A new identification control for generalized Julia sets*

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In this paper, we propose a new method to realize drive–response system synchronization control and parameter identification for a class of generalized Julia sets. By means of this method, the zero asymptotic sliding variables are applied to control the fractal identification. Furthermore, the problems of synchronization control are solved in the case of a drive system with unknown parameters, and the unknown parameters of the drive system can be identified in the asymptotic synchronization process. The results of simulation examples demonstrate the effectiveness of this new method. Particularly, the basic Julia set is also discussed.

Keywords: Julia set, synchronization, parameter identification

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1. Introduction

The fractal was discovered by Mandelbort.[1] Moreover, the fractal theory which describes fractal properties and corresponding applications was also proposed.[2] As a nonlinear forefront theory, the fractal theory has successfully explained a lot of nonlinear phenomena. At present, this theory has a variety of applications in particle diffusion, medicine cancer cell growth, image processing, cryptanalysis, and so on.[3–8] In recent years, researchers have paid much attention to the fractal sets of dynamical systems in fractal geometry. Singular attractors of dynamical systems belong to fractal sets that come from the analytic mapping iteration on the complex plane. The analytic mapping iteration divides the complex plane into two parts: the Fatou set and the Julia set. It has been found that the generalized Mandelbrot–Julia (M–J) sets which enjoy an important position in the fractal have a fine and complex structure. In 2004 and 2007, Wang *et al.* studied the application of the M–J sets in physics.[9,10] That is a typical Langevin problem, i.e., the analysis of the dynamics of a charged particle which is under the continuous influence of a constant impulse at one-dimensional discrete time points in a double-well potential and a time-dependent magnetic field. Based on the particle's dynamics characteristics, the physical meaning of the generalized M–J sets was discussed. Many mathematicians are interested in the properties, kinetic characteristics, and dimensions of the Julia set and its generalization form.[11–17] Importantly, people often have the requirements for determining the non-linear domain range and the collection parameters of the fractal collection, so it is very necessary and significant to study the synchronization control and parameter identification problems on nonlinear fractal sets. Nowadays, some significant achievements about fractal synchronous control have been reported, such as the nonlinear coupling methods and the gradient control laws, which are aiming to realize the region control of the stable domain and the synchronous control for complex system Julia sets.[18–21] It is pointed out that these methods are feasible only in the case of knowing the drive system parameters. However, the drive system parameters are usually unknown, so the drive–response system synchronization control is difficult to be solved by the former methods. In this paper, for the generalized Julia sets, we propose a new method to realize the drive–response system synchronization control and parameter identification. By applying the zero-asymptotical sliding variable of the discrete control system to the generalized Julia sets, the synchronous control of the drive–response system and the parameter identification of the drive system are achieved simultaneously under the conditions of the drive system with unknown parameters.

The paper is organized as follows. In Section 2, we design and theoretically prove a synchronization controller and a identifier for the generalized Julia sets. In Section 3, we similarly design a controller and a identifier for the basic Julia set. Finally, we draw the conclusion in Section 4.

2. Design of synchronization controller and parameter identifier for generalized Julia sets

2.1. Preparatory knowledge

The Julia set $J(f)$ is created by the iteration of a complex variable function f, which is defined to be the closure of the repelling periodic points of f. Using the Montel theorem, we can obtain the following equivalent definition about the Julia set of the complex polynomial f: $J(f) = J_0(f) = \{z \in \mathbf{C}$:

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the function family $\{f^k\}_{k\geq 0}$ is nonnormal at $z\}$. For the complex polynomial f, its Julia set has the following properties: (i) $J(f)$ is nonempty and bounded; (ii) $J(f)$ is fully invariant, i.e., $J(f) = f(J(f)) = f^{-1}(J(f))$; (iii) $J(f)$=$J(f^p)$ for any positive integer p; (iv) if ω is an attractive fixed point of f, then $\partial A(\omega) = J(f)$, where $A(\omega)$ is the attractive domain of the attractive fixed point ω. That is $A(\omega) = \{z \in \mathbf{C} : f^k(z) \to \omega, (k \to \infty)\}$, i.e., $\omega \to \infty$.

