Cloaking an object in a half space without reflection*

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(Received 15 August 2012; revised manuscript received 15 October 2012)

A novel cloaking scheme to hide an object in a half space from electromagnetic (EM) detection without reflection is firstly presented. The proposed cloaking scheme contains a couple of matching strips, which consist of an isotropic material layer and an anisotropic UPML layer, located right under the bottom surface of a semi-cylindrical cloaking shell. Simple expressions for the material parameters of the cloaking scheme are derived. Numerical simulations are also performed, and a good cloaking effect is achieved. The cloaking scheme is effective to hide the local object with strong scattering characters placed on mobile carriers, such as the radar antenna system on an aircraft.

Keywords: invisibility cloak, half space, matching strips, reflection

PACS: 41.20.Jb, 78.20.Ci **DOI:** 10.1088/1674-1056/22/5/054101

1. Introduction

Rendering an object invisible in free space has for long been the focus of research since the theory of invisibility cloak based on optics transformation was firstly proposed and experimentally verified by Pendry *et al*.[1–3] With the powerful means of optics transformation, the propagation paths of the electromagnetic wave can be manipulated artificially to exclude from the object embedded inside the cloak without perturbing the exterior fields. Due to its remarkable performance, i.e., absence of scattered field, a growing number of various invisibility cloaks have been studied intensively at both microwave and optical frequencies.[4–17]

In previous studies on invisibility cloaking, researchers initially focused on the sealed integrity cloak design.[4–11] Along with further research, cloaking an object in a half space has become a hot spot of this realm. Carpet cloaking[12–14] and other new cloaking concepts[15–17] were sequentially put forward to hide an object on a PEC or a dielectric half space. All these studies utilized the similar principle that makes the reflected field excited by the cloak be the same as that from bare flat ground as if the object does not exist. There still exist reflected fields in the half space. These existing cloaking schemes are effective to hide the object located on the ground. Even so, they also have limitations. When they are applied to hide the object placed on mobile carriers, such as a radar antenna system on an aircraft, the reflected fields excited by the cloaking scheme are definitely detectable.

In this paper, a novel cloaking scheme to hide an object in a half space without reflection is firstly presented. The proposed scheme is validated to hide the object placed on mobile carriers where absorbing materials cannot be used directly, such as a radar antenna system. The cloaking configuration contains a couple of matching strips located right under the bottom surface of a semi-cylindrical cloaking shell. Though the proposed scheme is similar to that described in Ref. [15], its working principle is entirely different. The matching strips in our scheme are designed to comprise two layers: an isotropic material layer and an anisotropic UMPL material layer, as shown in Fig. 1. The working principle of our cloaking scheme is given below. First, we guide the incident wave to exclude from the object by the semi-cylindrical cloaking shell, and then manipulate it to pass through the interface into the matching strips without reflection and absorb the incident wave inside the matching strips. Compared with the existing methods, there is no reflected fields in the half space with our cloaking scheme.

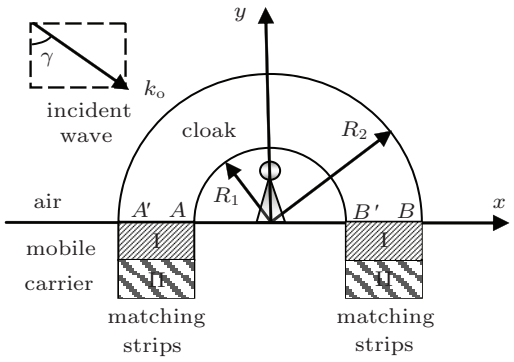


Fig. 1. Schematic diagram for the proposed cloaking configuration. I: isotropic material layer, II: anisotropic UPML material layer.

2. Theoretical derivation of the material parameters of the proposed cloaking scheme

To validate the feasibility of the proposed cloaking scheme, we start from the theoretical derivation of this scheme. For simplicity, we consider that the cloaking configuration is two-dimensional (2D) and the incident field is

*Project supported by the Program for New Century Excellent Talents in University of China (Grant No. NCET-10-0894).

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transverse electrically (TE) polarized. The expression of the time-harmonic incident wave is assumed as

$$E_{i,z} = E_0 \exp\left[-\mathrm{j}k_0\left(x\sin\gamma - y\cos\gamma\right)\right]\exp\left(\mathrm{j}\omega t\right), \quad (1)$$

where γ is the incident angle.

