Exponential synchronization of coupled memristive neural networks via pinning control*

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This paper is concerned with the exponential synchronization problem of coupled memristive neural networks. In contrast to general neural networks, memristive neural networks exhibit state-dependent switching behaviors due to the physical properties of memristors. Under a mild topology condition, it is proved that a small fraction of controlled subsystems can efficiently synchronize the coupled systems. The pinned subsystems are identified via a search algorithm. Moreover, the information exchange network needs not to be undirected or strongly connected. Finally, two numerical simulations are performed to verify the usefulness and effectiveness of our results.

Keywords: synchronization, memristor, coupled neural networks, pinning control

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1. Introduction

Since the memristor (short for memory resistor) was firstly postulated by Chua,[1] and then realized in Hewlett Packard laboratories,[2] considerable efforts[3–15] have been made to explore the nonlinear circuit element. Chua[1] pointed out that the memristor, with switchable memristance (electric resistance of memristor), provided a relation between electric charge and magnetic flux. Similar to the switching behaviors in dynamical systems caused by environmental disturbances, irregular changes of memristance result from abrupt changes of external voltage.[3] In addition, a memristor possesses a nonvolatile memory property. When the voltage applied on a memristor is turned off, the memristor retains its last active resistance. Combining the storage capability of a memristor[4] and the memory pattern of a human brain realized by adjusting the strength of synaptic connections between neurons,[16] the memristive neural network (MNN) was firstly proposed by Hu and Wang[5] to emulate a human brain. Based on the construction of cell neural networks, Wen *et al.*[8,9] introduced the circuit model of MNNs and derived corresponding state-dependent switching rules. Recently, Fang *et al.*[10] presented the SPICE model for multi-level memristors. By using non-smooth analysis technology,[17,18] the existing results mainly focused on the dynamical behaviors of MNNs.[5–9,11–13]

In addition, the collective behaviors of coupled neural networks have gained increasing research interest.[19–23] It was pointed out in Refs. [19] and [23] that an array of interacted neural networks could achieve higher-level information processing and exhibit more complicated and unpredictable dynamics. As a typical collective behavior in nature, synchronization can be employed to obtain some spontaneous properties,[24] and understand uncertain systems from the deterministic ones.[25] Therefore, studying the synchronization performances of coupled neural networks has both theoretical and practical significance.

For the reason that memristors cannot be fabricated with arbitrary size in engineering, a network composed of interconnected nanoscale memristors was established to achieve huge memory capacity.[15] The authors in Refs. [16] and [26] argued that the connections of neurons have a great influence on the memory capacity in neural networks. In general, the behaviors of the whole system can be determined by the way in which the parts receive and transmit information.[24] In addition, an external force is necessary under the condition of a prescribed objective. In view of the impracticality of imposing control signals on all nodes, pinning control[22,27–33] was proposed for general networks. In Refs. [22] and [28], as few controllers were adopted to achieve the desired synchronization for different network structures. Afterwards, a uniform pinning control criterion was derived for both directed and undirected networks.[30]

However, there is little work concerning the collective behaviors of coupled memristive neural networks (CMNNs), especially for directed and unconnected cases. Motivated by the previous studies mentioned above, we study the pinning synchronization problem of a memristive network, which has promising applications in massive data storage and retrieval.[4] With the aid of the set-valued map theory and the differential inclusion theory,[17,18] we aim to develop novel criteria that guarantee the exponential synchronization of CMNNs, in which only a small fraction of nodes are accessible from control signals. Moreover, the pinned nodes are identified via a

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search algorithm.

This paper is organized as follows. In Section 2, notations and model are introduced. Sufficient conditions that ensure the exponential synchronization of CMNNs are presented in Section 3. In Section 4, two numerical examples are given. Conclusion is made in Section 5.

2. Notations and model

Throughout this paper, $\mathbb{R}^n$ and $\boldsymbol{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of $m\times n$ real matrices, respectively. For real symmetric matrix $\boldsymbol{P}$, $\boldsymbol{P}<0$ ($\boldsymbol{P}>0$) means $\boldsymbol{P}$ is negative definite (positive definite). For vector $\boldsymbol{x}\in\mathbb{R}^n$, $\|\boldsymbol{x}\|=\sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$, $|\boldsymbol{x}|=(|x_1|,|x_2|,\ldots,|x_n|)^{\mathrm{T}}$. The $\mathscr{C}([-\tau,0],\mathbb{R}^n)$ denotes the family of continuous functions φ from $[-\tau,0]$ to $\mathbb{R}^n$ with norm $\|\varphi\|=\sup_{-\tau\le\xi\le0}|\varphi(\xi)|$. Let $\lambda_{\max}(\boldsymbol{P})$ and $\lambda_{\min}(\boldsymbol{P})$ be, respectively, the maximum and minimum eigenvalue of $\boldsymbol{P}$. The $\mathrm{diag}(p_1,p_2,\ldots,p_n)$ denotes a diagonal matrix. The $\mathrm{co}\{a,b\}$ stands for the closure of a convex hull generated by real numbers a and b. For matrices $\boldsymbol{X}=(x_{ml})_{n\times n}$, $\boldsymbol{Y}=(y_{ml})_{n\times n}$, and $\boldsymbol{Z}=(z_{ml})_{n\times n}$, $\boldsymbol{Z}\in\mathrm{co}\{\boldsymbol{X},\boldsymbol{Y}\}$ indicates that $\min\{x_{ml},y_{ml}\}\le z_{ml}\le\max\{x_{ml},y_{ml}\}$ holds for $m,l=1,2,\ldots,n$.

In this paper, solutions of all systems discussed below are intended in the sense of Filippov.[17] The definitions and rules of Filippov solutions can be consulted in Refs. [17], [18], and [25]. Based on the physical characteristics of a memristor and Refs. [5], [8], [9], and [11]–[13], MNNs are described by

$$\begin{aligned}\dot{y}_m(t) &= -c_m y_m(t)+\sum_{l=1}^{n}a_{ml}(\boldsymbol{y}(t))f_l(y_l(t))\\ &\quad+\sum_{l=1}^{n}b_{ml}(\boldsymbol{y}(t))g_l(y_l(t-\tau(t)))+I_m(t),\\ &\quad(m=1,2,\ldots,n),\end{aligned}\tag{1}$$

where $\boldsymbol{y}(t)=(y_1(t),y_2(t),\ldots,y_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ and $\boldsymbol{I}(t)=(I_1(t),I_2(t),\ldots,I_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ denote the voltage applied on capacitor and external input, respectively. The parameters $A(\boldsymbol{y}(t))=(a_{ml}(\boldsymbol{y}(t)))_{n\times n}$ and $B(\boldsymbol{y}(t))=(b_{ml}(\boldsymbol{y}(t)))_{n\times n}$ are memristive connection weights.[8,9] The self-feedback connection matrix is $\boldsymbol{C}=\mathrm{diag}(c_1,c_2,\ldots,c_n)>0$. Continuously differentiable function $\tau(t)$ is bounded, such that $0\le\tau(t)\le\tau$, $\dot{\tau}(t)\le u$, where τ and u are known constants. $\boldsymbol{f}(\boldsymbol{y}(t))=(f_1(y_1(t)),\ldots,f_n(y_n(t)))^{\mathrm{T}}\in\mathbb{R}^n$ and $\boldsymbol{g}(\boldsymbol{y}(t-\tau(t)))=(g_1(y_1(t-\tau(t))),\ldots,g_n(y_n(t-\tau(t))))^{\mathrm{T}}\in\mathbb{R}^n$ are, respectively, the bounded feedback functions without and with time delay between memristive connection weights and $\boldsymbol{y}(t)$. The initial state is given by $\boldsymbol{y}(t)=\varphi(t)\in\mathscr{C}([-\tau,0],\mathbb{R}^n)$.

Figure 1 shows the structure of a memristor,[2] whose resistance is determined by the length of the doped region $w(t)$ $(0\le w(t)\le D)$ in this device. The typical current–voltage characteristics[3] of memristor presented in Fig. 2(a) are caused by the nonlinearity of the memristor and external periodic voltage. Figure 2(b) shows the chaotic attractor of a memristor circuit.[3] The properties of chaotic systems have been efficiently applied to engineering, such as secure communication,[34,35] and image encryption.[36,37] The memristor circuit is obtained by replacing Chua's diode in the canonical Chua's oscillator[3] with a flux-controlled memristor. The $i_3(t)$ is the current through the inductance L, and $v_1(t)$ and $v_2(t)$ are the voltage of the capacity C_1 and C_2, respectively.

