# A thermal entangled quantum refrigerator based on a two-qubit Heisenberg model with Dzyaloshinskii–Moriya interaction in an external magnetic field*

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Based on an isotropic two spin-1/2 qubits Heisenberg model with the Dzyaloshinskii–Moriya interaction in an external magnetic field, we have constructed an entangled quantum refrigerator. Expressions for the basic thermodynamic quantities, i.e., the heat exchanged, the net work input, and the coefficient of performance, are derived. Some intriguing features and their qualitative explanations in zero and non zero magnetic fields are given. The influence of the thermal entanglement on the refrigerator is investigated. The results obtained here have general significance and will be helpful to understand the performance of an entangled quantum refrigerator.

**Keywords:** Heisenberg model, thermal entanglement, concurrence, entangled refrigerator

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## 1. Introduction

Quantum entanglement has attracted much attention not only because it is a powerful tool for testing the local hidden theory against quantum mechanics[1] but also because it is an important resource for quantum information processing.[2–8] The Heisenberg chain is valuable to quantum information processing; it has been used to describe quantum computers, quantum dots,[9,10] nuclear spins,[11] electronic spins,[12] and optical lattices.[13] The thermal entanglement of the Heisenberg chain has been widely studied. Nielsen[14] is the first person to study the entanglement in a two-qubit Heisenberg $XXX$ chain with the Hamiltonian $H = J\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + B\left(\sigma_z^1 + \sigma_z^2\right)$. He showed that the entanglement in such a system exists only for the antiferromagnatic $(J > 0)$ case below a threshold temperature $T_{\mathrm{C}}$.

Thermodynamics is a theory of impressive success and has a wide range of applications. The working substance in a quantum cycle may be a spin system, a harmonic oscillator system, a two-level or multilevel system, ideal quantum gases, photon gas, and so on.[15–24] Because of the quantum features of the working substance, many exotic properties that differ from their classical counterparts have been found. The influence of the quantum characteristics of the working fluid on the performance of the cycle must be considered. Recently, the quantum heat engine or refrigerator using a quantum entangled system as the working substance has been introduced.[25–33]

To further enrich the theory of the entangled quantum thermodynamic cycle, we propose a thermal entangled quantum Otto refrigerator based on a two-qubit Heisenberg model with the Dzyaloshinskii–Moriya interaction in an external magnetic field. The purpose of this paper is to analyze the effect of the quantum entanglement on the thermodynamic quantities of the quantum Otto engine. Some intriguing features and qualitative explanations are given. The validity of the second law of thermodynamics is confirmed in the system.

## 2. Two-qubit Heisenberg model with Dzyaloshinskii–Moriya interaction in an external magnetic field

The Hamiltonian for two isotropic spin-1/2 qubits with the Dzyaloshinskii–Moriya (DM) interaction in an external magnetic field $B$ can be given as

$$H = J\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + B(\sigma_1^z + \sigma_2^z) + \boldsymbol{D} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2), \tag{1}$$

where $J$ is the isotropic bilinear interaction between the spin pair, $\boldsymbol{D}$ is the component of the DM interaction, and $\boldsymbol{\sigma}_i$ $(i = 1, 2)$ are the well known Pauli spin matrices for spin-1/2. For simplicity, we choose $\boldsymbol{D} = D\boldsymbol{z}$, then the Hamiltonian becomes

$$H = J\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + B(\sigma_1^z + \sigma_2^z) + D\left(\sigma_1^x \sigma_2^y - \sigma_1^y \sigma_2^x\right). \tag{2}$$

The four eigenvalues of Eq. (2) are

$$\begin{cases} E_1 = -B, \\ E_2 = B, \\ E_3 = -J\sqrt{1+D^2}, \\ E_4 = J\sqrt{1+D^2}. \end{cases} \tag{3}$$

The state of the system at thermal equilibrium is described by the density matrix $\rho(T) = \exp(-\beta H)/Z$, the partition function is given by $Z = \mathrm{Tr}[\exp(-\beta H)]$, where $\beta = 1/k_{\mathrm{B}}T$, $k_{\mathrm{B}}$ is the Boltzmann constant and taken to be

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unity. To quantify the amount of entanglement associated with $\rho_{12}$, we adopt the thermal concurrence $C$ defined by Wootters[34]

$$C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \tag{4}$$

where $\lambda_i$ $(i = 1,2,3,4)$ are the square roots of the eigenvalues of density matrix operator $\rho = \rho_{12}\left(\sigma_1^y \otimes \sigma_2^y\right)\rho_{12}^*\left(\sigma_1^y \otimes \sigma_2^y\right)$ in decreasing order, * indicates the complex conjugation, and $\sigma^y$ is the Pauli matrix. The thermal concurrence varies from $C = 0$ for an unentangled state to $C = 1$ for a maximally entangled state. The square roots of the eigenvalues of matrix $\rho$ are

$$\lambda_1 = \lambda_2 = \frac{1}{2}\frac{1}{\cosh\left(\dfrac{B}{k_{\mathrm{B}}T}\right) + \cosh\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right)}, \tag{5}$$

$$\lambda_3 = \frac{1}{1 + \exp\left(\dfrac{2J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right) + 2\exp\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right)\cosh\left(\dfrac{B}{k_{\mathrm{B}}T}\right)}, \tag{6}$$

$$\lambda_4 = \frac{1}{2}\frac{\exp\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right)}{\cosh\left(\dfrac{B}{k_{\mathrm{B}}T}\right) + \cosh\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right)}. \tag{7}$$

