\sigma}_1=|2\rangle\langle 1|$ and $\hat{\sigma}_2=|3\rangle\langle 1|$ are the atomic transition operators. The Lamb–Dicke parameter $\eta=k_{\rm c}\sqrt{\hbar/2\mu\omega_{\rm m}}$, with $k_{\rm c}=\omega/c$ being the wave number of the cavity field and μ the atomic mass. In the Lamb–Dicke regime, $\eta\ll 1$, and the interaction picture Hamiltonian reads

$$\begin{aligned}\hat{H}_{\rm I}^{\rm I}(t) &= \hbar g\eta\left[\left(\hat{a}\hat{\sigma}_2^{+}+\hat{a}^{+}\hat{\sigma}_1\right)\left(\hat{b}\,{\rm e}^{-{\rm i}(\omega_{\rm m}-\Delta)t}+\hat{b}^{+}\,{\rm e}^{{\rm i}(\omega_{\rm m}+\Delta)t}\right)\right.\\ &\quad\left.+\left(\hat{a}^{+}\hat{\sigma}_2+\hat{a}\hat{\sigma}_1^{+}\right)\left(\hat{b}\,{\rm e}^{-{\rm i}(\omega_{\rm m}+\Delta)t}+\hat{b}^{+}\,{\rm e}^{{\rm i}(\omega_{\rm m}-\Delta)t}\right)\right], \quad (2)\end{aligned}$$

where $\Delta=(\omega_2-\omega_3)/2=\omega-(\omega_1-\omega_2)=(\omega_1-\omega_3)-\omega$. If $\Delta\approx\omega_{\rm m}$, we disregard the rapidly oscillating terms, and the Hamiltonian becomes

$$\hat{H}_{\rm I-eff}^{\rm I}=\hbar g\eta[\hat{a}^{+}\hat{\sigma}_1\hat{b}+\hat{a}\hat{\sigma}_1^{+}\hat{b}^{+}+\hat{a}^{+}\hat{\sigma}_2\hat{b}^{+}+\hat{a}\hat{\sigma}_2^{+}\hat{b}]. \quad (3)$$

The first (second) term in Eq. (3) describes the process in which the atom transits from $|1\rangle$ to $|2\rangle$ ($|2\rangle$ to $|1\rangle$), and one phonon of atomic center-of-mass vibrational motion annihilate (create) and one photon of the cavity field simultaneously create (annihilate). The third (last) term corresponds to the atomic transition process $|1\rangle\rightarrow|3\rangle$ ($|3\rangle\rightarrow|1\rangle$) with one phonon and one photon simultaneously creating (annihilating).

Since $\hat{H}_0$ and $\hat{H}_{\rm I-eff}^{\rm I}$ satisfy the commutation relation, the time-evolution operator can be expressed as

$$\hat{U}(t) = \hat{U}_0(t)\hat{U}_{\rm I}(t), \quad (4)$$

$$\begin{aligned}\hat{U}_0(t) &= \exp(-{\rm i}\hat{H}_0t/\hbar),\\ &= \exp\left[-{\rm i}\omega\left(\hat{a}^{+}\hat{a}+\frac{1}{2}\right)t\right]\exp\left[-{\rm i}\omega_{\rm m}\left(\hat{b}^{+}\hat{b}+\frac{1}{2}\right)t\right]\\ &\quad\times\begin{pmatrix}\exp(-{\rm i}\omega_1t) & 0 & 0\\ 0 & \exp(-{\rm i}\omega_2t) & 0\\ 0 & 0 & \exp(-{\rm i}\omega_3t)\end{pmatrix}, \quad (5)\end{aligned}$$

$$\hat{U}_{\rm I}(t)=\begin{pmatrix}\hat{U}_{11}^{\rm I} & \hat{U}_{12}^{\rm I} & \hat{U}_{13}^{\rm I}\\ \hat{U}_{21}^{\rm I} & \hat{U}_{22}^{\rm I} & \hat{U}_{23}^{\rm I}\\ \hat{U}_{31}^{\rm I} & \hat{U}_{32}^{\rm I} & \hat{U}_{33}^{\rm I}\end{pmatrix}. \quad (6)$$