From the definition of the Julia set and its properties, we can control the trajectory of the iterative points and achieve the control of the Julia set. Then we consider two complex systems with the same structure

$$x_{n+1} = f(x_n, a_i, c), \tag{1}$$

$$y_{n+1} = f(y_n, a_i, c_n). \tag{2}$$

We add a controller to system (2) to correlate it with system (1), that is

$$y_{n+1} = f(y_n, a_i, c_n) + u_n(x_n, y_n, a_i), \tag{3}$$

where a_i $(i = 1, 2, \ldots)$ are given complex numbers, and c is an unknown complex parameter to be identified. Then, system (1) is a drive system, and system (3) is a response system. The controller u_n of system (3) achieves the asymptotic synchronization of drive system (1) and response system (3). If parameters a_i, c, c_n are given, their corresponding Julia sets are also identified, which are written as J^* and J_n, respectively. If

$$\lim_{n\to\infty}(J_n \cup J^* - J_n \cap J^*) = \emptyset\ , \tag{4}$$

systems (1) and (3) are synchronized.

In fact, we know that the orbit of Julia set $J(f)$ and f are closely related by the definition of the Julia set. The Julia sets synchronize as soon as their orbits synchronize based on this theory, therefore, we design the following controller. When $n \to \infty$, systems (1) and (3) gradually synchronize with the same initial iterative value, in the meantime, the unknown parameter c of system (3) is identified, that is $c_n \to c$.

2.2. Design of synchronous controller and parameter identifier

For a class of generalized Julia sets

$$x_{n+1} = a_1 x_n^q + a_2 x_n^p + a_3 x_n + c, \tag{5}$$

where a_1, a_2, a_3 are complex constants, $q > 1, p > 1$, we discuss the synchronization problem and the identification of parameter c for the Julia sets.

We can obtain the Julia set by the iterative calculation of the points in a bounded region using property (i). Therefore, suppose in the bounded region D, we only consider the iteration of the mid-point of D. Using property (ii), we only need to calculate the points whose tracks are all in D. Because if there is a n_0 to make $f^{n_0}(z) \notin D$, using property (ii), we know $z \notin J$.

System (5) is the drive system, and the other system with the same structure of the generalized complex Julia sets is the response system

$$y_{n+1} = a_1 y_n^q + a_2 y_n^p + a_3 y_n + c_n + u_n, \tag{6}$$

where u_n is the synchronous controller.

We assume that the drive and the response systems are uncertainty fractal sets, i.e., the parameter c is unknown. Then we design an appropriate synchronization controller for the drive system in the case of the response system with unknown parameters. Hence, we realize the completely synchronization of systems (5) and (6), and identify c at the same time.

We take the following error variables between the drive and the response systems:

$$e_1(n) = x_n - y_n, \tag{7}$$

$$e_2(n) = c_n - c. \tag{8}$$

Next, we design the synchronous controller

$$\begin{aligned} u_n \ =\ & e_1(n)\left(a_1 \sum_{i=0}^{q-1} x_n^{q-1-i} y_n^i + a_2 \sum_{i=0}^{p-1} x_n^{p-1-i} y_n^i + a_3 + k_1 - 1\right) \\ & + x_{n+1} - a_1 x_n^q - a_2 x_n^p - a_3 x_n - c_n, \end{aligned} \tag{9}$$

where constant k_1 satisfies $0 < k_1 < 2$. And we design the parameter identification law

$$c_{n+1} = (1 - k_2)c_n + k_2(x_{n+1} - a_1 x_n^q - a_2 x_n^p - a_3 x_n), \tag{10}$$

where constant k_2 satisfies $0 < k_2 < 2$.

Lemma 1[22] If $\sigma(n)$ is a sliding variable of a single-input and single-output discrete variable structure control system, the discrete control variable satisfies

$$\sigma(n)\Delta\sigma(n) < -\frac{1}{2}[\Delta\sigma(n)]^2, \quad \sigma(n) \neq 0, \tag{11}$$

where $\Delta\sigma(n) = \sigma(n+1) - \sigma(n)$, $|\sigma(n+1)| < |\sigma(n)|$, and $\sigma(n)$ tends to zero when $n \to \infty$.

Theorem 1 If the synchronous controller of system (6) is described by Eq. (9) and its parameter identifier is described by Eq. (10), drive system (5) and response system (6) with an arbitrary initial value can achieve a global asymptotic synchronization, and the parameters of the Julia sets of the generalized complex systems are identified.