From Eqs. (4) and (7), the concurrence is given by

$$C = \max\left\{\frac{\sinh\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right) - 1}{\cosh\left(\dfrac{B}{k_{\mathrm{B}}T}\right) + \cosh\left(\dfrac{J\sqrt{1+D^2}}{k_{\mathrm{B}}T}\right)}, 0\right\}. \tag{8}$$

Then we know $C = 0$ if $\sinh\left(J\sqrt{1+D^2}/k_{\mathrm{B}}T\right) \leq 1$, i.e., the critical temperature is given by

$$\mathscr{T}_{\mathrm{C}} = \frac{|J|\sqrt{1+D^2}}{k_{\mathrm{B}}\operatorname{arcsinh}(1)} \approx 1.1346\frac{|J|\sqrt{1+D^2}}{k_{\mathrm{B}}}. \tag{9}$$

The entanglement vanishes for $T \geq \mathscr{T}_{\mathrm{C}}$. It is interesting to see that the critical temperature is dependent on the coupling constant $J$ and the DM interaction.

## 3. Model of thermal entangled quantum refrigerator

The entangled quantum refrigerator considered here is analogue to the classical Otto refrigerator. The working medium is two isotropic spin-1/2 qubits with the DM interaction in an external magnetic field $B$. A cycle sketch of the thermal entangled quantum refrigerator is shown in Fig. 1. The refrigerator cycle consists of four states labeled by A, B, C, and D, as illustrated in Fig. 1. The entangled quantum refrigerator includes two isothermal processes (1 and 3) and two quantum adiabatic processes (2 and 4).

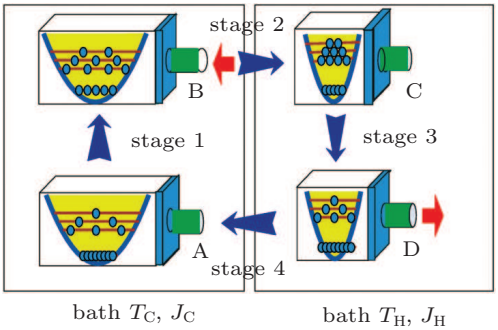



**Fig. 1.** (color online) Schematic diagram of the entangled quantum refrigerator based on two spin-1/2 qubits with the DM interaction in an external magnetic field.

**Stage 1** (A → B) The working medium of two spin-1/2 qubits with the coupling constant $J_{\mathrm{C}}$ and the external magnetic field $B$ is coupled to a cold bath of temperature $T_{\mathrm{C}}$ and its energy structure is kept fixed. In this isochoric process, the population of the four discrete levels is changed, from population $P_{i\mathrm{H}}$ to population $P_{i\mathrm{C}}$ $(i = 1,2,3,4)$

$$\begin{cases} P_{1\mathrm{H}} = \exp\left(\dfrac{B}{k_{\mathrm{B}}T_H}\right)/Z_H, \\ P_{2\mathrm{H}} = \exp\left(-\dfrac{B}{k_{\mathrm{B}}T_{\mathrm{H}}}\right)/Z_{\mathrm{H}}, \\ P_{3\mathrm{H}} = \exp\left(\dfrac{J_{\mathrm{H}}\sqrt{1+D^2}}{k_{\mathrm{B}}T_{\mathrm{H}}}\right)/Z_{\mathrm{H}}, \\ P_{4\mathrm{H}} = \exp\left(-\dfrac{J_{\mathrm{H}}\sqrt{1+D^2}}{k_{\mathrm{B}}T_{\mathrm{H}}}\right)/Z_{\mathrm{H}}, \end{cases} \tag{10}$$

$$\begin{cases} P_{1\mathrm{C}} = \exp\left(\dfrac{B}{k_{\mathrm{B}}T_{\mathrm{C}}}\right)/Z_{\mathrm{C}}, \\ P_{2\mathrm{C}} = \exp\left(-\dfrac{-B}{k_{\mathrm{B}}T_{\mathrm{C}}}\right)/Z_{\mathrm{C}}, \\ P_{3\mathrm{C}} = \exp\left(\dfrac{J_{\mathrm{C}}\sqrt{1+D^2}}{k_{\mathrm{B}}T_{\mathrm{C}}}\right)/Z_{\mathrm{C}}, \\ P_{4\mathrm{C}} = \exp\left(-\dfrac{J_{\mathrm{C}}\sqrt{1+D^2}}{k_{\mathrm{B}}T_{\mathrm{C}}}\right)/Z_{\mathrm{C}}, \end{cases} \tag{11}$$