The matrix elements of $\hat{U}_{\rm I}(t)$ can be written as

$$\begin{aligned}&\hat{U}_{11}^{\rm I}=\hat{C},\quad \hat{U}_{12}^{\rm I}=-{\rm i}\frac{\hat{S}}{\hat{\Omega}}\hat{a}\hat{b}^{+},\quad \hat{U}_{13}^{\rm I}=-{\rm i}\frac{\hat{S}}{\hat{\Omega}}\hat{a}\hat{b},\\ &\hat{U}_{21}^{\rm I}=-\hat{U}_{12}^{\rm I+},\quad \hat{U}_{22}^{\rm I}=\hat{a}^{+}\hat{b}\frac{\hat{C}-1}{\hat{\Omega}^2}\hat{a}\hat{b}^{+}+1,\\ &\hat{U}_{23}^{\rm I}=\hat{a}^{+}\hat{b}\frac{\hat{C}-1}{\hat{\Omega}^2}\hat{a}\hat{b},\quad \hat{U}_{31}^{\rm I}=-\hat{U}_{13}^{\rm I+},\quad \hat{U}_{32}^{\rm I}=\hat{U}_{23}^{\rm I+},\\ &\hat{U}_{33}^{\rm I}=\hat{a}^{+}\hat{b}^{+}\frac{\hat{C}-1}{\hat{\Omega}^2}\hat{a}\hat{b}+1,\end{aligned}$$

where $\hat{C}=\cos(g\eta\hat{\Omega}t)$, $\hat{S}=\sin(g\eta\hat{\Omega}t)$, and $\hat{\Omega}=\sqrt{\hat{a}\hat{a}^{+}(\hat{b}^{+}\hat{b}+\hat{b}\hat{b}^{+})}$.

Assuming that the atomic internal state $|\psi_{\rm a}(0)\rangle$ is in coherent superposition state, i.e., $|\psi_{\rm a}(0)\rangle=\sum_{i=1}^{3}c_i|i\rangle$, the cavity field and the atomic center-of-mass motion are both in the coherent superposition state of the number states: $|\psi_{\rm f}(0)\rangle=\sum_{n=0}^{\infty}F_n|n\rangle$ and $|\psi_{\rm m}(0)\rangle=\sum_{\rm m=0}^{\infty}G_{\rm m}|m\rangle$, and the initial state of the system can be written as

$$|\psi(0)\rangle=|\psi_{\rm a}(0)\rangle\otimes|\psi_{\rm f}(0)\rangle\otimes|\psi_{\rm m}(0)\rangle. \quad (7)$$

Using the time evolution operator, we can obtain the two-time correlation function as

$$\begin{aligned}&\left\langle[\hat{\sigma}_1^{+}(t_1)+\hat{\sigma}_2^{+}(t_1)][\hat{\sigma}_1(t_2)+\hat{\sigma}_2(t_2)]\right\rangle\\ &=\langle\psi(0)|\hat{U}^{+}(t_1)\left(\hat{\sigma}_1^{+}+\hat{\sigma}_2^{+}\right)\hat{U}(t_1)\\ &\quad\times\hat{U}^{+}(t_2)\left(\hat{\sigma}_1+\hat{\sigma}_2\right)\hat{U}(t_2)|\psi(0)\rangle. \quad (8)\end{aligned}$$

If the half-band width of the spectrometer Γ is much larger than the atomic decay rate, the time-dependent physical spectrum emitted by a harmonically trapped Λ-type three-level atom is given by:[26]