Proof From the previous discussion, if systems (5) and (6) realize a track synchronization, then the Julia sets of systems (5) and (6) realize a global asymptotic synchronization with an arbitrary initial value. We only need to prove that systems (5) and (6) can realize a track synchronization, that is, $|x_n - y_n| \to 0$ $(n \to \infty)$. It is equivalent to $|e_1(n)| \to 0$ $(n \to \infty)$.

From Eqs. (5)–(7), we have

$$e_1(n+1) - e_1(n)$$

$$= a_1x_n^q + a_2x_n^p + a_3x_n + c - a_1y_n^q - a_2y_n^p - a_3y_n - c_n - u_n - x_n + y_n. \quad (12)$$

Substituting Eq. (9) into Eq. (12), we obtain

$$\begin{aligned} & e_1(n+1) - e_1(n) \\ &= a_1x_n^q + a_2x_n^p + a_3x_n + c - a_1y_n^q - a_2y_n^p - a_3y_n \\ &\quad - c_n - x_n + y_n - \Bigg[e_1(n)\Bigg(a_1\sum_{i=0}^{q-1} x_n^{q-1-i}y_n^i \\ &\quad + a_2\sum_{i=0}^{p-1} x_n^{p-1-i}y_n^i + a_3 + k_1 - 1\Bigg) \\ &\quad + x_{n+1} - a_1x_n^q - a_2x_n^p - a_3x_n - c_n\Bigg] \\ &= -k_1e_1(n). \end{aligned} \quad (13)$$

According to the sliding variable property, we take $e_1(n)$ as the sliding variable. If $0 < k_1 < 2$ is satisfied , Eq. (13) leads to

$$\begin{aligned} & e_1(n)[e_1(n+1) - e_1(n)] = -k_1[e_1(n)]^2 \\ &< -\frac{1}{2}k_1^2[e_1(n)]^2 = -\frac{1}{2}[e_1(n+1) - e_1(n)]^2, \end{aligned} \quad (14)$$

that is

$$e_1(n)[e_1(n+1) - e_1(n)] < -\frac{1}{2}[e_1(n+1) - e_1(n)]^2. \quad (15)$$

Hence, we have $|e_1(n+1)| < |e_1(n)|$ according to Lemma 1, which is equivalent to $|e_1(n)| \to 0$ for $n \to \infty$. So the Julia sets of systems (5) and (6) have achieved a global asymptotic synchronization with an arbitrary initial value.

Similarly, according to the sliding variable property and Eq. (10), if $e_2(n)$ is the sliding variable and $0 < k_2 < 2$, we get

$$\begin{aligned} c_{n+1} - c_n &= c_{n+1} - c - (c_n - c) \\ &= e_2(n+1) - e_2(n) = -k_2e_2(n). \end{aligned} \quad (16)$$

That is

$$c_{n+1} - c_n = -k_2e_2(n). \quad (17)$$

If $0 < k_2 < 2$ is satisfied , Eq. (17) leads to

$$\begin{aligned} & e_2(n)[e_2(n+1) - e_2(n)] = -k_2[e_2(n)]^2 \\ &< -\frac{1}{2}k_2^2[e_2(n)]^2 = -\frac{1}{2}[e_2(n+1) - e_2(n)]^2, \end{aligned} \quad (18)$$

that is

$$e_2(n)[e_2(n+1) - e_2(n)] < -\frac{1}{2}[e_2(n+1) - e_2(n)]^2. \quad (19)$$

According to Lemma 1, we obtain $|e_2(n+1)| < |e_2(n)|$, i.e., $|e_2(n)| \to 0$. It is ensured that c_n asymptotically tends to the actual value of c when $n \to \infty$.

Example 1 Let $q = 3$, $p = 2$, $c = -0.25 + 0.6\mathrm{i}$, $a_i = 1$ $(i = 1,2,3)$, $k_1 = 0.2$, $k_2 = 0.1$, $c_0 = \mathrm{i}$ for target system (5) and response system (6). After n steps, the Julia sets of drive system (5) and response system (6) are shown in Fig. 1.