where

$$Z_{\rm H} = \exp\left(\frac{B}{k_{\rm B}T_{\rm H}}\right)+\exp\left(-\frac{B}{k_{\rm B}T_{\rm H}}\right) + \exp\left(\frac{J_{\rm H}\sqrt{1+D^2}}{k_{\rm B}T_{\rm H}}\right)+\exp\left(-\frac{J_{\rm H}\sqrt{1+D^2}}{k_{\rm B}T_{\rm H}}\right),$$

$$Z_{\rm C} = \exp\left(\frac{B}{k_{\rm B}T_{\rm C}}\right)+\exp\left(-\frac{B}{k_{\rm B}T_{\rm C}}\right) + \exp\left(\frac{J_{\rm C}\sqrt{1+D^2}}{k_{\rm B}T_{\rm C}}\right)+\exp\left(-\frac{J_{\rm C}\sqrt{1+D^2}}{k_{\rm B}T_{\rm C}}\right).$$

And only heat is transferred in this stage to yield changes in the occupation probabilities, no work is done as there is no change in the energy levels.

**Stage 2** (B $\rightarrow$ C) This is an adiabatic process. In this stage, if the compression rate is sufficiently slow according to the quantum adiabatic theorem, the occupation probabilities $P_{i\rm C}$ $(i=1,2,3,4)$ of the working medium will remain unchanged. The exchange constant changes from $J_{\rm C}$ to $J_{\rm H}$. The working medium is decoupled from the cold reservoir of temperature $T_{\rm C}$, and the energy structure is varied from $E_{i\rm C}$ to $E_{i\rm H}$ $(i=1,2,3,4)$

$$\begin{cases} E_{1\rm C}=-B, \\ E_{2\rm C}=B, \\ E_{3\rm C}=-J_{\rm C}\sqrt{1+D^2}, \\ E_{4\rm C}=J_{\rm C}\sqrt{1+D^2}, \end{cases} \tag{12}$$

$$\begin{cases} E_{1\rm H}=-B, \\ E_{2\rm H}=B, \\ E_{3\rm H}=-J_{\rm H}\sqrt{1+D^2}, \\ E_{4\rm H}=J_{\rm H}\sqrt{1+D^2}. \end{cases} \tag{13}$$

The refrigerator performs negative work when the energy spacing increases.

**Stage 3** (C $\rightarrow$ D) Stage 3 is almost the inverse process of stage 1. The working medium is coupled to a hot reservoir of temperature $T_{\rm H}$, and its energy structure $E_{i\rm H}$ $(i=1,2,3,4)$ is kept fixed. In this isochoric process, the population of the four discrete levels is changed from $P_{i\rm C}$ to $P_{i\rm H}$ $(i=1,2,3,4)$, and some heat is transferred, but no work is performed in this stage.

**Stage 4** (D $\rightarrow$ A) Stage 4 is also an adiabatic process. In this process, the occupation probability $P_{i\rm H}$ $(i=1,2,3,4)$ is kept fixed. The working medium is decoupled from the hot reservoir of temperature $T_{\rm H}$, the exchange constant changes from $J_{\rm H}$ back to $J_{\rm C}$, and the energy structure is varied from $E_{i\rm H}$ to $E_{i\rm C}$ $(i=1,2,3,4)$. In this stage, an amount of work is performed by the system.

The expectation value of the energy of the system with four coupling discrete energy levels is

$$U=\langle E\rangle=\sum_{i=1,}^{4}P_{ij}E_{ij},\quad (i=1,2,3,4;\ j={\rm C,H}), \tag{14}$$

where $E_{ij}$ are the energy levels, and $P_{ij}$ are the corresponding occupation probabilities. Infinitesimally,

$$\mathrm{d}U=\sum_{i=1}^{4}E_{ij}\,\mathrm{d}P_{ij}+P_{ij}\,\mathrm{d}E_{ij}, \tag{15}$$

from which we make the following identifications for infinitesimal heat transferred $\mathrm{d}Q$ and work done $\mathrm{d}W$

$$\begin{cases} \mathrm{d}Q=\sum\limits_{i=1}^{4}E_{ij}dP_{ij}, \\ \mathrm{d}W=\sum\limits_{i=1}^{4}P_{ij}dE_{ij}. \end{cases} \tag{16}$$