$$\begin{aligned}S(\nu) &= \Gamma\int_0^T{\rm d}t_1\int_0^T{\rm d}t_2\exp[-(\Gamma-{\rm i}\nu)(T-t_1)\\ &\quad-(\Gamma+{\rm i}\nu)(T-t_2)]\\ &\quad\times\langle[\hat{\sigma}_1^{+}(t_1)+\hat{\sigma}_2^{+}(t_1)][\hat{\sigma}_1(t_2)+\hat{\sigma}_2(t_2)]\rangle\\ &= \frac{\Gamma}{4}\sum_{n=1,m=0}^{\infty}\frac{1}{(2m+1)}\\ &\quad\times\{|\sqrt{m}[D_{n,m-1}^{+}\mathbb{F}(-\Delta,\Omega_{n+1,m-1},\Omega_{n,m})\\ &\quad+D_{n,m-1}^{-}\mathbb{F}(-\Delta,-\Omega_{n+1,m-1},\Omega_{n,m})]\\ &\quad+\sqrt{m+1}[D_{n,m+1}^{+}\mathbb{F}(\Delta,\Omega_{n+1,m+1},\Omega_{n,m})\\ &\quad+D_{n,m+1}^{-}\mathbb{F}(\Delta,-\Omega_{n+1,m+1},\Omega_{n,m})]|^2\\ &\quad+|\sqrt{m}[D_{n,m-1}^{+}\mathbb{F}(-\Delta,\Omega_{n+1,m-1},-\Omega_{n,m})\\ &\quad+D_{n,m-1}^{-}\mathbb{F}(-\Delta,-\Omega_{n+1,m-1},-\Omega_{n,m})]\\ &\quad+\sqrt{m+1}[D_{n,m+1}^{+}\mathbb{F}(\Delta,\Omega_{n+1,m+1},-\Omega_{n,m})\\ &\quad+D_{n,m+1}^{-}\mathbb{F}(\Delta,-\Omega_{n+1,m+1},\,-\Omega_{n,m})]\,|^2\\ &\quad+2|\sqrt{m+1}[D_{n,m-1}^{+}\mathbb{F}(-\Delta,\Omega_{n+1,m-1},0)\\ &\quad+D_{n,m-1}^{-}\mathbb{F}(-\Delta,-\Omega_{n+1,m-1},0)]\\ &\quad-\sqrt{m}[D_{n,m+1}^{+}{\rm F}(\Delta,\Omega_{n+1,m+1},0)\\ &\quad+D_{n,m+1}^{-}\mathbb{F}(\Delta,-\Omega_{n+1,m+1},0)]|^2\}\\ &\quad+\frac{\Gamma}{2}\sum_{\rm m=0}^{\infty}[|D_{0,m}^{+}\mathbb{F}(-\Delta,\Omega_{1,m},0)\\ &\quad+D_{0,m}^{-}\mathbb{F}(-\Delta,-\Omega_{1,m},0)|^2\\ &\quad+|D_{0,m+1}^{+}\mathbb{F}(\Delta,\Omega_{1,m+1},0)\\ &\quad+D_{0,m+1}^{-}\mathbb{F}(\Delta,-\Omega_{1,m+1},0)|^2]\\ &\quad+\frac{\Gamma}{2}\sum_{n=0}^{\infty}|D_{n,0}^{+}\mathbb{F}(\Delta,\Omega_{n+1,0},0)\\ &\quad+D_{n,0}^{-}{\rm F}(\Delta,-\Omega_{n+1,0},0)|^2, \quad (9)\end{aligned}$$

where

$$D_{n,m}^{\pm}=\frac{\sqrt{2}}{2}c_1F_nG_{\rm m}\pm\sqrt{\frac{m}{4m+2}}c_2F_{n+1}G_{m-1}$$

$$\pm\sqrt{\frac{m+1}{4m+2}}c_3F_{n+1}G_{m+1}, \tag{10}$$

$$\mathbb{F}(x,y,z) = \frac{\exp[\mathrm{i}(\nu-\omega-x-g\eta y-g\eta z)T]-\exp(-\Gamma T)}{\Gamma+\mathrm{i}(\nu-\omega-x-g\eta y-g\eta z)}, \tag{11}$$

$$\Omega_{n,m} = \sqrt{n(2m+1)}, \tag{12}$$

$$G_{-1} = G_{-2} = 0. \tag{13}$$

In the next section, we investigate the character of the emission spectrum for a trapped Λ-type three-level atom using Eqs. (9)–(13).

3. Results and discussion

If the atom is initially in the upper state $|1\rangle$, and the atomic center-of-mass motion and the cavity field are both in the Fock state, the emission spectrum $S(\upsilon)$ of a trapped Λ-type three-level atom is reduced into