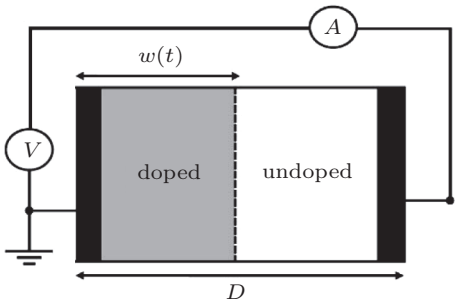


Fig. 1. Structure of a memristor reported by Hewlett Packard researchers.

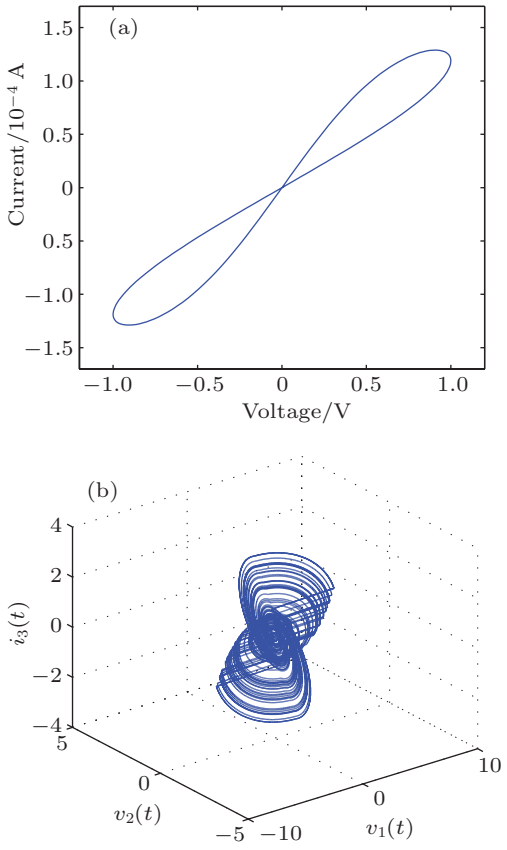


Fig. 2. (color online) (a) Typical current–voltage characteristics of a memristor; (b) chaotic attractor of a flux-controlled memristor circuit.

Based on the property of a memristor,[3] we know that memristance switches within a range according to the applied

voltage. Similar to Refs. [5], [8], and [9], we let

$$a_{ml}(\boldsymbol{y}(t)) = \begin{cases} a'_{ml}, & \varpi(\dot{y}(t)) \le 0, \\ a''_{ml}, & \varpi(\dot{y}(t)) > 0, \end{cases}$$
$$b_{ml}(\boldsymbol{y}(t)) = \begin{cases} b'_{ml}, & \vartheta(\dot{y}(t)) \le 0, \\ b''_{ml}, & \vartheta(\dot{y}(t)) > 0, \end{cases} \quad (2)$$

where $\varpi(\dot{y}(t))$ and $\vartheta(\dot{y}(t))$ obey memristive switching rules,[8,9] and a'_{ml}, a''_{ml}, b'_{ml}, b''_{ml} are known constants with respect to memristances.

For simplification, we assume the following representation: $\overline{a}_{ml} = \max\{a'_{ml}, a''_{ml}\}$, $\underline{a}_{ml} = \min\{a'_{ml}, a''_{ml}\}$, $\overline{\boldsymbol{A}} = (\overline{a}_{ml})_{n\times n}$, $\underline{\boldsymbol{A}} = (a_{ml})_{n\times n}$, $\overline{b}_{ml} = \max\{b'_{ml}, b''_{ml}\}$, $\underline{b}_{ml} = \min\{b'_{ml}, b''_{ml}\}$, $\overline{\boldsymbol{B}} = (\overline{b}_{ml})_{n\times n}$, $\underline{\boldsymbol{B}} = (b_{ml})_{n\times n}$, $\tilde{a} = \max_{m,l}\{|a'_{ml}|, |a''_{ml}|\}$, and $\tilde{b} = \max_{m,l}\{|b'_{ml}|, |b''_{ml}|\}$.

The linear combination of N MNNs (1) produces CMNNs. Thus, the dynamical behaviors of each MNN are determined by both its own property and the effects of its neighbors. The CMNNs can be well described by a general network, in which each MNN is considered as a node, and the i-th MNN can influence the j-th MNN, if there exists a directed path from node i to node j. Each path is a sequence of order links of the form $(i,i_1),\ldots,(i_{j-1},j)$. The controlled CMNNs is given by

$$\begin{aligned} \dot{\boldsymbol{x}}_i(t) = & -\boldsymbol{C}\boldsymbol{x}_i(t) + \boldsymbol{A}(\boldsymbol{x}_i(t))\boldsymbol{f}(\boldsymbol{x}_i(t)) \\ & + \boldsymbol{B}(\boldsymbol{x}_i(t))\boldsymbol{g}(\boldsymbol{x}_i(t-\tau(t))) + \boldsymbol{I}(t) \\ & + c\sum_{j=1}^{N} w_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t) + \boldsymbol{u}_i(t), \quad (i = 1,2,\ldots,N), \end{aligned} \quad (3)$$

where $\boldsymbol{x}_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ and $\boldsymbol{u}_i(t) = (u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ are, respectively, the state variable and control input of the i-th node. Constant matrix $\boldsymbol{W} = (w_{ij})_{N\times N}$ represents the coupling structure of CMNNs, and the elements w_{ij} are defined as follows: if there is a directed link from node j to node i, then $w_{ij} = 1$; otherwise, $w_{ij} = 0$ $(i \ne j)$. Diffusive coupling condition $w_{ii} = -\sum_{j=1, j\ne i}^{N} w_{ij}$ is satisfied for $i = 1,2,\ldots,N$. Positive diagonal matrix $\boldsymbol{\Gamma} = \mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$ stands for the inner coupling between two connected MNNs. Positive constant c is the coupling strength of CMNNs. The $\boldsymbol{A}(x_i(t))$ and $B(x_i(t))$ are defined by Eq. (2). The initial values of CMNNs are given by $\boldsymbol{x}_i(t) = \boldsymbol{\varphi}_i(t) \in \mathscr{C}([-\tau, 0], \mathbb{R}^n)$.

Remark 1 $\boldsymbol{A}(\boldsymbol{x}_i(t))$ and $\boldsymbol{B}(\boldsymbol{x}_i(t))$ switch according to the state of each subsystem, and hence the controlled CMNNs (3) is a state-dependent switching system. If they are constants, system (3) is converted into the coupled deterministic neural networks investigated in Refs. [19]–[21], and if they obey Markovian switching rules, system (3) is transformed into the coupled stochastic neural networks studied in Ref. [23].

To study the pinning synchronization of CMNNs (3), we employ the following assumptions.

Assumption 1 Suppose there exist positive constants $F_1, F_2, \ldots, F_n$, and $G_1, G_2, \ldots, G_n$, such that $\|\boldsymbol{f}_m(\boldsymbol{x}_m(t)) - \boldsymbol{f}_m(\boldsymbol{y}_m(t))\| \le F_m\|\boldsymbol{x}_m(t) - \boldsymbol{y}_m(t)\|$, $\|\boldsymbol{g}_m(\boldsymbol{x}_m(t)) - \boldsymbol{g}_m(\boldsymbol{y}_m(t))\| \le G_m\|\boldsymbol{x}_m(t) - \boldsymbol{y}_m(t)\|$ hold for $\boldsymbol{x}_m(t), \boldsymbol{y}_m(t), \boldsymbol{f}_m(\boldsymbol{x}_m(t)), \boldsymbol{f}_m(\boldsymbol{y}_m(t)) \in \mathbb{R}$ $(m = 1,2,\ldots,n)$.