According to the quantum interpretations of heat transferred and work input in Eq. (16), the heat transferred $Q_{\rm C}$ in stage 1 and $Q_{\rm H}$ in stage 3 are given by

$$Q_{\rm C} = \sum_{i=1}^{4}E_{i\rm C}\,(P_{i\rm C}-P_{i\rm H}) = \alpha_{\rm C}-\alpha_{\rm H}+\sqrt{1+D^2}\,(\beta_{\rm C}-\beta_{\rm H})\,J_{\rm C}, \tag{17}$$

$$Q_{\rm H} = \left|\sum_{i=1}^{4}E_{i\rm H}\,(P_{i\rm H}-P_{i\rm C})\right| = \alpha_{\rm C}-\alpha_{\rm H}+\sqrt{1+D^2}\,(\beta_{\rm C}-\beta_{\rm H})\,J_{\rm H}, \tag{18}$$

where

$$\alpha_{\rm C}=\frac{B\sinh\left(\frac{B}{k_{\rm B}T_{\rm C}}\right)}{\cosh\left(\frac{B}{k_{\rm B}T_{\rm C}}\right)+\cosh\left(\sqrt{1+D^2}\,\frac{J_{\rm C}}{k_{\rm B}T_{\rm C}}\right)},$$

$$\alpha_{\rm H}=\frac{B\sinh\left(\frac{B}{k_{\rm B}T_{\rm H}}\right)}{\cosh\left(\frac{B}{k_{\rm B}T_{\rm H}}\right)+\cosh\left(\sqrt{1+D^2}\,\frac{J_{\rm H}}{k_{\rm B}T_{\rm H}}\right)},$$

$$\beta_{\rm C}=\frac{\sinh\left(\sqrt{1+D^2}\,\frac{J_{\rm C}}{k_{\rm B}T_{\rm C}}\right)}{\cosh\left(\frac{B}{k_{\rm B}T_{\rm C}}\right)+\cosh\left(\sqrt{1+D^2}\,\frac{J_{\rm C}}{k_{\rm B}T_{\rm C}}\right)},$$

$$\beta_{\rm H}=\frac{\sinh\left(\sqrt{1+D^2}\,\frac{J_{\rm H}}{k_{\rm B}T_{\rm H}}\right)}{\cosh\left(\frac{B}{k_{\rm B}T_{\rm H}}\right)+\cosh\left(\sqrt{1+D^2}\,\frac{J_{\rm H}}{k_{\rm B}T_{\rm H}}\right)}.$$

From the conservation of energy, we find that the net work input in the two quantum adiabatic processes, i.e., stages 2 and 4, is

$$W=Q_{\rm H}-Q_{\rm C}=\sqrt{1+D^2}\,(J_{\rm H}-J_{\rm C})\,(\beta_{\rm C}-\beta_{\rm H}). \tag{19}$$

The coefficient of performance (COP) $\varepsilon$ of the entangled quantum refrigerator reads

$$\varepsilon=\frac{Q_{\rm C}}{W}=\frac{\alpha_{\rm C}-\alpha_{\rm H}+\sqrt{1+D^2}\,(\beta_{\rm C}-\beta_{\rm H})\,J_{\rm C}}{\sqrt{1+D^2}\,(J_{\rm H}-J_{\rm C})\,(\beta_{\rm C}-\beta_{\rm H})}. \tag{20}$$

## 4. Thermodynamic quantities

The entanglement under our consideration is that of two thermal equilibrium states at the end of stage 1 (stage 3) denoted by $C_1$ ($C_2$). They are given by

$$C_1 = \begin{cases} 0, & T_{\rm C} \geq \sqrt{1+D^2}\dfrac{J_{\rm C}}{k_{\rm B}\,\mathrm{arcsinh}\,(1)}, \\ \dfrac{\sinh\left(\sqrt{1+D^2}\dfrac{J_{\rm C}}{k_{\rm B}T_{\rm C}}\right)-1}{\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm C}}\right)+\cosh\left(\sqrt{1+D^2}\dfrac{J_{\rm C}}{k_{\rm B}T_{\rm C}}\right)}, & T_{\rm C} < \sqrt{1+D^2}\dfrac{J_{\rm C}}{k_{\rm B}\,\mathrm{arcsinh}\,(1)}, \end{cases} \tag{21}$$

$$C_2 = \begin{cases} 0, & T_{\rm H} \geq \sqrt{1+D^2}\dfrac{J_{\rm H}}{k_{\rm B}\,\mathrm{arcsinh}\,(1)}, \\ \dfrac{\sinh\left(\sqrt{1+D^2}\dfrac{J_{\rm H}}{k_{\rm B}T_{\rm H}}\right)-1}{\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm H}}\right)+\cosh\left(\sqrt{1+D^2}\dfrac{J_{\rm H}}{k_{\rm B}T_{\rm H}}\right)}, & T_{\rm H} < \sqrt{1+D^2}\dfrac{J_{\rm H}}{k_{\rm B}\,\mathrm{arcsinh}\,(1)}. \end{cases} \tag{22}$$