2.1. Design of the semi-cylindrical cloaking shell

The semi-cylindrical cloaking shell we designed here is considered as a truncated half of the ideal cylindrical cloak. Applying the approach proposed in Ref. [3], the relative permittivity and permeability tensors of the cloaking shell can be derived by mapping the following coordinate transformation:

$$r = \begin{cases} r'\dfrac{(R_2 - R_1)}{R_2 + R_1}, & (r' \le R_2), \\ r', & (r' > R_2), \end{cases}$$
$$\phi = \phi', z = z'. \quad (2)$$

A complete set of material parameters of the semi-cylindrical cloaking shell in cylindrical coordinates are given by

$$\overline{\overline{\varepsilon_{\mathrm{r}}}} = \overline{\overline{\mu_{\mathrm{r}}}} = \mathrm{diag}\left(\frac{r-R_1}{r}, \frac{r}{r-R_1}, \left(\frac{R_2}{R2-R_1}\right)^2\frac{r-R_1}{r}\right), \quad (3)$$

where R_1 and R_2 are the inner and outer radius of the cloaking shell, respectively.

2.2. Design of the isotropic material layer of the matching strips

As is well known, after propagating through the outer boundary of the semi-cylindrical cloaking shell, the incident wave is guided without any reflection until reaching the interfaces AA' and BB'. Based on the above-mentioned coordinate transformation, the electric field inside the cloaking shell becomes

$$E_{c,z} = E_0 \exp\left[-\mathrm{j}k_{\mathrm{o}}\left(x'\sin\gamma - y'\cos\gamma\right)\right]\exp\left(\mathrm{j}\omega t\right). \quad (4)$$

Due to the anisotropic and inhomogeneous characters of the semi-cylindrical cloaking shell, here we locally consider the electromagnetic wave around an arbitrary point $P(x_{\mathrm{p}},0)$ on the interface AA' or BB' as a plane wave to simplify the derivation of the material parameters. According to the deduction in Ref. [15], the electromagnetic field around an arbitrary point $P(x_{\mathrm{p}},0)$ on the interface AA' or BB' is obtained as

$$\overline{E}_{c,z} = E_0 \exp\left(-\mathrm{j}k_{\mathrm{o}}\left\{(x-x_{\mathrm{p}})\sin\gamma\left[\frac{R_2}{R_2-R_1}\right] - y\cos\gamma\left(\frac{r'_{\mathrm{p}}}{r_{\mathrm{p}}}\right) + x'_{\mathrm{p}}\sin\gamma\right\}\right)\exp(\mathrm{j}\omega t). \quad (5)$$

Now consider the cloaking configuration shown in Fig. 1. When such a local plane wave impinges on the interface AA' or BB', the local reflection coefficient can be derived as

$$\Gamma_{\mathrm{c}} = \frac{\mu_{\mathrm{r}}^{\mathrm{m1}}\cos\gamma - \sqrt{\varepsilon_{\mathrm{r}}^{\mathrm{m1}}\mu_{\mathrm{r}}^{\mathrm{m1}}\left[R_2/(R_2-R1)\right]^2 - \sin^2\gamma}}{\mu_{\mathrm{r}}^{\mathrm{m1}}\cos\gamma + \sqrt{\varepsilon_{\mathrm{r}}^{\mathrm{m1}}\mu_{\mathrm{r}}^{\mathrm{m1}}\left[R_2/(R_2-R_1)\right]^2 - \sin^2\gamma}}, \quad (6)$$

where $\varepsilon_{\mathrm{r}}^{m1}$ and $\mu_{\mathrm{r}}^{\mathrm{m1}}$ are the relative permittivity and permeability tensors of the isotropic material layer. To make the incident wave propagate through the interface without reflection, the local reflection coefficient should be zero. Thus, we have

$$(\mu_{\mathrm{r}}^{\mathrm{m1}2} - 1)\cos^2\gamma + 1 = \varepsilon_{\mathrm{r}}^{\mathrm{m1}}\mu_{\mathrm{r}}^{\mathrm{m1}}\left[\frac{R_2}{R_2-R_1}\right]^2. \quad (7)$$