Assumption 2 For $\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{f}(\boldsymbol{x}(t)), f(\boldsymbol{y}(t)), g(\boldsymbol{x}(t)), \boldsymbol{g}(\boldsymbol{y}(t)) \in \mathbb{R}^n$, the following statements hold:

$$\begin{aligned} & \mathrm{co}\{\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}\}\boldsymbol{f}(\boldsymbol{x}(t)) - \mathrm{co}\{\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}\}f(\boldsymbol{y}(t)) \\ \subseteq\ & \mathrm{co}\{\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}\}[\boldsymbol{f}(\boldsymbol{x}(t)) - f(\boldsymbol{y}(t))], \\ & \mathrm{co}\{\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}\}g(\boldsymbol{x}(t-\tau(t))) - \mathrm{co}\{\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}\}\boldsymbol{g}(\boldsymbol{y}(t-\tau(t))) \\ \subseteq\ & \mathrm{co}\{\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}\}[g(\boldsymbol{x}(t-\tau(t))) - \boldsymbol{g}(\boldsymbol{y}(t-\tau(t)))]. \end{aligned}$$

Assumption 2 is satisfied if the following inequalities hold for $m = 1,2,\ldots,n$:

$$\begin{aligned} & \min\left\{\sum_{l=1}^{n} a_{ml}[\boldsymbol{f}_l(\boldsymbol{x}_l(t)) - \boldsymbol{f}_l(\boldsymbol{y}_l(t))], \right. \\ & \left. \sum_{l=1}^{n} \overline{a}_{ml}[\boldsymbol{f}_l(\boldsymbol{x}_l(t)) - \boldsymbol{f}_l(\boldsymbol{y}_l(t))]\right\} \\ \le\ & \min\left\{\sum_{l=1}^{n} a_{ml}(\boldsymbol{f}_l(\boldsymbol{x}_l(t))), \sum_{l=1}^{n} \overline{a}_{ml}(\boldsymbol{f}_l(\boldsymbol{x}_l(t)))\right\} \\ & - \max\left\{\sum_{l=1}^{n} a_{ml}(\boldsymbol{f}_l(\boldsymbol{y}_l(t))), \sum_{l=1}^{n} \overline{a}_{ml}(\boldsymbol{f}_l(\boldsymbol{y}_l(t)))\right\}; \\ & \max\left\{\sum_{l=1}^{n} a_{ml}(\boldsymbol{f}_l(\boldsymbol{x}_l(t))), \sum_{l=1}^{n} \overline{a}_{ml}(\boldsymbol{f}_l(\boldsymbol{x}_l(t)))\right\} \\ & - \min\left\{\sum_{l=1}^{n} a_{ml}(\boldsymbol{f}_l(\boldsymbol{y}_l(t))), \sum_{l=1}^{n} \overline{a}_{ml}(\boldsymbol{f}_l(\boldsymbol{y}_l(t)))\right\} \\ \le\ & \max\left\{\sum_{l=1}^{n} a_{ml}[\boldsymbol{f}_l(\boldsymbol{x}_l(t)) - \boldsymbol{f}_l(\boldsymbol{y}_l(t))], \right. \\ & \left. \sum_{l=1}^{n} \overline{a}_{ml}[\boldsymbol{f}_l(\boldsymbol{x}_l(t)) - \boldsymbol{f}_l(\boldsymbol{y}_l(t))]\right\}. \end{aligned}$$

Assumption 3 In this network, a small fraction of nodes have paths to most of the others.

Remark 2 Assumption 3 states that most nodes in the network can be influenced by few nodes directly or indirectly. If all nodes can receive information from one node, this one is the root node of a spanning tree. However, disjoint subgroups and isolated nodes may exist in networks. For instance, it is common that many biological and social networks are spontaneously divided into some subgroups due to specified applications in engineering.[31] In each subgroup, there exist a large number of internal links relative to external ones.[38] Subsequently, the root nodes in each subgroup are called the local root nodes of the whole network. This kind of network is exactly what we are concerned with in this paper.

Definition 1[18] Suppose $\mathscr{E} \subseteq \mathbb{R}^n$. Then, $x \multimap \mathscr{F}(x)$ is called a set-valued map defined on $\mathscr{E}$, if there is a nonempty set $\mathscr{F}(x) \subseteq \mathbb{R}^n$ for each point $x \in \mathscr{E}$. If for any open set $\mathscr{N}$ containing $\mathscr{F}(x_0)$, there exists a neighborhood $\mathscr{M}$ of x_0 such that

$\mathscr{F}(\mathscr{M}) \subset \mathscr{N}$, then a set-valued map $\mathscr{F}$ with non-empty values is said to be upper-semi-continuous at $x_0 \in \mathscr{E}$. The $\mathscr{F}(x)$ is said to have a closed (convex, compact) image, if $\mathscr{F}(x)$ is closed (convex, compact) for each $x \in \mathscr{E}$.

Based on Definition 1, set-valued map

$$\begin{aligned}&\boldsymbol{x}_i(t) \multimap -\boldsymbol{C}\boldsymbol{x}_i(t)+\mathrm{co}\{\underline{\boldsymbol{A}},\overline{\boldsymbol{A}}\}\boldsymbol{f}(\boldsymbol{x}_i(t))\\&+\mathrm{co}\{\underline{\boldsymbol{B}},\overline{\boldsymbol{B}}\}\boldsymbol{g}(\boldsymbol{x}_i(t-\tau(t)))\\&+\boldsymbol{I}(t)+c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)+\boldsymbol{u}_i(t)\end{aligned}$$

is upper-semi-continuous with non-empty compact convex values. It follows from Assumption 1 and generalized Halanay inequalities[25] that the solutions to Eq. (3) are bounded. Under the framework of Filippov, these solutions with any initial values are absolutely continuous in compact interval $[0,+\infty)$ and differentiable almost everywhere. Before further deriving our main results, the following lemmas are required.

Lemma 1[31] For matrix $\boldsymbol{B}=(b_{ml})_{p\times q}$, assume $\alpha(\boldsymbol{B})=\frac{1}{2}\max\{\mathrm{p},\mathrm{q}\}\max_{m,l}\{|b_{ml}|\}$, then $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{y}\le\alpha(\boldsymbol{B})(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y})$ holds for all $\boldsymbol{x}\in\mathbb{R}^p$, $\boldsymbol{y}\in\mathbb{R}^q$.

Lemma 2[39] For any constant positive definite matrix $\boldsymbol{P}\in\boldsymbol{R}^{n\times n}$, scalar $0<\gamma(t)<\gamma$, vector function $x:[0,\gamma]\to\mathbb{R}^n$, such that the integrations concerned are well defined, then $(\int_0^{\tau(t)}\boldsymbol{x}(\xi)\mathrm{d}\xi)^{\mathrm{T}}\boldsymbol{P}(\int_0^{\tau(t)}\boldsymbol{x}(\xi)\mathrm{d}\xi)\le\tau(t)\int_0^{\tau(t)}\boldsymbol{x}^{\mathrm{T}}(\xi)\boldsymbol{P}\boldsymbol{x}(\xi)\mathrm{d}\xi$.

3. Main results

In this section, the pinning control method is adopted to synchronize the controlled CMNNs (3) to a given trajectory (1). Based on the theory of set-valued maps and differential inclusions,[17,18] it follows from Eqs. (1) and (3) that

$$\begin{aligned}\dot{\boldsymbol{y}}(t)\ \in\ &-\boldsymbol{C}\boldsymbol{y}(t)+\mathrm{co}\{\underline{\boldsymbol{A}},\overline{\boldsymbol{A}}\}f(\boldsymbol{y}(t))\\&+\mathrm{co}\{\underline{\boldsymbol{B}},\overline{\boldsymbol{B}}\}\boldsymbol{g}(\boldsymbol{y}(t-\tau(t)))+I(t),\qquad(4)\\\dot{\boldsymbol{x}}_i(t)\ \in\ &-\boldsymbol{C}\boldsymbol{x}_i(t)+\mathrm{co}\{\underline{\boldsymbol{A}},\overline{\boldsymbol{A}}\}\boldsymbol{f}(\boldsymbol{x}_i(t))\\&+\mathrm{co}\{\underline{\boldsymbol{B}},\overline{\boldsymbol{B}}\}\boldsymbol{g}(\boldsymbol{x}_i(t-\tau(t)))+\boldsymbol{I}(t)\\&+c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}\boldsymbol{x}_j(t)+\boldsymbol{u}_i(t)\qquad(5)\end{aligned}$$

hold for $t\in[0,+\infty)$.

Defining synchronization error $\boldsymbol{e}_i(t)=\boldsymbol{x}_i(t)-\boldsymbol{y}(t)$ $(i=1,2,\ldots,N)$ and subtracting Eq. (4) from Eq. (5), we obtain the following system:

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t)\ \in\ &-\boldsymbol{C}\boldsymbol{e}_i(t)+\mathrm{co}\{\underline{\boldsymbol{A}},\overline{\boldsymbol{A}}\}\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t))\\&+\mathrm{co}\{\underline{\boldsymbol{B}},\overline{\boldsymbol{B}}\}\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t)))\\&+c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t)+\boldsymbol{u}_i(t),\qquad(6)\end{aligned}$$

where

$$\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t))=\boldsymbol{f}(\boldsymbol{x}_i(t))-f(\boldsymbol{y}(t)),$$

$$\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t)))=\boldsymbol{g}(\boldsymbol{x}_i(t-\tau(t)))-\boldsymbol{g}(\boldsymbol{y}(t-\tau(t))).$$

Or equivalently, there exist $\boldsymbol{A}_i\in\mathrm{co}\{\underline{\boldsymbol{A}},\overline{\boldsymbol{A}}\}$, $B_i\in\mathrm{co}\{\underline{\boldsymbol{B}},\overline{\boldsymbol{B}}\}$, such that

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t)\ =\ &-\boldsymbol{C}\boldsymbol{e}_i(t)+\boldsymbol{A}_i\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t))+\boldsymbol{B}_i\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t)))\\&+c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t)+\boldsymbol{u}_i(t),\qquad(7)\end{aligned}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ depend on the initial values of the i-th subsystem and time t.