$$\begin{aligned}
S(\upsilon) = {} & \frac{\Gamma(1-\delta_{n0})}{8}\left\{\frac{m+1}{2m+3}|\mathbb{F}(-\Delta,\Omega_{n+1,m},\Omega_{n,m+1})\right. \\
& +\mathbb{F}(-\Delta,-\Omega_{n+1,m},\Omega_{n,m+1})|^2 \\
& +\frac{m+1}{2m+3}|\mathbb{F}(-\Delta,\Omega_{n+1,m},-\Omega_{n,m+1}) \\
& +\mathbb{F}(-\Delta,-\Omega_{n+1,m},-\Omega_{n,m+1})|^2 \\
& +\frac{2(m+2)}{2m+3}|\mathrm{F}(-\Delta,\Omega_{n+1,m},0)+\mathbb{F}(-\Delta,-\Omega_{n+1,m},0)|^2 \\
& +\frac{m}{2m-1}|[\mathbb{F}(\Delta,\Omega_{n+1,m},\Omega_{n,m-1}) \\
& +\mathbb{F}(\Delta,-\Omega_{n+1,m},\Omega_{n,m-1})]|^2 \\
& +\frac{m}{2m-1}|\mathbb{F}(\Delta,\Omega_{n+1,m},-\Omega_{n,m-1}) \\
& +\mathrm{F}(\Delta,-\Omega_{n+1,m},-\Omega_{n,m-1})|^2 \\
& \left.+\frac{2(m-1)}{2m-1}|\mathbb{F}(\Delta,\Omega_{n+1,m},0)+\mathbb{F}(\Delta,-\Omega_{n+1,m},0)|^2\right\} \\
& +\frac{\Gamma\delta_{n0}}{4}[|\mathrm{F}(-\Delta,\Omega_{1,m},0)+\mathbb{F}(-\Delta,-\Omega_{1,m},0)|^2 \\
& +|\mathbb{F}(\Delta,\Omega_{1,m},0)+\mathbb{F}(\Delta,-\Omega_{1,m},0)|^2].
\end{aligned} \tag{14}$$

In general, the emission spectra of a trapped Λ-type three-level atom have six pairs of peaks, as shown in Table 1. The peaks are divided into two groups which are symmetrically located at about $\omega-\Delta$ and $\omega+\Delta$, respectively. The peak spacing in each group is proportional to atom-field coupling constant g and the Lamb–Dicke parameter η. With the increase in the photon number n and the phonon number m, the four sidebands around $\omega-\Delta$ move away from each other, so do the other four sidebands around $\omega+\Delta$. When $2g\eta|\Omega_{n+1,m}-\Omega_{n,m+1}|<\Gamma$, the two peaks located at $\omega-\Delta-g\eta(\Omega_{n+1,m}-\Omega_{n,m+1})$ and $\omega-\Delta+g\eta(\Omega_{n+1,m}-\Omega_{n,m+1})$ combine with each other, leading to a single peak at $\omega-\Delta$, as shown in Fig. 2. The relative peak height is only relevant to the phonon number m. These spectrum characteristics are quite different from one of the stationary Λ-type three-level atom.[20,22] The difference stems from variations of the system's energy levels and eigenvectors.

Table 1. Position and relative height of peaks in the emission spectra.

Position of peak	Relative height of peak
$\omega-\Delta\pm g\eta(\sqrt{n(2m+3)}+\sqrt{(n+1)(2m+1)})$	$(m+1)/(2m+3)$
$\omega-\Delta\pm g\eta(\sqrt{n(2m+3)}-\sqrt{(n+1)(2m+1)})$	$(m+1)/(2m+3)$
$\omega-\Delta\pm g\eta\sqrt{(n+1)(2m+1)}$	$(2m+4)/(2m+3)$
$\omega+\Delta\pm g\eta(\sqrt{(n+1)(2m+1)}+\sqrt{n(2m-1)})$	$m/(2m-1)$
$\omega+\Delta\pm g\eta(\sqrt{(n+1)(2m+1)}-\sqrt{n(2m-1)})$	$m/(2m-1)$
$\omega+\Delta\pm g\eta\sqrt{(n+1)(2m+1)}$	$(2m-2)/(2m-1)$

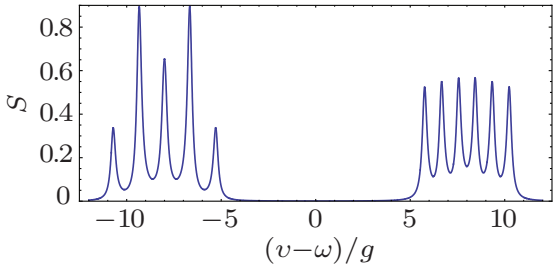


Fig. 2. (color online) Atomic emission spectrum $S(\nu)$ with $n=3$, $m=2$, $C_1=1$, $C_2=C_3=0$, $\Gamma=0.15$ g, $T=60$ g^{-1}, $\eta=0.3$, and $\Delta=8$ g.