From Eqs. (21) and (22), we find

$$J_{\rm C} = \frac{k_{\rm B}T_{\rm C}}{\sqrt{1+D^2}}\,\mathrm{arccosh}\left[\frac{C_1 + C_1^2\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm C}}\right) + \sqrt{2-C_1^2+2C_1\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm C}}\right)+C_1^2\cosh^2\left(\dfrac{B}{k_{\rm B}T_{\rm C}}\right)}}{1-C_1^2}\right], \tag{23}$$

$$J_{\rm H} = \frac{k_{\rm B}T_{\rm H}}{\sqrt{1+D^2}}\,\mathrm{arccosh}\left[\frac{C_2 + C_2^2\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm H}}\right) + \sqrt{2-C_2^2+2C_2\cosh\left(\dfrac{B}{k_{\rm B}T_{\rm H}}\right)+C_2^2\cosh^2\left(\dfrac{B}{k_{\rm B}T_{\rm H}}\right)}}{1-C_2^2}\right]. \tag{24}$$

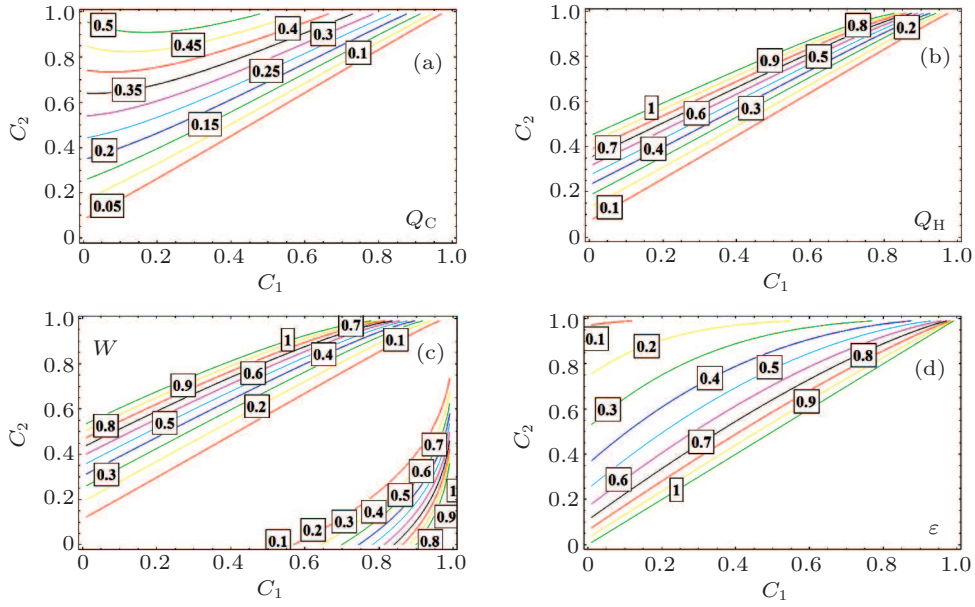



**Fig. 2.** (color online) Variations of heat transferred (a) $Q_{\rm C}$, (b) $Q_{\rm H}$, (c) net work input $W$, and (d) COP $\varepsilon$ of the entangled quantum refrigerator with variables $C_1$ and $C_2$ in isoline map for parameters $k_{\rm B}T_{\rm C} = 1$, $k_{\rm B}T_{\rm H} = 2$, and $B = 0$ T.

Now, we explore the relation between entanglement and the basic thermodynamics quantities of the entangled quantum refrigerator. We first explore the case of zero external magnetic field $B = 0$. Equations (23) and (24) become

$$J_{\rm C} = \frac{k_{\rm B}T_{\rm C}}{\sqrt{1+D^2}}\,\mathrm{arccosh}\left(\frac{C_1 + C_1^2 + \sqrt{2+2C_1}}{1-C_1^2}\right), \tag{25}$$

$$J_{\rm H} = \frac{k_{\rm B}T_{\rm H}}{\sqrt{1+D^2}}\,\mathrm{arccosh}\left(\frac{C_2 + C_2^2 + \sqrt{2+2C_2}}{1-C_2^2}\right). \tag{26}$$

By simple deduction, equations (17)–(20) yield

$$Q_{\rm C} = \sqrt{1+D^2}\left[\tanh\left(\sqrt{1+D^2}\frac{J_{\rm C}}{2k_{\rm B}T_{\rm C}}\right)\right.$$

$$- \tanh\left(\sqrt{1+D^2}\frac{J_{\rm H}}{2k_{\rm B}T_{\rm H}}\right)\Bigg]J_{\rm C}, \tag{27}$$

$$Q_{\rm H} = \sqrt{1+D^2}\Bigg[\tanh\left(\sqrt{1+D^2}\frac{J_{\rm C}}{2k_{\rm B}T_{\rm C}}\right) - \tanh\left(\sqrt{1+D^2}\frac{J_{\rm H}}{2k_{\rm B}T_{\rm H}}\right)\Bigg]J_{\rm H}, \tag{28}$$

$$W = Q_{\rm H} - Q_{\rm C} = \sqrt{1+D^2}\Bigg[\tanh\left(\sqrt{1+D^2}\frac{J_{\rm C}}{2k_{\rm B}T_{\rm C}}\right) - \tanh\left(\sqrt{1+D^2}\frac{J_{\rm H}}{2k_{\rm B}T_{\rm H}}\right)\Bigg](J_{\rm C}-J_{\rm H}), \tag{29}$$