Actually, there exist arbitrary constants $0\le\zeta_i\le1$ and $0\le\iota_i\le1$, such that $\boldsymbol{A}_i=\zeta_i\underline{\boldsymbol{A}}+(1-\zeta_i)\overline{\boldsymbol{A}}$, and $\boldsymbol{B}_i=\iota_i\underline{\boldsymbol{B}}+(1-\iota_i)\overline{\boldsymbol{B}}$.

In order to achieve the expected synchronization, control inputs are imposed on partial nodes and the pinned nodes should be identified prior to synchronization analysis. From Assumption 3, we know that the global information exchange benefits from the local root nodes. Therefore, under certain topology condition, the whole network can be steered to desired states if all these local root nodes are identified and controlled.

In this work, the set of network nodes $\mathscr{V}$ is divided into two disjoint subsets: the set of pinned nodes $\tilde{\mathscr{V}}$ and the set of uncontrolled nodes $\mathscr{V}\setminus\tilde{\mathscr{V}}$. Thus, the external controllers are given by

$$\boldsymbol{u}_i(t)=\begin{cases}-cd_i\boldsymbol{\Gamma}\boldsymbol{e}_i(t), & i\in\tilde{\mathscr{V}},\\0, & i\in\mathscr{V}\setminus\tilde{\mathscr{V}},\end{cases}\qquad(8)$$

If $i\in\tilde{\mathscr{V}}$, then $d_i>0$; otherwise, $d_i=0$. In addition, we define that $\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\ldots,d_N)$.

The synchronization error system (7), together with control strategy (8), yields the error system as follows:

$$\begin{cases}\dot{\boldsymbol{e}}_i(t)=\boldsymbol{z}_i(t),\\\boldsymbol{z}_i(t)=-\boldsymbol{C}\boldsymbol{e}_i(t)+\boldsymbol{A}_i\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t))+\boldsymbol{B}_i\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t)))\\\qquad+c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}\boldsymbol{e}_j(t)-cd_i\boldsymbol{\Gamma}\boldsymbol{e}_i(t).\end{cases}\qquad(9)$$

According to Lemma 1, relations of $\alpha(\boldsymbol{A}_i)=\frac{1}{2}n\tilde{a}$ and $\alpha(\boldsymbol{B}_i)=\frac{1}{2}n\tilde{b}$ hold for $i=1,2,\ldots,N$ and $t\in[0,\infty)$. Denote $\tilde{\boldsymbol{e}}_k(t)=(e_{1k}(t),e_{2k}(t)\ldots,e_{Nk}(t))^{\mathrm{T}}\in\mathbb{R}^N$, $\tilde{\boldsymbol{e}}_k(t-\tau(t))=(e_{1k}(t-\tau(t)),e_{2k}(t-\tau(t)),\ldots,e_{Nk}(t-\tau(t)))^{\mathrm{T}}\in\mathbb{R}^N$, $\tilde{\boldsymbol{e}}_k(t-\tau)=(e_{1k}(t-\tau),e_{2k}(t-\tau),\ldots,e_{Nk}(t-\tau))^{\mathrm{T}}\in\mathbb{R}^N$, $\tilde{\boldsymbol{z}}_k(t)=(z_{1k}(t),z_{2k}(t),\ldots,z_{Nk}(t))^{\mathrm{T}}\in\mathbb{R}^N$, $\boldsymbol{F}=\mathrm{diag}(F_1,F_2,\ldots,F_n)$, $\boldsymbol{G}=\mathrm{diag}(G_1,G_2,\ldots,G_n)$.

Obviously, the exponential synchronization of controlled CMNNs (3) can be realized if error system (9) is exponentially stable in the sense of Filippov. Then, we will consider the dynamical behaviors of system (9).

Theorem 1 Suppose that Assumptions 1–3 hold. For given positive constant ε, the exponential synchronization of

controlled CMNNs (3) is achieved by system (8), if there exist positive diagonal matrix $\boldsymbol{P} = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, positive definite matrices $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{S}$, $\boldsymbol{T} \in \boldsymbol{R}^{N\times N}$, arbitrary diagonal matrices $\boldsymbol{U} = \mathrm{diag}(u_1, u_2, \ldots, u_n)$, $\boldsymbol{V} = \mathrm{diag}(v_1, v_2, \ldots, v_n)$, and positive constants d_i $(i \in \tilde{\mathscr{V}})$, such that the following inequalities hold for $k = 1, 2, \ldots, n$,:

$$\varepsilon\tau - 1 < 0, \tag{10}$$

$$\varepsilon\boldsymbol{Q} + \varepsilon\boldsymbol{R} + (\varepsilon\tau - 1)\boldsymbol{T} < 0, \tag{11}$$

$$\boldsymbol{\Xi}_k = \begin{pmatrix} \boldsymbol{\Xi}_{11}^k & \boldsymbol{\Xi}_{12}^k & 0 & \boldsymbol{\Xi}_{14}^k & p_k\boldsymbol{I} & u_k\boldsymbol{I} & 0 \\ * & \boldsymbol{\Xi}_{22}^k & \boldsymbol{\Xi}_{23}^k & 0 & 0 & 0 & 0 \\ * & * & \boldsymbol{\Xi}_{33}^k & 0 & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Xi}_{44}^k & 0 & 0 & v_k\boldsymbol{I} \\ * & * & * & * & \boldsymbol{\Xi}_{55}^k & 0 & 0 \\ * & * & * & * & * & \boldsymbol{\Xi}_{66}^k & 0 \\ * & * & * & * & * & * & \boldsymbol{\Xi}_{77}^k \end{pmatrix} < 0, \tag{12}$$

where

$$\begin{aligned}
\boldsymbol{\Xi}_{11}^k &= (\varepsilon p_k + 2u_kc_k - 2p_kc_k + 3n\tilde{a}F_k^2)\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{R} \\
&\quad - (1-\varepsilon\tau)\boldsymbol{S} + \tau\boldsymbol{T} + cp_k\gamma_k(\boldsymbol{W} + \boldsymbol{W}^{\mathrm{T}} - 2\boldsymbol{D}) \\
&\quad - cu_k\gamma_k(\boldsymbol{W} + \boldsymbol{W}^{\mathrm{T}} - 2\boldsymbol{D}), \\
\boldsymbol{\Xi}_{12}^k &= (1-\varepsilon\tau)\boldsymbol{S}, \\
\boldsymbol{\Xi}_{14}^k &= (u_k + v_kc_k)\boldsymbol{I} - cv_k\gamma_k(\boldsymbol{W}^{\mathrm{T}} - \boldsymbol{D}), \\
\boldsymbol{\Xi}_{22}^k &= 3n\tilde{b}G_k^2\boldsymbol{I} - (1-u)\boldsymbol{Q} - 2(1-\varepsilon\tau)\boldsymbol{S}, \\
\boldsymbol{\Xi}_{23}^k &= (1-\varepsilon\tau)\boldsymbol{S}, \\
\boldsymbol{\Xi}_{33}^k &= -\boldsymbol{R} - (1-\varepsilon\tau)\boldsymbol{S}, \\
\boldsymbol{\Xi}_{44}^k &= \tau^2 S + 2v_k\boldsymbol{I}, \\
\boldsymbol{\Xi}_{55}^k &= \boldsymbol{\Xi}_{66}^k = \boldsymbol{\Xi}_{77}^k = -\frac{1}{n\tilde{a} + n\tilde{b}}\boldsymbol{I},
\end{aligned}$$

$\boldsymbol{I}$ and $\boldsymbol{0}$ are $N \times N$ identity and null matrix, respectively.