From Eq. (7), we know that the solution ($\varepsilon_{\mathrm{r}}^{\mathrm{m1}}$, $\mu_{\mathrm{r}}^{\mathrm{m1}}$) is not unique. To make the solution ($\varepsilon_{\mathrm{r}}^{\mathrm{m1}}$, $\mu_{\mathrm{r}}^{\mathrm{m1}}$) independent of the incident angle γ, among all of the possibilities we choose the following typical solution:

$$\varepsilon_{\mathrm{r}}^{\mathrm{m1}} = \left[\frac{R_2}{R_2-R1}\right]^2, \quad \mu_{\mathrm{r}}^{\mathrm{m1}} = 1. \quad (8)$$

It is notable that both the relative permittivity and permeability of the isotropic material layer are real numbers, which means that the isotropic material layer is lossless. Hence, we conclude that the incident wave can propagate through the interfaces into the isotropic material layer without reflection, whereas apparently it cannot be absorbed inside it.

2.3. Design of the anisotropic UPML layer of the matching strips

To absorb the electromagnetic wave inside the isotropic material layer, another layer matching strip is designed. Here we introduce the concept of the anisotropic uniaxial perfect matching layer (UPML)[18,19] into the matching strips. Since the material parameters of the UPML are derived directly from Maxwell's equations, the electromagnetic fields inside the UPML still satisfy them.

Now consider the regions of the matching strips shown in Fig. 2. The plane wave inside region I is assumed to impinge on the interface between the two layers, which is given by

$$E_{i,z}^{\mathrm{m1}} = E_0 \exp\left[-\mathrm{j}\left(k_{\mathrm{m1},x}x - k_{\mathrm{m1},y}y\right)\right]\exp\left(\mathrm{j}\omega t\right). \quad (9)$$

The relative permittivity and permeability tensors of this anisotropic UPML material layer are defined as follows:

$$\overline{\overline{\varepsilon_{\mathrm{r}}^{\mathrm{m2}}}} = \varepsilon_{\mathrm{r}}^{\mathrm{m1}}\begin{bmatrix} a & & \\ & b & \\ & & a \end{bmatrix} = \left(\frac{R_2}{R_2-R1}\right)^2\begin{bmatrix} a & & \\ & b & \\ & & a \end{bmatrix},$$
$$\overline{\overline{\mu_{\mathrm{r}}^{\mathrm{m2}}}} = \mu_{\mathrm{r}}^{\mathrm{m1}}\begin{bmatrix} c & & \\ & d & \\ & & c \end{bmatrix} = \begin{bmatrix} c & & \\ & d & \\ & & c \end{bmatrix}, \quad (10)$$

where a, b, c, and d are the unknown quantities that we need to work out.

In the region II of Fig. 2, the fields excited within the anisotropic material layer still satisfy Maxwell's equations, and the wave equation is expressed as

$$\begin{bmatrix} k_{\mathrm{m1}}^2 a - \dfrac{k_{\mathrm{m2},y}^2}{c} & -\dfrac{k_{\mathrm{m2},x}k_{\mathrm{m2},y}}{c} & 0 \\ -\dfrac{k_{\mathrm{m2},x}k_{\mathrm{m2},y}}{c} & k_{\mathrm{m1}}^2 c - \dfrac{k_{\mathrm{m2},x}^2}{c} & 0 \\ 0 & 0 & k_{\mathrm{m1}}^2 a - \dfrac{k_{\mathrm{m2},y}^2}{c} - \dfrac{k_{\mathrm{m2},x}^2}{d} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ E_z^{\mathrm{m2}} \end{bmatrix} = 0. \tag{11}$$

Consequently, we have

$$k_{\mathrm{m1}}^2 a - k_{\mathrm{m2},y}^2 c^{-1} - k_{\mathrm{m2},x}^2 d^{-1} = 0, \tag{12}$$

where $k_{\mathrm{m1}}^2 = \omega^2 \varepsilon_{\mathrm{r}}^{\mathrm{m1}} \mu_{\mathrm{r}}^{\mathrm{m1}} \varepsilon_{\mathrm{o}} \mu_{\mathrm{o}} = k_{\mathrm{m1,x}}^2 + k_{\mathrm{m1},y}^2$, and $k_{\mathrm{m2}}^2 = k_{\mathrm{m2,x}}^2 + k_{\mathrm{m2},y}^2$.