$$\varepsilon = \frac{Q_{\rm C}}{W} = \left(\frac{J_{\rm H}}{J_{\rm C}}-1\right)^{-1} = \left(\frac{\operatorname{arccosh}\left(\frac{C_2+C_2^2+\sqrt{2+2C_2}}{1-C_2^2}\right)}{\operatorname{arccosh}\left(\frac{C_1+C_1^2+\sqrt{2+2C_1}}{1-C_1^2}\right)}\frac{T_{\rm H}}{T_{\rm C}}-1\right)^{-1} \leq \varepsilon_{\rm C} = \frac{T_{\rm C}}{T_{\rm H}-T_{\rm C}}. \tag{30}$$

It can be verified from Eq. (30) that the COP $\varepsilon$ of the entangled quantum refrigerator in the case of zero external magnetic field cannot reach the Carnot COP $\varepsilon_{\rm C}$.

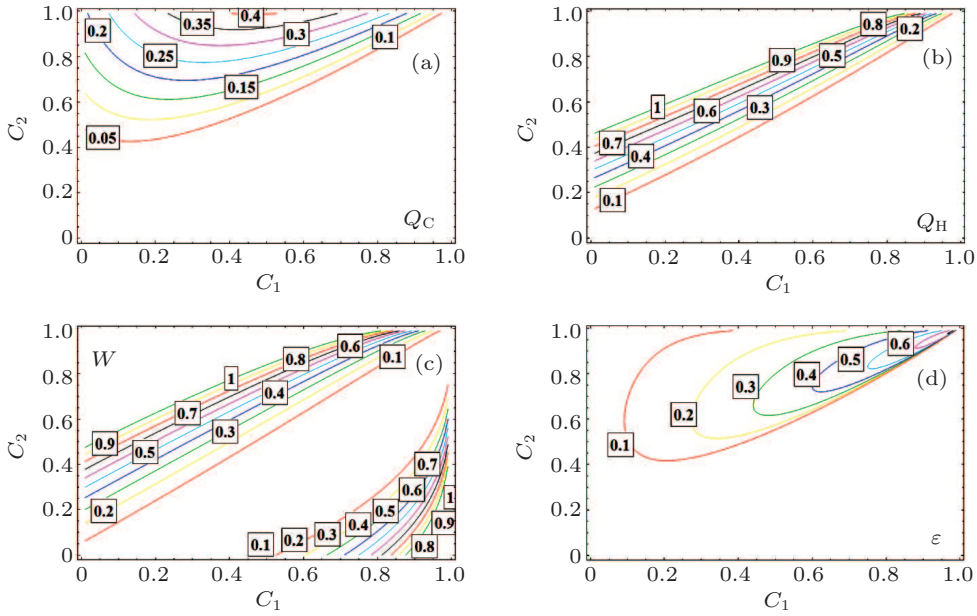



**Fig. 3.** (color online) Variations of heat transferred (a) $Q_{\rm C}$, (b) $Q_{\rm H}$, (c) net work input $W$, and (d) COP $\varepsilon$ of the entangled quantum refrigerator with variables $C_1$ and $C_2$ in isoline map for parameters $k_{\rm B}T_{\rm C}=1$, $k_{\rm B}T_{\rm H}=2$, and $B=1$ T.