Proof Consider the following functional:

$$\boldsymbol{V}(t) = \exp(\varepsilon t)\tilde{\boldsymbol{V}}(t), \tag{13}$$

where

$$\begin{aligned}
\tilde{\boldsymbol{V}}(t) &= \boldsymbol{V}_1(t) + \boldsymbol{V}_2(t) + \boldsymbol{V}_3(t) + \boldsymbol{V}_4(t) + \boldsymbol{V}_5(t), \\
\boldsymbol{V}_1(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t), \\
\boldsymbol{V}_2(t) &= \sum_{k=1}^{n}\int_{t-\tau(t)}^{t} \tilde{\boldsymbol{e}}_k^{\mathrm{T}}(\xi)\boldsymbol{Q}\tilde{\boldsymbol{e}}_k(\xi)\,\mathrm{d}\xi, \\
\boldsymbol{V}_3(t) &= \sum_{k=1}^{n}\int_{t-\tau}^{t} \tilde{\boldsymbol{e}}_k^{\mathrm{T}}(\xi)\boldsymbol{R}\tilde{\boldsymbol{e}}_k(\xi)\,\mathrm{d}\xi, \\
\boldsymbol{V}_4(t) &= \sum_{k=1}^{n}\tau\int_0^{\tau}\int_{t-\xi}^{t} \tilde{\boldsymbol{z}}_k^{\mathrm{T}}(\theta)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(\theta)\,\mathrm{d}\theta\,\mathrm{d}\xi, \\
\boldsymbol{V}_5(t) &= \sum_{k=1}^{n}\int_0^{\tau}\int_{t-\xi}^{t} \tilde{\boldsymbol{e}}_k^{\mathrm{T}}(\theta)\boldsymbol{T}\tilde{\boldsymbol{e}}_k(\theta)\,\mathrm{d}\theta\,\mathrm{d}\xi.
\end{aligned}$$

Calculating the upper right derivative of $\boldsymbol{V}(t)$ along the trajectory (9) gives

$$\begin{aligned}
\boldsymbol{D}^+\boldsymbol{V}(t) &= \exp(\varepsilon t)\Big\{\sum_{i=1}^{N}[e_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{z}_i(t) + \boldsymbol{z}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t)] \\
&\quad + \sum_{k=1}^{n}[\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t)\boldsymbol{Q}\tilde{\boldsymbol{e}}_k(t) + \tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t)\boldsymbol{R}\tilde{\boldsymbol{e}}_k(t) \\
&\quad - (1-\dot{\tau}(t))\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}\tilde{\boldsymbol{e}}_k(t-\tau(t)) \\
&\quad - \tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t-\tau)\boldsymbol{R}\tilde{\boldsymbol{e}}_k(t-\tau) + \tau^2\tilde{\boldsymbol{z}}_k^{\mathrm{T}}(t)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(t) \\
&\quad - \tau\int_{t-\tau}^{t}\tilde{\boldsymbol{z}}_k^{\mathrm{T}}(\xi)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(\xi)\,\mathrm{d}\xi + \tau\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t)\boldsymbol{T}\tilde{\boldsymbol{e}}_k(t) \\
&\quad - \int_{t-\tau}^{t}\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(\xi)\boldsymbol{T}\tilde{\boldsymbol{e}}_k(\xi)\,\mathrm{d}\xi] + \varepsilon\tilde{\boldsymbol{V}}(t)\Big\}.
\end{aligned} \tag{14}$$

According to the definition of $\tilde{\boldsymbol{V}}(t)$, we have

$$\begin{aligned}
\varepsilon\tilde{\boldsymbol{V}}(t) &\le \sum_{i=1}^{N}\varepsilon\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t) + \sum_{k=1}^{n}\Big[\int_{t-\tau}^{t}\varepsilon\tau^2\tilde{\boldsymbol{z}}_k^{\mathrm{T}}(\xi)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(\xi)\,\mathrm{d}\xi \\
&\quad + \int_{t-\tau}^{t}\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(\xi)(\varepsilon\boldsymbol{Q} + \varepsilon\boldsymbol{R} + \varepsilon\tau\boldsymbol{T})\tilde{\boldsymbol{e}}_k(\xi)\,\mathrm{d}\xi\Big].
\end{aligned} \tag{15}$$

With Lemma 1, we can derive that

$$\begin{aligned}
&\sum_{i=1}^{N}[\varepsilon e_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t) + e_i^{\mathrm{T}}(t)P\boldsymbol{z}_i(t) + z_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t)] \\
&\le \sum_{i=1}^{N}\{e_i^{\mathrm{T}}(t)[\varepsilon\boldsymbol{P} + (2\alpha(\boldsymbol{A}_i) + 2\alpha(\boldsymbol{B}_i))\boldsymbol{P}\boldsymbol{P} - 2\boldsymbol{P}\boldsymbol{C}]\boldsymbol{e}_i(t) \\
&\quad + 2\alpha(\boldsymbol{A}_i)\tilde{f}^{\mathrm{T}}(\boldsymbol{e}_i(t))\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t)) \\
&\quad + 2\alpha(\boldsymbol{B}_i)\tilde{\boldsymbol{g}}^{\mathrm{T}}(\boldsymbol{e}_i(t-\tau(t)))\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t)))\} \\
&\quad + 2c\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\Gamma}\boldsymbol{e}_j(t) \\
&\quad - 2c\sum_{i=1}^{N}d_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\Gamma}\boldsymbol{e}_i(t).
\end{aligned} \tag{16}$$

Based on Assumption 1, the following inequalities hold for positive diagonal matrices $\boldsymbol{F}$ and $\boldsymbol{G}$:

$$0 \le -\tilde{\boldsymbol{f}}^{\mathrm{T}}(\boldsymbol{e}_i(t))\tilde{\boldsymbol{f}}(\boldsymbol{e}_i(t)) + \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{F}\boldsymbol{F}\boldsymbol{e}_i(t), \tag{17}$$

$$\begin{aligned}
0 &\le -\tilde{\boldsymbol{g}}^{\mathrm{T}}(\boldsymbol{e}_i(t-\tau(t)))\tilde{\boldsymbol{g}}(\boldsymbol{e}_i(t-\tau(t))) \\
&\quad + \boldsymbol{e}_i^{\mathrm{T}}(t-\tau(t))\boldsymbol{G}\boldsymbol{G}\boldsymbol{e}_i(t-\tau(t)).
\end{aligned} \tag{18}$$

Hence, substituting Eqs. (17) and (18) into Eq. (16) yields

$$\begin{aligned}
&\sum_{i=1}^{N}[\varepsilon\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t) + \boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{z}_i(t) + \boldsymbol{z}_i^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_i(t)] \\
&\le \sum_{k=1}^{n}\{\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t)[(\varepsilon p_k + (n\tilde{a} + n\tilde{b})p_k^2 - 2p_kc_k + n\tilde{a}F_k^2)\boldsymbol{I} \\
&\quad + cp_k\gamma_k(\boldsymbol{W} + \boldsymbol{W}^{\mathrm{T}} - 2\boldsymbol{D})]\tilde{\boldsymbol{e}}_k(t) \\
&\quad + n\tilde{b}G_k^2\tilde{\boldsymbol{e}}_k^{\mathrm{T}}(t-\tau(t))\tilde{\boldsymbol{e}}_k(t-\tau(t))\}.
\end{aligned} \tag{19}$$

From Lemma 2 and condition (10), we know that

$$\begin{aligned}
&(\varepsilon\tau^2 - \tau)\sum_{k=1}^{n}\int_{t-\tau}^{t}\tilde{\boldsymbol{z}}_k^{\mathrm{T}}(\xi)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(\xi)\,\mathrm{d}\xi \\
&= -(1-\varepsilon\tau)\tau\sum_{k=1}^{n}\Big[\int_{t-\tau}^{t-\tau(t)}\tilde{\boldsymbol{z}}_k^{\mathrm{T}}(\xi)\boldsymbol{S}\tilde{\boldsymbol{z}}_k(\xi)\,\mathrm{d}\xi
\end{aligned}$$

$$+\int_{t-\tau(t)}^{t}\tilde{z}_k^{\mathrm{T}}(\xi)\boldsymbol{S}\tilde{z}_k(\xi)\mathrm{d}\xi\Big]$$
$$\leq(\varepsilon\tau-1)\sum_{k=1}^{n}\Big[\Big(\int_{t-\tau}^{t-\tau(t)}\tilde{z}_k(\xi)\mathrm{d}\xi\Big)^{\mathrm{T}}\boldsymbol{S}\Big(\int_{t-\tau}^{t-\tau(t)}\tilde{z}_k(\xi)\mathrm{d}\xi\Big)$$
$$+\Big(\int_{t-\tau(t)}^{t}\tilde{z}_k(\xi)\mathrm{d}\xi\Big)^{\mathrm{T}}\boldsymbol{S}\Big(\int_{t-\tau(t)}^{t}\tilde{z}_k(\xi)\mathrm{d}\xi\Big)\Big]. \tag{20}$$