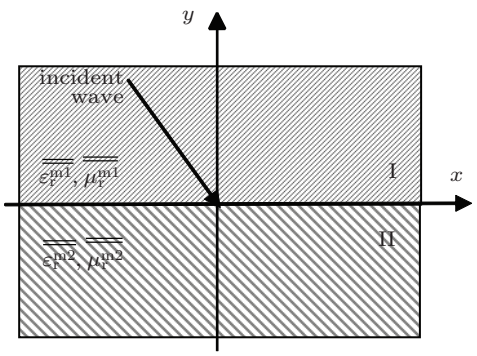


Fig. 2. Amplified diagram for the two-layer matching strips. I: isotropic material layer, II: anisotropic UPML material layer.

Based on Maxwell's equations, the reflected field can be easily derived from Eq. (9). From the continuity of the tangential fields across the interface, we obtain the reflection coefficient Γ

$$\Gamma = \frac{k_{\mathrm{m1},y} - k_{\mathrm{m2},y} c^{-1}}{k_{\mathrm{m1},y} + k_{\mathrm{m2},y} c^{-1}}. \tag{13}$$

When the incident wave completely propagates into the anisotropic UPML material layer for all incidence angles, the reflection coefficient Γ should be zero, so we have

$$k_{\mathrm{m2},y} = c k_{\mathrm{m1},y}. \tag{14}$$

Due to the phase matching across the interface, we also have

$$k_{\mathrm{m2},x} = k_{\mathrm{m1},x}. \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (12), we obtain the following relationship:

$$a = c, \quad a = d^{-1}. \tag{16}$$

According to the duality of the electromagnetic theory, a similar formula for the TM incidence can also be obtained

$$c = a, \quad c = b^{-1}. \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (10) results in

$$\overline{\overline{\varepsilon_{\mathrm{r}}^{\mathrm{m2}}}} = \left(\frac{R_2}{R_2 - R_1}\right)^2 \begin{bmatrix} a & & \\ & a^{-1} & \\ & & a \end{bmatrix},$$
$$\overline{\overline{\mu_{\mathrm{r}}^{\mathrm{m2}}}} = \begin{bmatrix} a & & \\ & a^{-1} & \\ & & a \end{bmatrix}, \tag{18}$$

where $a = 1 + \sigma/(\mathrm{j}\omega\varepsilon_o)$, and σ is the conductivity of the anisotropic UPML material layer.

Substituting $a = 1 + (\sigma/\mathrm{j}\omega\varepsilon_{\mathrm{o}})$ into Eq. (14) results in

$$\begin{aligned} k_{\mathrm{m2},y} &= c k_{\mathrm{m1},y} = \left(1 + \frac{\sigma}{\mathrm{j}\omega\varepsilon_{\mathrm{o}}}\right) k_{\mathrm{m1},y} \\ &= k_{\mathrm{m1},y} - \mathrm{j}\frac{\sigma}{\omega\varepsilon_{\mathrm{o}}} k_{\mathrm{m1},y}. \end{aligned} \tag{19}$$

Substituting Eqs. (15) and (19) into the formula of the electric field inside the anisotropic UPML material layer, we find that

$$\begin{aligned} E_{\mathrm{i},z}^{\mathrm{m2}} &= E_0 \exp(-\alpha_y y) \exp\left[-\mathrm{j}\left(k_{\mathrm{m1},x} x - k_{\mathrm{m1},y} y\right)\right] \\ &\quad \times \exp(\mathrm{j}\omega t), \end{aligned} \tag{20}$$

where $\alpha_y = \sigma k_{\mathrm{m1},y}/\omega\varepsilon_{\mathrm{o}}$.

From Eq. (20), we know that the electromagnetic wave inside the anisotropic UPML layer is evanescent mode, and will be totally absorbed inside it.

Through the above theoretical derivation, we have obtained all of the material parameters of the proposed cloaking scheme. With this cloaking scheme, the goal of rendering an object invisible on a half space without any reflection can be achieved.

3. Numerical simulations of the proposed cloaking scheme

In order to verify the cloaking effect of the cloaking scheme, numerical simulations using the COMSOL software have been carried out. The frequency of the incident wave is 7.5 GHz. The dimensions of the cloaking shell are set as $R_1 = 0.1$ m and $R_2 = 0.2$ m, and its relative permittivity and permeability tensors are obtained with Eq. (3) (specified in Cartesian coordinates). The material parameters of the matching strips are obtained from Eqs. (8) and (18), and the thickness of each layer of the matching strips is set as 0.04 m. Perfectly matched layers are used to terminate the computational domain.