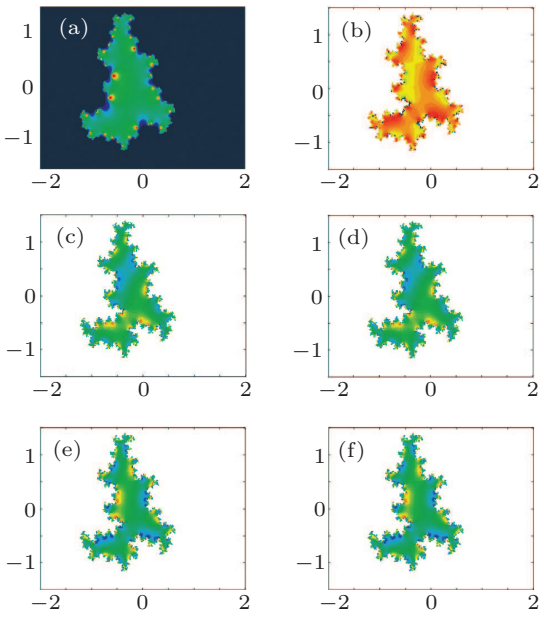


Fig. 1. (color online) The Julia sets of response system (6) after (a) 5 steps, (b) 20 steps, (c) 50 steps, (e) 100 steps; and the Julia sets of drive system (5) after (d) 50 steps, (f) 100 steps; where $k_1 = 0.2$ and $k_2 = 0.1$.

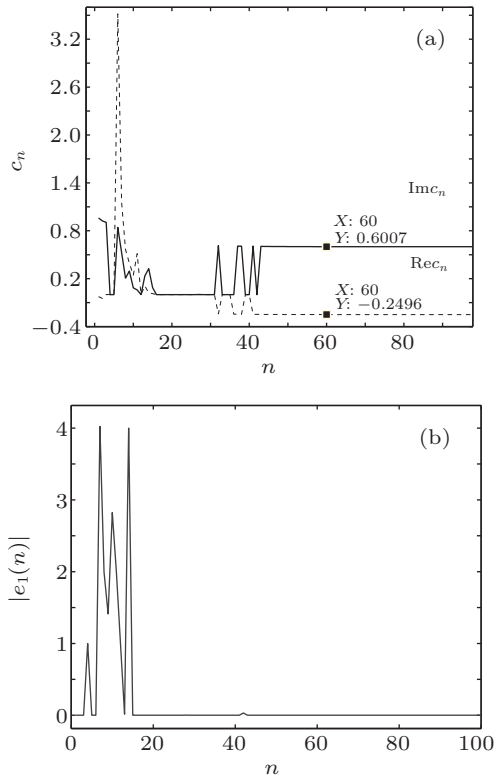


Fig. 2. The (a) c_n of system (6) and (b) $|e_1(n)|$ changing with n, where $k_1 = 0.2$ and $k_2 = 0.1$.

The figures show that response system (6) synchronizes with drive system (5) after 50 steps. Moreover, response system (6) keeps the synchronization with drive system (5) after 100 steps. The identification process of c_n is shown in Fig. 2(a). The $|e_1(n)|$ between drive system (5) and response system (6) changing with n is shown in Fig. 2(b).

From the simulation results, we find that in the iterative process, the real and the imaginary parts of c_n are stable

at -0.25 and 0.6, respectively, and the synchronization error modulus $|e_1(n)|$ tends to zero. Hence, response system (6) and drive system (5) achieve the synchronization, and unknown c of drive system (5) is identified.

As the values of k_1 and k_2 increase from 0 to 2, the changing of c_n and $|e_1(n)|$ is shown in Figs. 3 and 4.

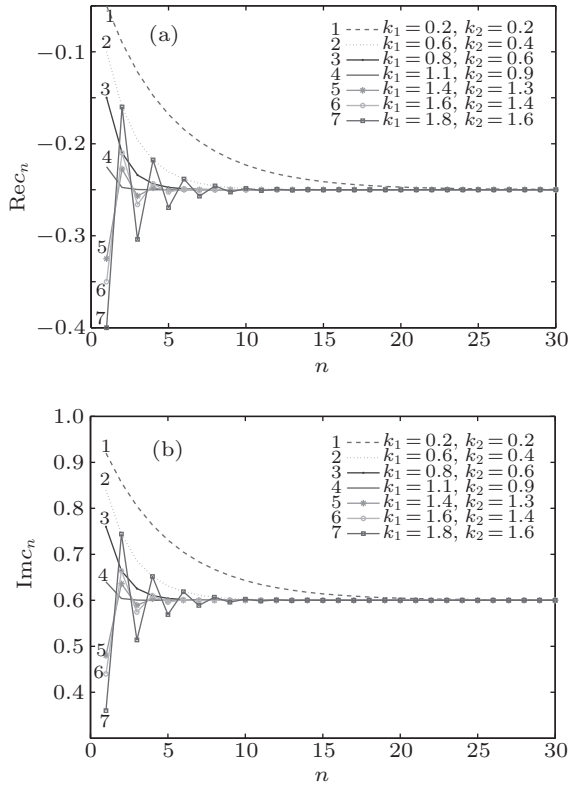


Fig. 3. Identification processes of (a) real and (b) imaginary parts of $|c_n|$ for different k_1 and k_2.