It then follows from the Newton–Leibniz formula that

$$\tilde{e}_k(t)-\tilde{e}_k(t-\tau(t))=\int_{t-\tau(t)}^{t}\tilde{z}_k(\xi)\mathrm{d}\xi, \tag{21}$$
$$\tilde{e}_k(t-\tau(t))-\tilde{e}_k(t-\tau)=\int_{t-\tau}^{t-\tau(t)}\tilde{z}_k(\xi)\mathrm{d}\xi. \tag{22}$$

In addition, for arbitrary diagonal matrices $\boldsymbol{U}$ and $\boldsymbol{V}$, the following statement is true:

$$\begin{aligned}0 &= 2\exp(\varepsilon t)\sum_{i=1}^{N}(e_i^{\mathrm{T}}(t)\boldsymbol{U}+z_i^{\mathrm{T}}(t)\boldsymbol{V})[z_i(t)+\boldsymbol{C}e_i(t)\\ &\quad-\boldsymbol{A}_i\tilde{\boldsymbol{f}}(e_i(t))-B_i\tilde{\boldsymbol{g}}(e_i(t-\tau(t)))\\ &\quad-c\sum_{j=1}^{N}w_{ij}\boldsymbol{\Gamma}e_j(t)+cd_i\boldsymbol{\Gamma}e_i(t)]\\ &\leq\exp(\varepsilon t)\sum_{k=1}^{n}\{(2v_k+(n\tilde{a}+n\tilde{b})v_k^2)\tilde{z}_k^{\mathrm{T}}(t)\tilde{z}_k(t)\\ &\quad+((n\tilde{a}+n\tilde{b})u_k^2+2u_kc_k+2n\tilde{a}F_k^2)\tilde{e}_k^{\mathrm{T}}(t)\tilde{e}_k(t)\\ &\quad+2\tilde{e}_k^{\mathrm{T}}(t)[(u_k+v_kc_k)\boldsymbol{I}-cv_k\gamma_k(\boldsymbol{W}^{\mathrm{T}}-\boldsymbol{D})]\tilde{z}_k(t)\\ &\quad-cu_k\gamma_k\tilde{e}_k^{\mathrm{T}}(t)(\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}-2\boldsymbol{D})\tilde{e}_k(t)\\ &\quad+2n\tilde{b}G_k^2\tilde{e}_k^{\mathrm{T}}(t-\tau(t))\tilde{e}_k(t-\tau(t))\}.\end{aligned} \tag{23}$$

According to Eqs. (14)–(23), we can conclude that

$$\begin{aligned}&\boldsymbol{D}^{+}\boldsymbol{V}(t)\\ &\leq\exp(\varepsilon t)\sum_{k=1}^{n}\{\eta_k^{\mathrm{T}}(t)\Xi_k\eta_k(t)\\ &\quad+\int_{t-\tau}^{t}\tilde{e}_k^{\mathrm{T}}(\xi)[\varepsilon\boldsymbol{Q}+\varepsilon\boldsymbol{R}+(\varepsilon\tau-1)\boldsymbol{T}]\tilde{e}_k(\xi)\mathrm{d}\xi\},\end{aligned} \tag{24}$$

where

$$\eta_k(t)=(\tilde{e}_k^{\mathrm{T}}(t),\tilde{e}_k^{\mathrm{T}}(t-\tau(t)),\tilde{e}_k^{\mathrm{T}}(t-\tau),\tilde{z}_k^{\mathrm{T}}(t))^{\mathrm{T}}, \tag{25}$$
$$\Xi_k=\begin{pmatrix}\tilde{\Xi}_{11}^k & (1-\varepsilon\tau)\boldsymbol{S} & 0 & \tilde{\Xi}_{14}^k\\ * & \tilde{\Xi}_{22}^k & (1-\varepsilon\tau)\boldsymbol{S} & 0\\ * & * & \tilde{\Xi}_{33}^k & 0\\ * & * & * & \tilde{\Xi}_{44}^k\end{pmatrix}, \tag{26}$$

$$\begin{aligned}\tilde{\Xi}_{11}^k &= (\varepsilon p_k+(n\tilde{a}+n\tilde{b})p_k^2+(n\tilde{a}+n\tilde{b})u_k^2\\ &\quad-2p_kc_k+2u_kc_k+3n\tilde{a}F_k^2)\boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{R}\\ &\quad-(1-\varepsilon\tau)\boldsymbol{S}+\tau\boldsymbol{T}+cp_k\gamma_k(\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}-2\boldsymbol{D})\\ &\quad-cu_k\gamma_k(\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}-2\boldsymbol{D}),\\ \tilde{\Xi}_{14}^k &= (u_k+v_kc_k)\boldsymbol{I}-cv_k\gamma_k(\boldsymbol{W}^{\mathrm{T}}-\boldsymbol{D}),\\ \tilde{\Xi}_{22}^k &= 3n\tilde{b}G_k^2I-(1-u)\boldsymbol{Q}-2(1-\varepsilon\tau)\boldsymbol{S},\\ \tilde{\Xi}_{33}^k &= -\boldsymbol{R}-(1-\varepsilon\tau)\boldsymbol{S},\\ \tilde{\Xi}_{44}^k &= \tau^2S+(2v_k+(n\tilde{a}+n\tilde{b})v_k^2)\boldsymbol{I}.\end{aligned}$$

By Schur complement theorem,[40] $\tilde{\boldsymbol{\Xi}}_k<0$ can be transformed into linear matrix inequality (LMI) (12). Thus, conditions (10)–(12) indicate that $\boldsymbol{V}(t)\leq\boldsymbol{V}(0)$ always holds. Furthermore, we can derive from Eq. (13) that $\boldsymbol{V}(0)\leq\tilde{\theta}$, where

$$\begin{aligned}\tilde{\theta} &= [\lambda_{\max}(\boldsymbol{P})+\lambda_{\max}(\boldsymbol{Q})\tau+\lambda_{\max}(\boldsymbol{R})\tau+\lambda_{\max}(\boldsymbol{T})\tau^2]\\ &\quad\times\sum_{i=1}^{N}\sup_{-\tau\leq\xi\leq0}e_i^{\mathrm{T}}(\xi)e_i(\xi)+\lambda_{\max}(S)\tau^3\\ &\quad\times\sum_{i=1}^{N}\sup_{-\tau\leq\xi\leq0}z_i^{\mathrm{T}}(\xi)z_i(\xi).\end{aligned}$$

Therefore, the facts that $\exp(\varepsilon t)\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\boldsymbol{P}e_i(t)\leq\boldsymbol{V}(t)$ and $\boldsymbol{V}(t)\leq\tilde{\theta}$ lead to $\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)\leq\theta\exp(-\varepsilon t)$, where $\theta=\tilde{\theta}\cdot\lambda_{\min}^{-1}(\boldsymbol{P})$. Thus, the exponential synchronization of controlled CMNNs (3) is achieved. The proof is completed.

The exponential synchronization criteria in terms of LMIs (10)–(12) can be efficiently solved by the interior point method. In the following, we will focus on some topology-based conditions.

Corollary 1 Suppose that Assumptions 1–3 hold. For given positive constant ε, the exponential synchronization of controlled CMNNs (3) with $u<1$ is achieved by system (8), if there exist positive constants d_i $(i\in\tilde{\mathscr{V}})$, such that the following inequalities hold for $k=1,2,\ldots,n$:

$$\varepsilon\tau-1<0, \tag{27}$$
$$\delta_k+c\gamma_k\lambda_{\max}(\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}-2\boldsymbol{D})<0, \tag{28}$$

where

$$\delta_k=\varepsilon+n\tilde{a}+n\tilde{b}-2c_k+n\tilde{a}F_k^2+\frac{n\tilde{b}G_k^2}{(1-\varepsilon\tau)(1-u)}.$$

Proof Consider the functional

$$\begin{aligned}\boldsymbol{U}(t) &= \exp(\varepsilon t)\Big\{\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t)\\ &\quad+\sum_{k=1}^{n}\Big[\alpha_k\int_{t-\tau(t)}^{t}\tilde{e}_k^{\mathrm{T}}(\xi)\tilde{e}_k(\xi)\mathrm{d}\xi\\ &\quad+\beta_k\int_{0}^{\tau}\int_{t-\xi}^{t}\tilde{e}_k^{\mathrm{T}}(\theta)\tilde{e}_k(\theta)\mathrm{d}\theta\mathrm{d}\xi\Big]\Big\},\end{aligned} \tag{29}$$

where

$$\alpha_k=\frac{n\tilde{b}G_k^2}{1-u},\quad\beta_k=\frac{\varepsilon n\tilde{b}G_k^2}{(1-u)(1-\varepsilon\tau)},\quad(k=1,2,\ldots,n).$$

A similar calculation as in Theorem 1 gives $\boldsymbol{D}^{+}\boldsymbol{U}(t)\leq\exp(\varepsilon t)\sum_{k=1}^{n}\tilde{e}_k^{\mathrm{T}}(t)[\delta_k\boldsymbol{I}+c\gamma_k(\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}-2\boldsymbol{D})]\tilde{e}_k(t)$. Therefore, condition (28) implies that $\boldsymbol{D}^{+}\boldsymbol{U}(t)\leq0$ holds almost

everywhere. The rest is similar to that of Theorem 1 and omitted.