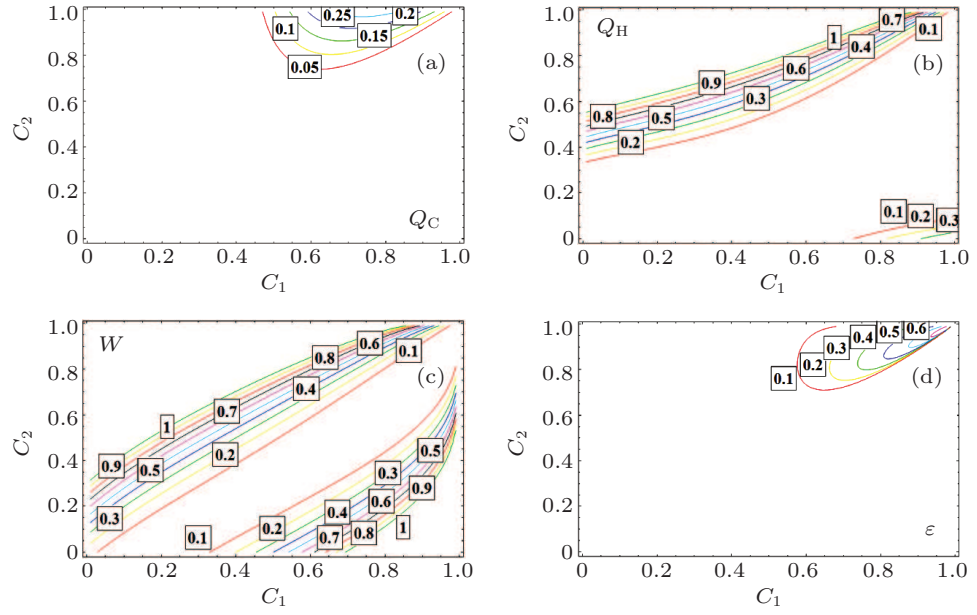



**Fig. 4.** (color online) Variations of heat transferred (a) $Q_{\rm C}$, (b) $Q_{\rm H}$, (c) net work input $W$, and (d) COP $\varepsilon$ of the entangled quantum refrigerator with variables $C_1$ and $C_2$ in isoline map for parameters $k_{\rm B}T_{\rm C}=1$, $k_{\rm B}T_{\rm H}=2$, and $B=3$ T.

Equations (27)–(30) analytically give the expressions of the entangled quantum refrigerator in the case of zero external magnetic field in terms of two thermal concurrences $C_1$ and $C_2$. Figure 2 shows the isolines of heat exchanged $Q_C$, $Q_H$, net work input $W$, and COP $\varepsilon$ in the case of zero magnetic field. It can be found that the isolines are open lines. That is to say, the larger the thermal concurrence $C_1$, the larger $Q_C$, $Q_H$, $W$, and $\varepsilon$, and the smaller the thermal concurrence $C_2$, the larger $Q_C$, $Q_H$, $W$, and $\varepsilon$.

Combining Eqs. (17)–(20), we can calculate heat exchanged $Q_C$, $Q_H$, net work input $W$, and COP $\varepsilon$ of the entangled refrigerator in the case of nonzero magnetic field ($B \neq 0$). By numerical calculations we can plot the variations of these basic thermodynamic quantities with $C_1$ and $C_2$ for $B = 1.0$ T, $B = 3.0$ T, $B = 7.0$ T, and $B = 10.0$ T, as shown in Figs. 3–6, respectively.

It is clear from Figs. 3–6 that in a high enough magnetic field $B$, the isolines of the heat exchanged $Q_C$, $Q_H$, net work input $W$, and COP $\varepsilon$ also appear when $C_1 < C_2$, whereas for the zero external magnetic field case, it seems that only thermal concurrence $C_1 > C_2$ is relevant. This can be explained by that the entanglement can increase with the increase of temperature in certain magnetic field $B$. Thereby $C_1 < C_2$ would possibly occur for some magnetic field $B$.

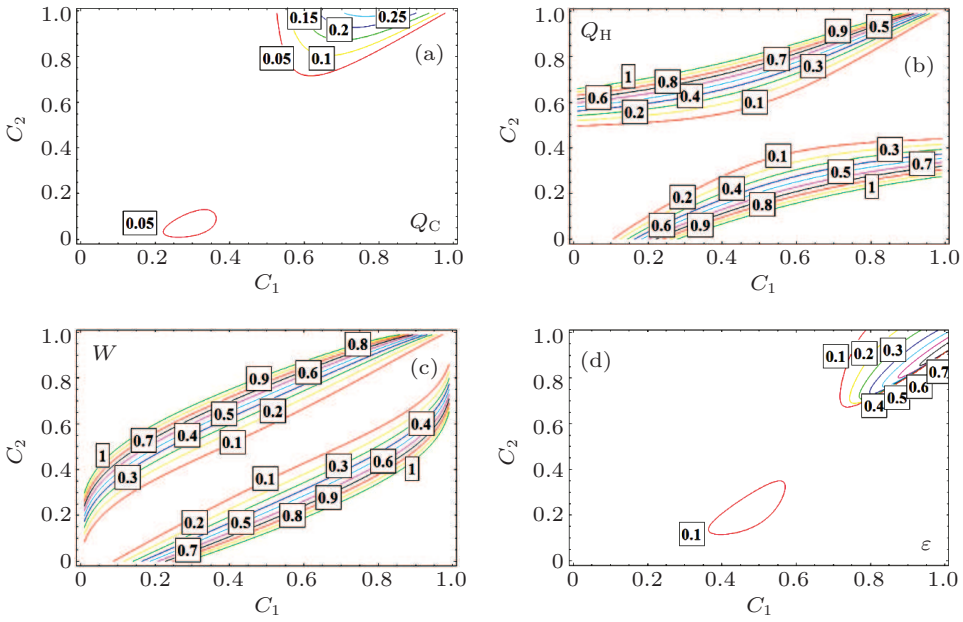



**Fig. 5.** (color online) Variations of heat transferred (a) $Q_C$, (b) $Q_H$, (c) net work input $W$, and (d) COP $\varepsilon$ of the entangled quantum refrigerator with variables $C_1$ and $C_2$ in isoline map for parameters $k_B T_C = 1$, $k_B T_H = 2$, and $B = 7$ T.