The energy eigenvectors of the system, i.e., eigenvectors of $\hat{H}_0+\hat{H}^{\mathrm{I}}_{\mathrm{I-eff}}$, are

$$\begin{aligned}
|E^0_{n,m}\rangle = {} & \sqrt{\frac{m+1}{2m+1}}|2,n+1,m-1\rangle \\
& -\sqrt{\frac{m}{2m+1}}|3,n+1,m+1\rangle,
\end{aligned} \tag{15}$$

$$\begin{aligned}
|E^{\pm}_{n,m}\rangle = {} & \frac{\sqrt{2}}{2}|1,n,m\rangle\pm\frac{\sqrt{2}}{2}\left(\sqrt{\frac{m}{2m+1}}|2,n+1,m-1\rangle\right. \\
& \left.+\sqrt{\frac{m+1}{2m+1}}|3,n+1,m+1\rangle\right).
\end{aligned} \tag{16}$$

The corresponding energy levels are

$$E^0_{n,m}=\left(n+\frac{3}{2}\right)\hbar\omega+\left(m+\frac{1}{2}\right)\hbar\Delta, \tag{17}$$

$$E^{\pm}_{n,m}=E^0_{n,m}\pm\hbar g\eta\sqrt{(n+1)(2m+1)}, \tag{18}$$

where the subscripts $n\geq 0$ and $m>0$. When an atom transits from $|1\rangle$ to $|2\rangle$ ($|3\rangle$), the system accordingly transits from $|E^{\pm}_{n+1,m}\rangle$ to $|E^x_{n,m-1}\rangle$ ($|E^x_{n,m+1}\rangle$) ($x=+,-,0$), as shown in Fig. 3. The atomic spectrum peak position is determined by the energy level difference between $|E^{\pm}_{n+1,m}\rangle$ and $|E^x_{n,m-1}\rangle$ ($|E^x_{n,m+1}\rangle$). The relative peak height depends on the expansion coefficients for the system initial state vector in terms of the energy eigenvectors.

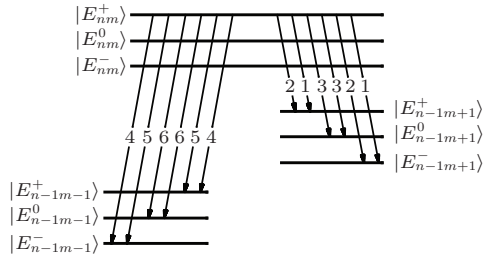


Fig. 3. Energy level scheme of the system. The number from 1 to 6 denote the line number in Table 1.

For the subscript $n \geq 0$, there is only one energy eigenvector $|E_{n,-1}\rangle = |3, n+1, 0\rangle$ in the case of $m = -1$, and two energy eigenvectors $|E^{\pm}_{n,0}\rangle = \sqrt{2}/2(|1, n, 0\rangle \pm |3, n+1, 1\rangle)$ under the condition of m=0. Accordingly, there are only two transition processes

$$|E^{\pm}_{n+1,0}\rangle \xrightarrow{\sigma_2} |E_{n,-1}\rangle,$$

and four transition processes

$$|E^{\pm}_{n+1,1}\rangle \xrightarrow{\sigma_2} |E^{\pm}_{n,0}\rangle.$$

We only see two peaks and four peaks around $\omega + \Delta$ in Figs. 4(a) and 4(b), respectively.

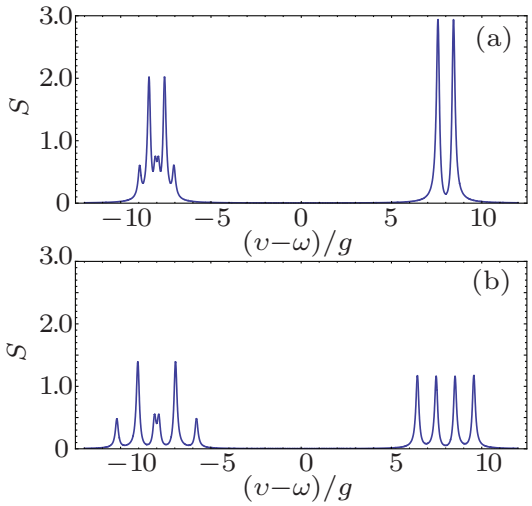


Fig. 4. (color online) Same as Fig.2 except $\Gamma = 0.1$ g. (a) $n = 1$, $m = 0$; (b) $n = 3$, $m = 1$.

If the cavity field is in the vacuum state, the system has only one energy level $E_{-1,m} = \hbar[\omega + (2m+1)\Delta]/2$, so there are only four peaks in Fig. 5(a), termed the vacuum Rabi plitting.