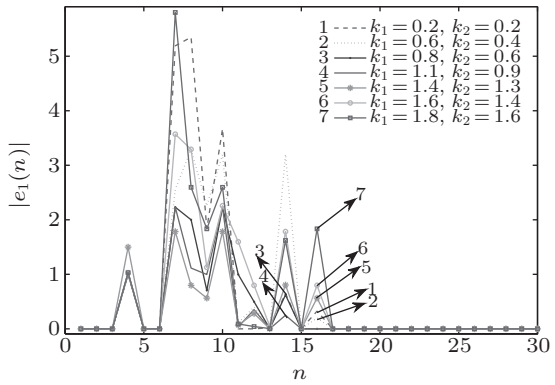


Fig. 4. The $|e_1(n)|$ between systems (5) and (6) for different k_1 and k_2.

From the results, we know that as the values of k_1 and k_2 increase from 0 to 1, the identification of c_n becomes easier and $|e_1(n)|$ tends to 0 more quickly. When the values of k_1 and k_2 continue to increase from 1 to 2, the identification of c_n is more difficult, and $|e_1(n)|$ tends to 0 more slowly. As a result, when the values of k_1 and k_2 are near 1, the identification effect of c_n is the best, and the speed of $|e_1(n)|$ tending to 0 is the fast. That is to say, the synchronization speed of the response system and the target system is the fastest.

For $k_1 = 2.5$, $k_2 = 2.5$, the identification process of c_n is shown in Fig. 5(a). The corresponding $|e_1(n)|$ between drive system (5) and response system (6) is shown in Fig. 5(b). From Fig. 5, we know that $|e_1(n)|$ between the two systems and $|c_n|$ of system (6) tend to infinite gradually. Therefore, the synchronization of the two systems is not possible, and unknown c of target system (5) cannot be identified.

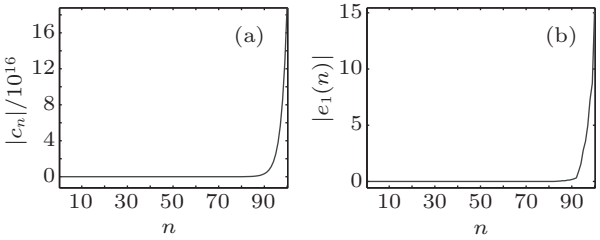


Fig. 5. The (a) $|c_n|$ of system (6) and (b) $|e_1(n)|$ for $k_1 = 2.5$, $k_2 = 2.5$.

3. Design of synchronization controller and parameter identifier for the basic Julia set

In this section, we discuss the synchronous controller and parameter identifier for the most basic Julia set $z_{n+1} = z_n^q + c$ $(q > 1)$. Therefore, let $a_1 = 1$, $a_2 = 0$, $a_3 = 0$ in target system (5) and response system (6), then systems (5) and (6) become

$$x_{n+1} = x_n^q + c, \tag{20}$$

$$y_{n+1} = y_n^q + c_n + u_n. \tag{21}$$

The errors between drive system (20) and response system (21) are

$$e_3(n) = x_n - y_n, \tag{22}$$

$$e_4(n) = c_n - c. \tag{23}$$

Hence, the design of the synchronous controller u_n is

$$u_n = e_3(n)\left(\sum_{i=0}^{q-1} x_n^{q-1-i} y_n^i + k_3 - 1\right) + x_{n+1} - x_n^q - c_n, \tag{24}$$

where constant k_3 satisfies $0 < k_3 < 2$. Moreover, the design of the parameter identifier is

$$c_{n+1} = (1 - k_4)c_n + k_4(x_{n+1} - x_n^q), \tag{25}$$

where constant k_4 satisfies $0 < k_4 < 2$.