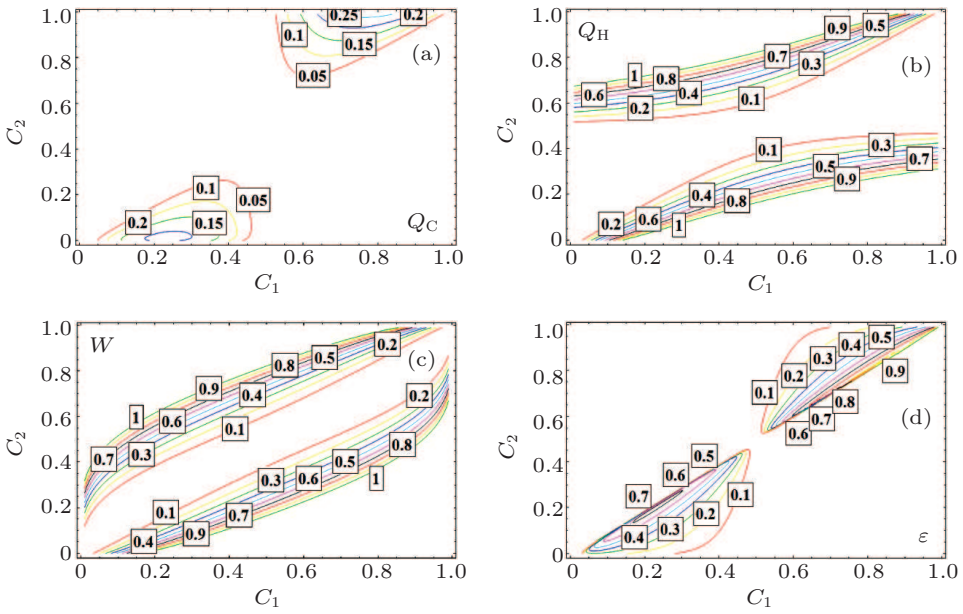



**Fig. 6.** (color online) Variations of heat transferred (a) $Q_C$, (b) $Q_H$, (c) net work input $W$, and (d) COP $\varepsilon$ of the entangled quantum refrigerator with variables $C_1$ and $C_2$ in isoline map for parameters $k_B T_C = 1$, $k_B T_H = 2$, and $B = 10$ T.

It is clear from Figs. 3–6 that the amount of heat transferred $Q_C$ decreases (increases) monotonically with increasing $C_1$ when $C_2$ is fixed in the case of $C_1 < C_2$ ($C_1 > C_2$). Moreover, the amount of heat transferred $Q_H$ increases (decreases) monotonically with increasing $C_2$ when $C_1$ is fixed in the case of $C_1 < C_2$ ($C_1 > C_2$). Furthermore, the isoline of COP $\varepsilon$ is always the loop isoline in the case of non zero magnetic field. Thus, COP $\varepsilon$ no longer monotonically changes with $C_1$ or $C_2$. This indicates that COP $\varepsilon$ exists a maximum $\varepsilon_{\max}$ when the thermal concurrences $C_1$ and $C_2$ attain certain values. That is to say, COP $\varepsilon$ is doubtlessly affected by non zero magnetic field $B\,(B \neq 0)$.

Comparing Fig. 2 with Figs. 3–6, we find that COP $\varepsilon$ and the maximum COP $\varepsilon_{\max}$ decrease with the increase of magnetic field $B$ in the case of $C_1 > C_2$. Furthermore, COP $\varepsilon$ and the maximum COP $\varepsilon_{\max}$ increase with the increase of magnetic field $B$ in the case of $C_1 < C_2$. The magnetic field not only affects the shape of COP $\varepsilon$ but also affects its size. The loop isolines of COP $\varepsilon$ become more and obvious in the case of $C_1 > C_2$, and less and obvious in the case of $C_1 < C_2$ when the magnetic field increases. We also find that $\varepsilon_C = \frac{T_C}{T_H - T_C}$ is not achievable in these figures. Therefore, the second law of thermodynamics holds in all cases.

## 5. Conclusion

Based on the isotropic two spin-1/2 qubits Heisenberg model with the DM interaction in an external magnetic field, we have constructed an entangled quantum refrigerator. Expressions for heat exchange, the net work input, and the COP are derived. It is clearly seen that COP $\varepsilon$ and the maximum COP $\varepsilon_{\max}$ decrease with the increase of magnetic field $B$ in the case of thermal concurrence $C_1 > C_2$. Furthermore, COP $\varepsilon$ and the maximum COP $\varepsilon_{\max}$ increase with the increase of magnetic field $B$ in the case of $C_1 < C_2$. The magnetic field not only affects the shape of COP $\varepsilon$ but also affects its size. The loop isolines of COP $\varepsilon$ become more and obvious in the case of $C_1 > C_2$, and becomes less and obvious in the case of $C_1 < C_2$ when the magnetic field increases. We also find that the second law of thermodynamics holds in all cases. The results obtained here have general significance and will be helpful to understand the performance of an entangled quantum refrigerator.

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