Remark 3 The network structure in our model needs not be undirected or strongly connected, even the spanning tree is not necessary. Undirected network and strongly connected network corresponds to symmetric coupling matrix[20] and irreducible coupling matrix,[22] respectively. If sufficient condition (28) is well satisfied by using appropriate control strategy and strong coupling strength, similar results can be derived for undirected, directed, strongly connected and weak connected network structure.

From Assumption 3, we know that the local root nodes can influence the others through direct or indirect paths. Moreover, the number of such local root nodes is much smaller than the network size, especially for large-scale network. It is worthwhile to note that, if node i cannot influence any other one, all elements in the i-th column of coupling matrix $\boldsymbol{W}$ are zero except w_{ii}. Thus, scaling down the search scope for the local root nodes benefits from appropriate operations on coupling matrix $\boldsymbol{W}$. The pinned nodes can be identified from these local root nodes by the following steps.

(I) Calculate the eigenvalues of coupling matrix $\boldsymbol{W} \in \boldsymbol{R}^{N\times N}$ for a large-scale network $\mathscr{G}$. The multiplicity (d) of the zero eigenvalue is equal to the minimum number of directed trees.[24]

(II) Remove the i-th row and the i-th column of matrix $\boldsymbol{W}$ if there exists only one nonzero element in the i-th column. Repeat this process until there is no column containing only one nonzero element. Then, the modified network $\tilde{\mathscr{G}}$ (with modified coupling matrix $\tilde{\boldsymbol{W}} \in \boldsymbol{R}^{\tilde{N}\times\tilde{N}}$) composed of $\tilde{N}$ ($\tilde{N} \leq N$) nodes is generated.

(III) Check the number (k) of nodes whose row and column elements in $\tilde{\boldsymbol{W}}$ are all zeros. The k nodes, which should be controlled firstly, are either the isolated nodes or the local root nodes of the original network $\mathscr{G}$. If $k = d$, all pinned nodes are identified and the pinned nodes coupling matrix is $\overline{\boldsymbol{W}} = \tilde{\boldsymbol{W}}$; otherwise switches to step IV.

(IV) By deep first search, identify the remaining $d-k$ pinned nodes and obtain the pinned nodes coupling matrix $\overline{\boldsymbol{W}}$ from the modified coupling matrix $\tilde{\boldsymbol{W}}$.

Remark 4 It is worth mentioning that, in step (II), the operation on the i-th column in which only one element is negative and all others are zero precedes the operation on the j-th column that contains only one positive element and zero elsewhere. The former implies that the i-th node's out-degree is zero, whereas the latter indicates that the j-th node can influence another one. In case there exist some disjoint subgroups in the network, we will face with this choice. Based on the aggregated algorithm,[32] our goal is to exclude the leaf nodes, obtain the local root nodes, and identify the pinned nodes in a small-scale network. More specifically, steps (I)–(III) narrow the search scope for the pinned nodes and avoid deep first search in the whole network. However, if the network is randomly generated or composed of numerous local root nodes, then global search is inevitable. Therefore, it is still a challenging problem to effectively identify the pinned nodes for a general network.

It is clear that exponential synchronization can be guaranteed if the control effects are strong enough. However, the feedback gains employed above may be larger than the needed. Thus, the adaptive method is adopted to track their actual values and control strategy (8) is modified as

$$\boldsymbol{u}_i(t) = \begin{cases} -c\boldsymbol{d}_i(t)\boldsymbol{\Gamma}\boldsymbol{e}_i(t), & (i \in \tilde{\mathscr{V}}), \\ 0, & (i \in \mathscr{V} \setminus \tilde{\mathscr{V}}), \end{cases} \tag{30}$$

$$\dot{\boldsymbol{d}}_i(t) = \exp(\varepsilon t)c\rho_i\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{e}_i(t), \tag{31}$$

where ρ_i are positive constants.

Corollary 2 Suppose Assumptions 1–3 hold. For given positive constant ε, the exponential synchronization of controlled CMNNs (3) with $u < 1$ is achieved using Eqs. (30) and (31), if there exist positive constants d_i^* ($i \in \tilde{\mathscr{V}}$), such that equations (27) and (28) hold for $k = 1,2,\ldots,n$, where diagonal matrix $\boldsymbol{D} = \mathrm{diag}(d_1^*,\ldots,d_N^*)$ in Eq. (28) is redefined as: if $i \in \tilde{\mathscr{V}}$, then $d_i^* > 0$; otherwise, $d_i^* = 0$.

Proof Consider the functional $\boldsymbol{W}(t) = \boldsymbol{U}(t) + \sum_{i=1}^{N} \rho_i^{-1}(d_i(t) - d_i^*)^2$, where $\boldsymbol{U}(t)$ is defined in Eq. (29). The rest is similar to that of Theorem 1 and omitted.

4. Numerical simulations

In this section, two numerical examples are given to verify our results in Section 3.

Consider an array of CMNNs where $\boldsymbol{x}_i(t) = (x_{i1}(t), x_{i2}(t))^{\mathrm{T}} \in \mathbb{R}^2$, $\boldsymbol{f}(\boldsymbol{x}_i(t)) = \boldsymbol{g}(\boldsymbol{x}_i(t)) = ((|x_{i1}(t)+1| - |x_{i1}(t)-1|)/2,\ (|\boldsymbol{x}_{i2}(t)+1| - |\boldsymbol{x}_{i2}(t)-1|)/2)^{\mathrm{T}} \in \mathbb{R}^2$, $\boldsymbol{I}(t) = (2,1)^{\mathrm{T}} \in \mathbb{R}^2$, $\boldsymbol{L} = \mathrm{diag}(1,1)$. Let the synchronization rate be $\varepsilon = 0.1$.

Based on the circuit model of MNNs,[8,9] we let

$$a_{11}(x_i(t)) = \begin{cases} -0.5, & -\dfrac{\mathrm{d}f_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} \leq 0, \\ -0.4, & -\dfrac{\mathrm{d}f_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$a_{12}(x_i(t)) = \begin{cases} 0.3, & \dfrac{\mathrm{d}f_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} \leq 0, \\ 0.2, & \dfrac{\mathrm{d}f_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$a_{21}(x_i(t)) = \begin{cases} 0.2, & \dfrac{\mathrm{d}f_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} \leq 0, \\ 0.3, & \dfrac{\mathrm{d}f_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$a_{22}(x_i(t)) = \begin{cases} -0.3, & -\dfrac{\mathrm{d}f_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} \le 0, \\ -0.4, & -\dfrac{\mathrm{d}f_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$b_{11}(x_i(t)) = \begin{cases} -0.4, & -\dfrac{\mathrm{d}g_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} \le 0, \\ -0.45, & -\dfrac{\mathrm{d}g_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$b_{12}(x_i(t)) = \begin{cases} 0.4, & \dfrac{\mathrm{d}g_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} \le 0, \\ 0.35, & \dfrac{\mathrm{d}g_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i1}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$b_{21}(x_i(t)) = \begin{cases} 0.3, & \dfrac{\mathrm{d}g_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} \le 0, \\ 0.45, & \dfrac{\mathrm{d}g_1(x_{i1}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$$b_{22}(x_i(t)) = \begin{cases} -0.4, & -\dfrac{\mathrm{d}g_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} \le 0, \\ -0.5, & -\dfrac{\mathrm{d}g_2(x_{i2}(t))}{\mathrm{d}t} - \dfrac{\mathrm{d}x_{i2}(t)}{\mathrm{d}t} > 0, \end{cases}$$

$\boldsymbol{C} = \mathrm{diag}(0.5, 0.5)$.

Example 1 Firstly, we discuss the controlled CMNNs (3) composed of three MNNs with control strategy (8), time-varying delay $\tau(t) = 0.25(1-\cos(t))$, and coupling strength $c = 5$. When $t \in [-\tau, 0]$ ($\tau = 0.5$), initial states are randomly chosen within $[-5, 5]$. The original coupling matrix is

$$\boldsymbol{W} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The eigenvalues of coupling matrix $\boldsymbol{W}$ are -2, 0, and 0, and hence the minimum number of directed trees is $d = 2$. In the second column of coupling matrix $\boldsymbol{W}$, there is only one negative element -2, and all others are 0. By deleting the second column and row of $\boldsymbol{W}$, we obtain the modified coupling matrix

$$\tilde{\boldsymbol{W}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

in which no column contains only one non-zero element. Moreover, all elements of $\tilde{\boldsymbol{W}}$ are zero, so $k = d = 2$, and $\overline{W} = \tilde{\boldsymbol{W}}$. In this way, the pinned nodes (nodes 1 and 3) are identified from the original coupling network.