Theorem 2 If the synchronous controller of system (21) is described by Eq. (24) and its parameter identifier is described by Eq. (25), drive system (20) and response system (21) with an arbitrary initial value can achieve a global asymptotic synchronization, and the parameters of the basic Julia set can be identified.

Proof If systems (20) and (21) realize a track synchronization with an arbitrary initial value, then the Julia sets of

systems (20) and (21) realize a global asymptotic synchronization. We only need to prove that systems (20) and (21) realize a track synchronization, that is, $|x_n - y_n| \to 0$ $(n \to \infty)$. It is equivalent to $|e_3(n)| \to 0$ $(n \to \infty)$.

From Eqs. (20)–(22), we obtain

$$e_3(n+1) - e_3(n) = x_n^q + c - y_n^q - c_n - u_n - x_n + y_n. \quad (26)$$

Next, substituting Eq. (24) into Eq. (26), we have

$$\begin{aligned} & e_3(n+1) - e_3(n) \\ &= x_n^q + c - y_n^q - c_n - x_n + y_n \\ &\quad - \left[e_3(n) \left(\sum_{i=0}^{q-1} x_n^{q-1-i} y_n^i + k_3 - 1 \right) + x_{n+1} - x_n^q - c_n \right] \\ &= -k_3 e_3(n). \end{aligned} \quad (27)$$

According to the sliding variable property, we have $e_3(n)$ as the sliding variable. If $0 < k_3 < 2$ is satisfied, equation (27) leads to

$$\begin{aligned} & e_3(n)[e_3(n+1) - e_3(n)] = -k_3[e_3(n)]^2 \\ & < -\frac{1}{2}k_3^2[e_3(n)]^2 = -\frac{1}{2}[e_3(n+1) - e_3(n)]^2, \end{aligned} \quad (28)$$

that is

$$e_3(n)[e_3(n+1) - e_3(n)] < -\frac{1}{2}[e_3(n+1) - e_3(n)]^2. \quad (29)$$

Hence, we have $|e_3(n+1)| < |e_3(n)|$ according to Lemma 1, which is equivalent to $|e_3(n)| \to 0$ when $n \to \infty$. So the Julia sets of systems (20) and (21) can achieve a global asymptotic synchronization with an arbitrary initial value.

Similarly, from the sliding variable properties, the error $e_4(n)$ is the sliding variable. If $0 < k_4 < 2$ is satisfied, Eq. (25) leads to

$$\begin{aligned} c_{n+1} - c_n &= c_{n+1} - c - (c_n - c) \\ &= e_4(n+1) - e_4(n) = -k_4 e_4(n). \end{aligned} \quad (30)$$

Next, from Eq. (30), we obtain

$$\begin{aligned} & e_4(n)[e_4(n+1) - e_4(n)] = -k_4[e_4(n)]^2 \\ & < -\frac{1}{2}k_4^2[e_4(n)]^2 = -\frac{1}{2}[e_4(n+1) - e_4(n)]^2. \end{aligned} \quad (31)$$

Obviously, that is

$$e_4(n)[e_4(n+1) - e_4(n)] < -\frac{1}{2}[e_4(n+1) - e_4(n)]^2. \quad (32)$$

According to Lemma 1, we obtain $|e_4(n+1)| < |e_4(n)|$, i.e., $|e_4(n)| \to 0$. It is ensured that c_n asymptotically tends to the actual value of c when $n \to \infty$.

Example 2 Let $q = 2$, $c = -0.8 + 0.15\mathrm{i}$, $k_3 = 0.2$, $k_4 = 0.1$, $c_0 = \mathrm{i}$ in drive system (20) and response system (21). After n steps, the Julia sets of drive system (20) and response system (21) are shown in Fig. 6.

The figures show that response system (21) synchronizes with drive system (20) after 50 steps. Moreover, the response system (21) keeps the synchronization with drive system (20) after 90 steps. The identification process of c_n of response system (21) is shown in Fig. 7(a). The changing of $|e_3(n)|$ between drive system (20) and response system (21) is shown in Fig. 7(b).