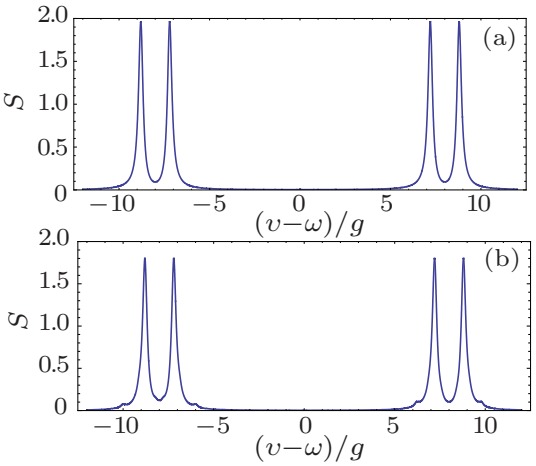


Fig. 5. (color online) (a) Same as Fig. 2 except $n = 0$ and $m = 3$. (b) Same as Fig. 5(a) except that the cavity field is coherent state, and $\bar{n} = 0.1$.

If the atomic center-of-mass motion is in thermal state and the cavity field is in the vacuum state, regardless of the average phonon number, the atomic spectrum remains four-peak structure. Because the phonon number variation increases with the average phonon number, the peaks become wider and lower, as shown in Fig. 6. Provided that the photon number is enhanced, the atomic spectrum changes dramatically with the average phonon number, as shown in Fig. 7.

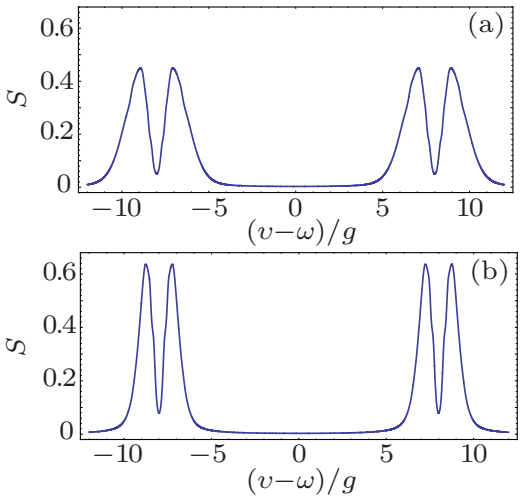


Fig. 6. (color online) Same as Fig. 2 except that the atomic center-of-mass motion is in the thermal state, $n = 0$, and the average phonon number equals (a) 3 and (b) 10.

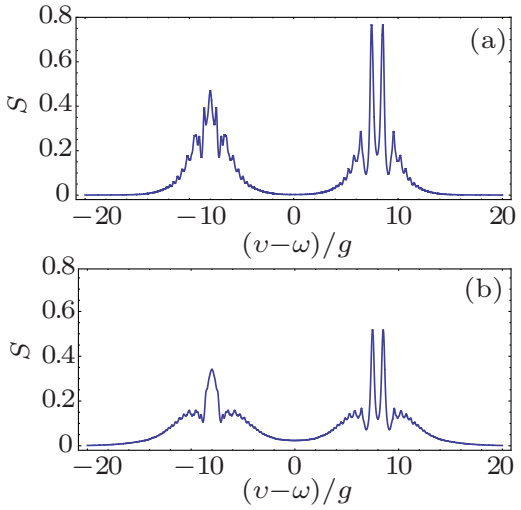


Fig. 7. (color online) Same as Fig. 6 except $n = 3$.

When the cavity field is initially in a coherent state, but the atomic center-of-mass motion is in the Fock state, if the average photon number $\bar{n}$ is very small, the atomic spectrum also has a four-peak structure due to the larger relative weight of the vacuum state, as shown in Fig. 5(b). With the increase in the average photon number $\bar{n}$, the emission spectra evolve into a complicated multi-peak structure. For the cavity field intensity is sufficiently large, a twelve-peak spectrum emerges. Compared with the emission spectra for the cavity field initially in the Fock state, the sidebands around $\omega + \Delta$ and $\omega - \Delta$ are obviously broader and lower, as illustrated in Fig. 8.