It is worthwhile to note that we do not divide the network into two subgroups and clarify each node belongs to which subgroup. This operation is impracticable for large-scale networks.

The constant feedback gains are $d_1 = 2$, $d_2 = 0$, and $d_3 = 2$. The feasible solutions to LMIs (10)–(12) are

$$\boldsymbol{P} = \mathrm{diag}(4.3281, 4.3281),$$
$$\boldsymbol{U} = \mathrm{diag}(-0.2795, -0.2795),$$
$$\boldsymbol{V} = \mathrm{diag}(-0.1036, -0.1036),$$

$$\boldsymbol{Q} = \begin{pmatrix} 13.5277 & -5.0155 & -0.3281 \\ -5.0155 & 14.1125 & -5.0155 \\ -0.3281 & -5.0155 & 13.5277 \end{pmatrix},$$

$$\boldsymbol{R} = \begin{pmatrix} 10.4952 & -4.9895 & -0.4226 \\ -4.9895 & 10.9909 & -4.9895 \\ -0.4226 & -4.9895 & 10.4952 \end{pmatrix},$$

$$\boldsymbol{S} = \begin{pmatrix} 0.1186 & 0.0068 & -0.0074 \\ 0.0068 & 0.1418 & 0.0068 \\ -0.0074 & 0.0068 & 0.1186 \end{pmatrix},$$

$$\boldsymbol{T} = \begin{pmatrix} 15.3588 & -7.0181 & -0.6755 \\ -7.0181 & 15.9943 & -7.0181 \\ -0.6755 & -7.0181 & 15.3588 \end{pmatrix}.$$

From Figs. 3(a) and 3(b), it can be seen clearly that the pinning synchronization is well achieved.

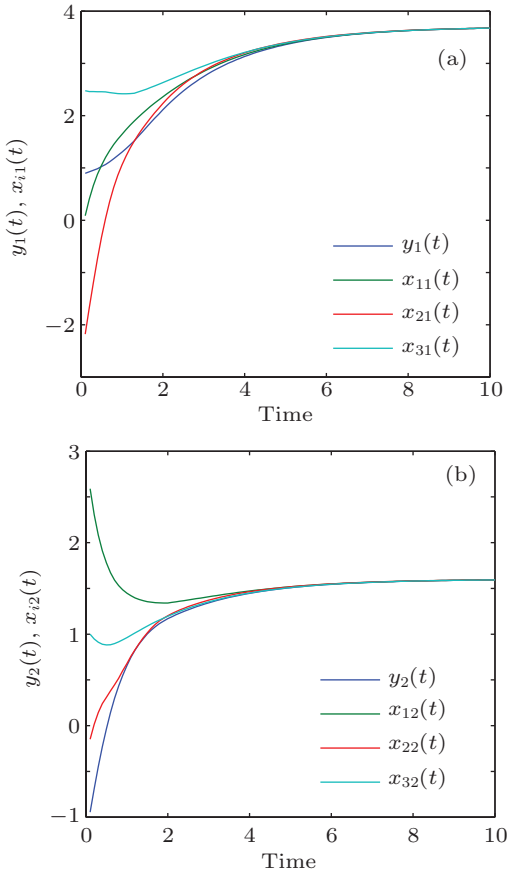


Fig. 3. (color online) In Example 1, the time evolution of state variables of $\boldsymbol{y}(t)$ and all nodes ($i = 1, 2, 3$) : (a) $y_1(t)$, $x_{i1}(t)$; (b) $y_2(t)$, $x_{i2}(t)$.

Example 2 In this example, we consider the controlled CMNNs (3) composed of 100 nodes with control strategy (30) and (31) and coupling strength $c = 10$.

By referring to the Watts–Strogatz (WS) model,[41] this directed network is constructed as follows: each node initially connects K neighbors, and then, with probability p, reconnect each link from a fixed node to any other one and remove the original link. There exists at most a directed link between any

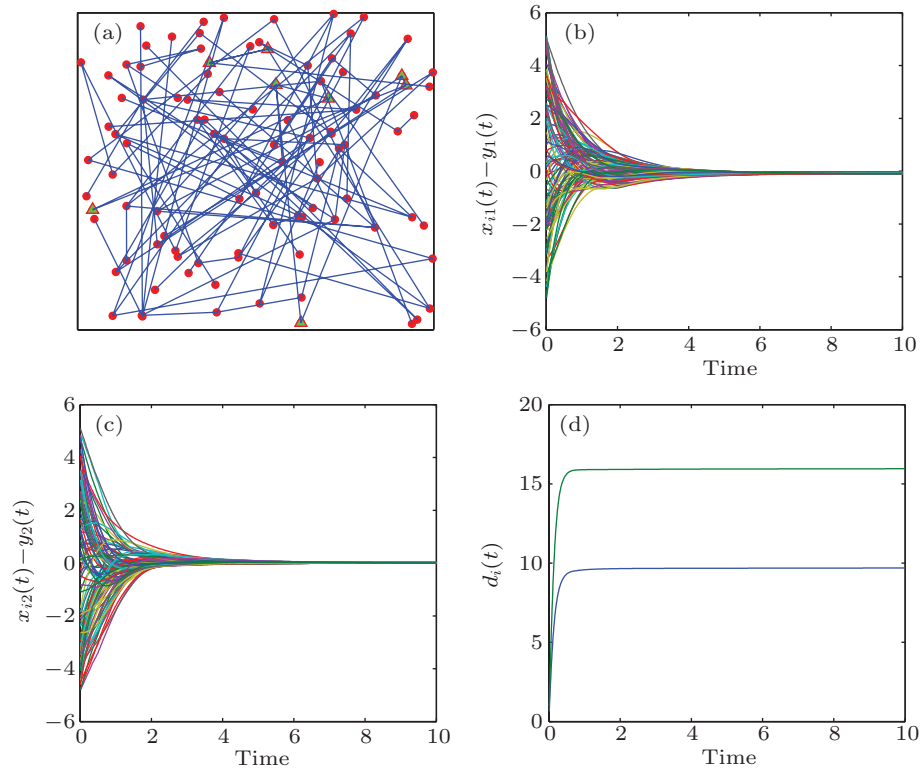


Fig. 4. (color online) (a) Network topology with the local root nodes marked with a green triangle and other nodes marked with a red circle. The time evolution of error states of all nodes ($i = 1, 2, \ldots, 100$): (b) $e_{i1}(t) = x_{i1}(t) - y_1(t)$; (c) $e_{i2}(t) = x_{i2}(t) - y_2(t)$. (d) Time-varying feedback gains $d_i(t)$ of the pinned nodes 7 and 65.

two nodes and each node cannot connect with itself. Besides, we let this construction satisfy Assumption 3.

The network generated by parameters $K = 1$ and $p = 0.8$ is presented in Fig. 4(a). The multiplicity of the zero eigenvalue of $\boldsymbol{W}$ is 2, which implies that there exist at least two directed trees in the underlying network. Following the same steps in Example 1, two disjoint sets of local root nodes $\mathcal{V}_1 = \{16, 17, 53, 54, 55, 65\}$ and $\mathcal{V}_2 = \{7, 29\}$ marked with a green triangle in Fig. 4(a) are obtained. The nodes in each set are strongly connected, and hence any two nodes (one in each set) can synchronize the whole network. With aforementioned parameters and controlled nodes 7 and 65, the time evolution of error states in Figs. 4(b) and 4(c) implies that the expected synchronization is realized. Figure 4(d) depicts the time-varying feedback gains of the pinned nodes.

5. Conclusion

In this paper, the exponential synchronization of CMNNs with time-varying delays is studied. Under the framework of Filippov's solutions, some theoretical results in terms of LMIs are derived to ensure the exponential synchronization of CMNNs. The pinning control method is employed to steer all subsystems towards a given trajectory and a search algorithm is designed to identify the controlled subsystems in such coupled systems. Moreover, our methods are suitable for undirected or strongly connected network structures.

We have taken inspirations from Ref. [42] concerning the synchronization control of coupled Chua's systems and its circuit realization. Our present work mainly focuses the synchronization criteria of CMMNs, and we will study its physical implementations in the near future.

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