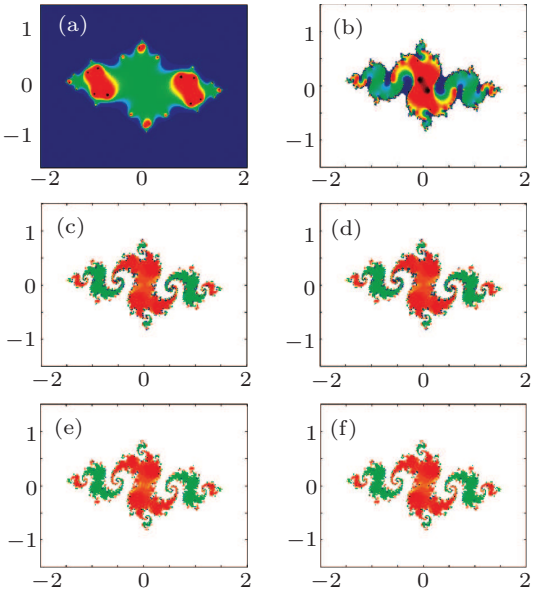


Fig. 6. (color online) The Julia sets of response system (21) after (a) 5 steps, (b) 20 steps, (c) 50 steps, (e) 90 steps; and the Julia sets of drive system (20) after (d) 50 steps, (f) 90 steps; where $k_3 = 0.2$ and $k_4 = 0.1$.

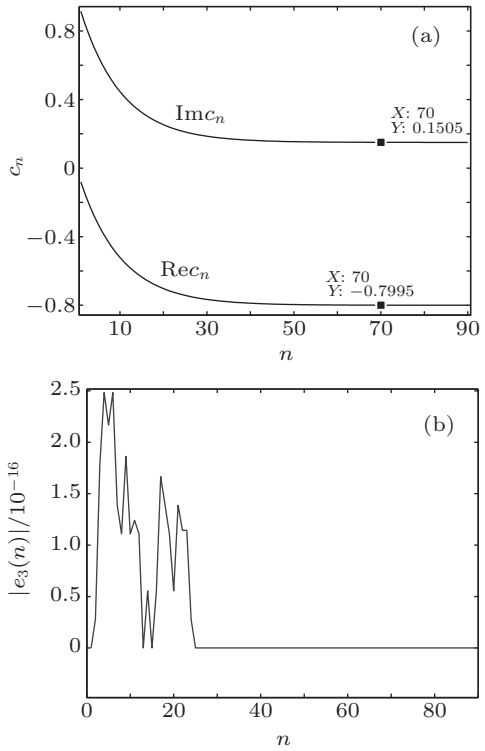


Fig. 7. The (a) c_n of system (21) and (b) $|e_3(n)|$ changing with n, where $k_3 = 0.2$ and $k_4 = 0.1$.

From the simulation results, we find that the real and the imaginary parts of c_n are stable at -0.8 and 0.15 respectively

in the iterative process and the synchronization error modulus $|e_3(n)|$ tends to zero. Hence, response system (21) and drive system (20) achieve the synchronization, and unknown c of drive system (20) is identified.

As the values of k_3 and k_4 increase from 0 to 2, the changings of c_n and $|e_3(n)|$ are shown in Figs. 8 and 9. From the results, we know that as the values of k_3 and k_4 increase from 0 to 1, the identification of c_n becomes easier and $|e_3(n)|$ tends to 0 more quickly. When the values of k_3 and k_4 continue to increase from 1 to 2, the identification of c_n becomes difficult, and $|e_3(n)|$ tends to 0 more slowly. As a result, when the values of k_3 and k_4 are near 1, the identification effect of c_n is the best, and the speed of $|e_3(n)|$ tending to 0 is the fastest, so the synchronization speed of the response system and the target system is the fastest.