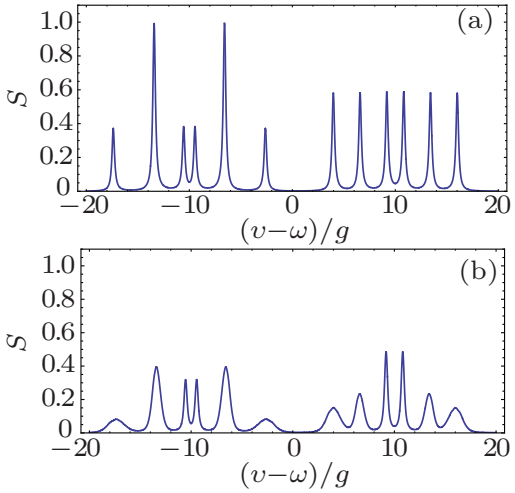


Fig. 8. (color online) (a) Same as Fig. 2 except $\Delta = 10$ g, $n = 25$, and $m = 2$. (b) Same as Fig. 8(a) except that the cavity field is in coherent state and $\bar{n} = 25$.

Supposing that the atomic internal state is initially prepared in the coherent superposition of its lower states $\cos\theta|2\rangle + \sin\theta\, e^{i\varphi}|3\rangle$, the atomic center-of-mass motion is in the Fock state or the thermal state, and the atomic spectrum is independent of the phase ϕ. However, the phase of the atomic dipole quantum plays an important role, ignoring the atomic center-of-mass motion.[20] The atomic center-of-mass motion is involved in the interaction between atom and cavity field, we can know from which lower state the atom jumps to the upper state according to the phonon number of the system final states, which leads to the loss of quantum interference effect. If the atomic internal and external freedoms are entangled like $\cos\vartheta|2,m\rangle + \sin\vartheta\, e^{i\varphi}|3,m+2\rangle$, the quantum interference between the two transition processes $|2,m\rangle \to |1,m+1\rangle$ and $|3,m+2\rangle \to |1,m+1\rangle$ makes the atomic emission spectrum sensitive to the phase φ, as shown in Fig. 9. Specially, when the parameters of the atomic initial state satisfy

$$\vartheta = \arctan\sqrt{(m+1)/(m+2)}, \quad \varphi = \pi, \tag{19}$$

atomic state remains unchanged because of the completely destructive interference between the transition processes $|2,m\rangle \to |1,m+1\rangle$ and $|3,m+2\rangle \to |1,m+1\rangle$. Meanwhile, all the peaks of the atomic emission spectrum disappear.

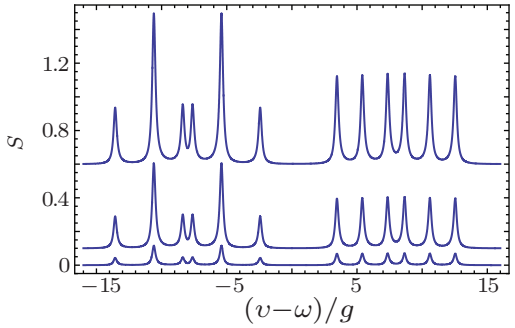


Fig. 9. (color online) Same as Fig. 2 except that the initial atomic state is in the entangled state $(1/2)|2,m\rangle + (\sqrt{3}/2)\, e^{i\varphi}|3,m+2\rangle$, $n = 15$, and $m = 1$. (a) $S(\upsilon)+0.6$, $\varphi = 0$; (b) $S(\upsilon)+0.1$, $\varphi = \pi/2$; (c) $S(\upsilon)$, $\varphi = \pi$.

4. Conclusions

As a Λ-type three-level atom is trapped in the node of the standing wave, the effective Hamiltonian of the system shows that the atomic internal state varies with the atomic center-of-mass motion, so the atomic center-of-mass motion plays a vital role in the atomic emission spectra. If the atom is initially in the upper state $|1\rangle$, the atomic center-of-mass motion and the cavity field are both in the Fock state, and the relative height of the peaks for atomic emission spectrum only depends on the phonon number, but has nothing to do with the photon number and the distance between the atomic lower levels. The atomic emission spectrum shows phase sensitivity only when the atomic internal and external freedoms are entangled, so the atomic center-of-mass motion not only directly affects the peak number, position, and height in atomic emission spectrum, but also influences the effects of the cavity field and the atomic initial state on atomic emission spectrum.

In summary, the shape of the emission spectrum for a harmonically trapped Λ-type three-level atom driven by a single-mode field may be controlled via the atomic center-of-mass motion.

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