In simulations, the object to be hidden is assumed to be the local region with strong scattering characters on mobile carrier, such as the radar antenna system. The other regions of the mobile carrier are covered with radar absorbing materials (RAM) to eliminate the reflection from them. An incident

wave excited by a line current source is adopted. For comparison, three cases are simulated with the same excitation and snapshots of the electric field distributions are shown in Fig. 3. Considering that the proposed cloaking scheme should work for an incident wave with all angles, we set normal incidence and oblique incidence for each case. From the field distributions in Figs. 3(a) and 3(b), it can be clearly seen that the field is severely distorted by the object without any cloak covering it. In Figs. 3(c) and 3(d), the object is embedded inside a semi-cylindrical cloaking shell without the matching strips. Although the cloaking shell can guide the incident wave to exclude it from the object, there still exists obvious perturbation on the exterior field because of the truncation of the semi-cylindrical cloaking shell. Figures 3(e) and 3(f) present the field distribution when the proposed cloaking scheme is implemented. One can clearly see that the electric field outside the cloaking shell is hardly perturbed in both of the conditions. Thus, we can draw the conclusion that the proposed cloaking scheme is effective to hide the object with strong scattering characters on mobile carriers where radar absorbing materials (RAMs) cannot be used directly.

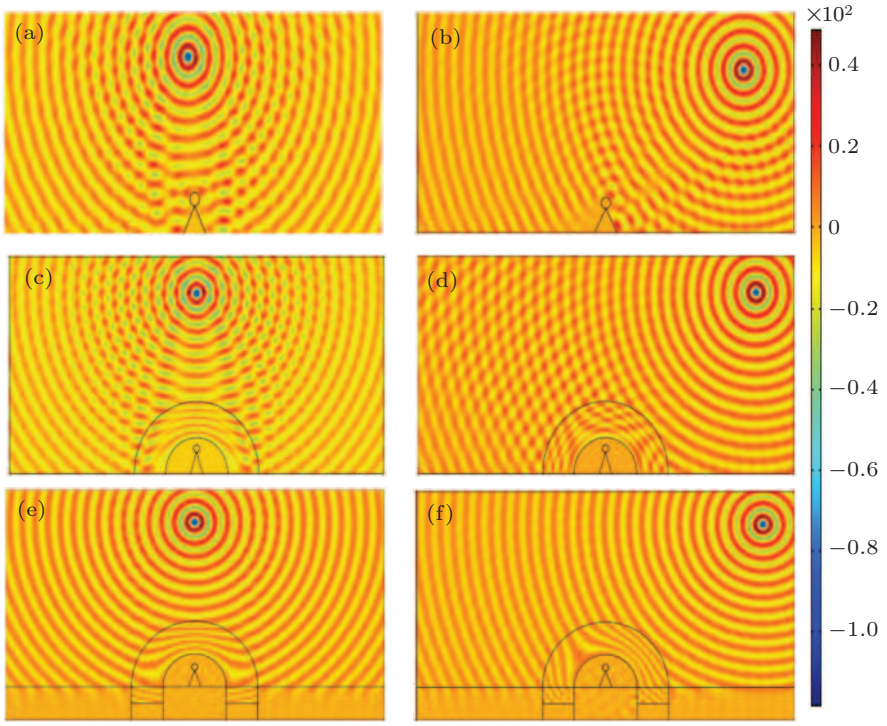


Fig. 3. Snapshots of the electric field distributions of a line current source for normal incidence and oblique incidence. (a), (b) Only an object; (c), (d) an object covered by a semi-cylindrical cloaking shell; (e), (f) an object covered by the proposed cloaking scheme.

4. Conclusions

In summary, a novel approach to hide an object in a half space without reflection is presented. The proposed cloaking scheme can guide the incident wave to exclude from the object and pass through the interface into the isotropic material layer without reflection, and absorb the electromagnetic wave in the anisotropic UPML layer. Numerical simulations have been performed to validate its cloaking effect. This novel cloaking scheme has enormous potential applications in hiding the local strong scattering sources on mobile carriers where radar absorbing materials (RAMs) cannot be used directly.

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