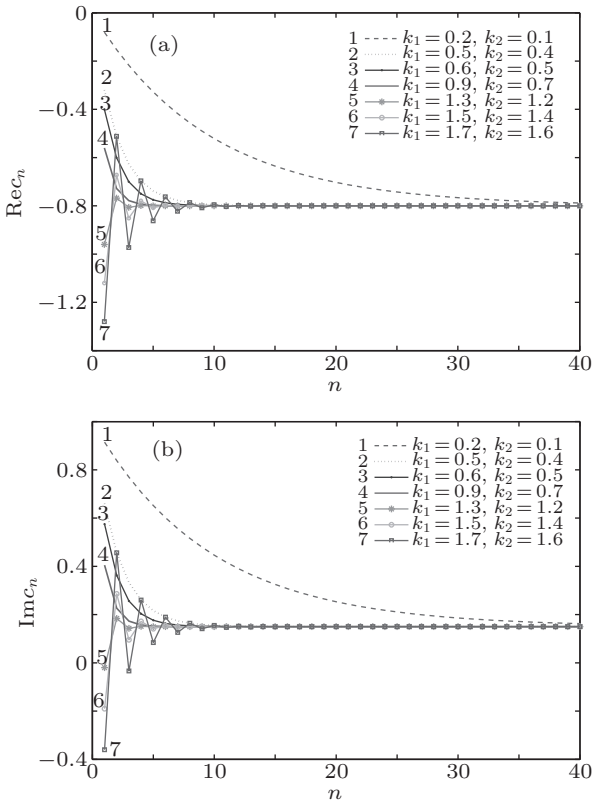


Fig. 8. Identification of (a) real and (c) imaginary parts of c_n for different k_3 and k_4.

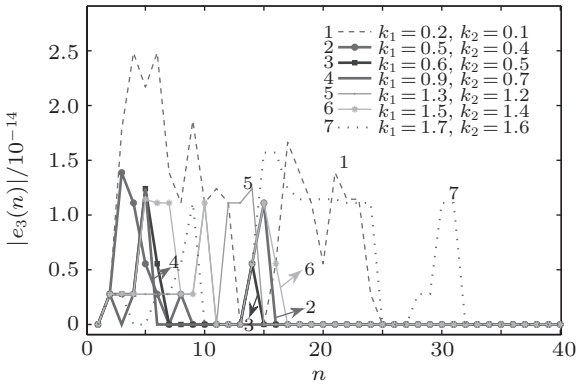


Fig. 9. The $|e_3(n)|$ for different k_3 and k_4.

When $k_3 = 2.5$ and $k_4 = 3$, the identification process of $|c_n|$ is shown in Fig. 10(a). The $|e_3(n)|$ between drive system (20) and response system (21) is shown in Fig. 10(b). From Fig. 10, we know that $|e_3(n)|$ between the two systems and $|c_n|$ of system (21) tend to infinity gradually. Therefore the synchronization of the two systems cannot be achieved, and unknown c of drive system (20) cannot be identified.

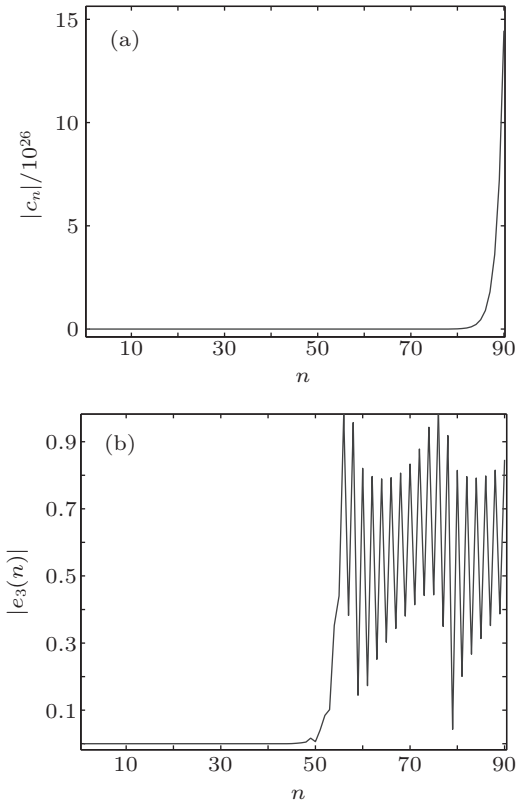


Fig. 10. The (a) $|c_n|$ of system (21) and (b) $|e_3(n)|$ for $k_3 = 2.5$ and $k_4 = 3$.

4. Conclusion

In conclusion, a new method is put forward to realize the synchronization control of the drive–response system and the parameter identification of the generalized Julia sets. The zero asymptotic property of the sliding variable of the discrete control system is applied to realize the identification control of the generalized Julia sets. With this method, we successfully solve the problem of synchronization control of the drive–response system and the parameter identification in the case of the drive system with unknown parameters. Meanwhile, we design an adaptive synchronization controller and a parameter identifier. In addition, it is found that this new method is effective for the parameter identification and the synchronization control of the basic Julia sets, which is significant for future important applications of the Julia sets.

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