Spectroscopy via adiabatic covariant action for the Bañados–Teitelboim–Zanelli (BTZ) black hole*

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Very recently, via the covariant form of the adiabatic invariant $I = \oint p_i \mathrm{d}q_i$ instead of $I = \int p_i \mathrm{d}q_i$, an equally spaced spectroscopy of a Schwarzschild black hole was derived. The emphasis was given to the covariant of results. In this paper, we extend that work in a spherically symmetric spacetime to the case of a rotating Bañados–Teitelboim–Zanelli (BTZ) black hole. It is noteworthy that the adiabatic covariant action $I = \oint p_i \mathrm{d}q_i$ gives the same value for the black hole spectroscopy in different coordinates. The result shows that the area spectrum is $\Delta A = 8\pi l_{\mathrm{P}}^2$, which confirms Bekenstein's initial proposal. And the result is consistent with that already obtained by other methods.

Keywords: spectroscopy, adiabatic covariant action, rotating Bañados–Teitelboim–Zanelli (BTZ) black hole

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1. Introduction

Recently, without the quasinormal mode, Majhi and Vagenas proposed a new approach in the context of adiabatic invariant quantities to investigate the black hole spectroscopy. In their work, the adiabatic invariant quantity combines the Hamilton of a black hole with the outgoing radial null path $\dot{r}$. By utilizing solely the adiabatic invariant quantity[1] $\int p_i \mathrm{d}q_i = nh$, they derived the equally spacing area and entropy spectra from the more general class of spherically symmetric static black holes, and they also recovered Bekenstein's original result $\Delta A = 8\pi l_{\mathrm{P}}^2$. Majhi and Vagenas's work is interesting, and their method is simpler than the quasinormal mode method. However, very recently, Jiang and Han argued that the adiabatic invariant quantity $I = \int p_i \mathrm{d}q_i$ is not canonically invariant[2] and its resulting quantum of the area spectrum apparently depends on the type of coordinates. Based on the idea that the closed contour integral $\oint p_i \mathrm{d}q_i$ is invariant under coordinate transformations,[3,4] Jiang and Han pointed out that the adiabatic invariant quantity should be of the covariant form $I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i = nh$. By combining the black hole property of adiabaticity with the oscillating velocity of the black hole horizon, Jiang and Han described the spectroscopy of a Schwarzschild black hole. An equally spaced spectroscopy was always given by $\Delta A = 8\pi l_{\mathrm{P}}^2$ in agreement with Bekenstein's original proposal.[5,6] The emphasis of that method was given to the covariant of results. Motivated by Jiang and Han's idea, we attempt to extend that work in a spherically symmetric spacetime to the case of a rotating Bañados–Teitelboim–Zanelli (BTZ) black hole.

This paper is organized as follows. In Section 2, we briefly review the rotating BTZ black hole and the revised adiabatic invariant quantity in the spherically symmetric spacetime. Then, we derive the expression of the adiabatic covariant action for the rotating black hole. In Sections 3 and 4, via the modified adiabatic covariant action and the Bohn–Sommerfeld quantization rule, the spectroscopy of the rotating BTZ black hole is presented in two different dragged reference frames. Finally we give a brief conclusion in Section 5.

2. Rotating BTZ black hole and adiabatic covariant action in dragging coordinate system

In recent years, one of the important developments in general relativity is that Bañados, Teitelboim, and Zanelli found the $(2+1)$-dimensional BTZ black hole.[7] The classical and the quantum properties of the $(2+1)$-dimensional BTZ black hole have been reviewed in detail by Carlip.[8] Its structure is simple enough to allow a number of exact computations, particularly in the quantum realm, wihch are impractical in 3+1 dimensions. Therefore, it is very interesting to investigate the spectroscopy of the $(2+1)$-dimensional BTZ black hole. The rotating BTZ black hole solution in a $(2+1)$-dimensional spacetime[7] is derived from a three-dimensional theory of gravity with a negative cosmological constant $(\Lambda = -1/l^2)$

$$S = \int \mathrm{d}x^3 \sqrt{-g}\,(R + 2\Lambda). \tag{1}$$

The corresponding line element is

$$\mathrm{d}s^2 = -\Delta(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\Delta(r)} + r^2\left(\mathrm{d}\phi - \frac{J}{2r^2}\,\mathrm{d}t\right)^2, \tag{2}$$

where the lapse function reads

$$\Delta(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \tag{3}$$

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with M and J being the ADM mass and the angular momentum of the BTZ black hole, respectively, and $-\infty < t < +\infty$, $0 \leqslant r < +\infty$. This metric is stationary and axially symmetric, with two Killing vectors $(\partial/\partial_t)^\mu$ and $(\partial/\partial_\phi)^\mu$. The event horizon is determined by the equation

$$\Delta(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} = 0, \tag{4}$$

and the corresponding outer horizon r_+ is

$$r_+^2 = \frac{l^2}{2}\left(M + \sqrt{M^2 - \frac{J^2}{l^2}}\right). \tag{5}$$

The angular velocity and the surface gravity at the outer horizon are

$$\Omega_+ = -\left.\frac{g_{t\phi}}{g_{\phi\phi}}\right|_{r=r_+} = \frac{J}{2r_+^2}, \tag{6}$$

$$\kappa = \frac{1}{2}\left.\frac{\mathrm{d}\Delta(r)}{\mathrm{d}r}\right|_{r=r_+} = \frac{1}{2}\Delta_r(r_+). \tag{7}$$

For the rotating BTZ black hole, to avoid the dragging effect, we should perform a dragging coordinate transformation

$$\mathrm{d}\phi = \frac{J}{2r^2}\,\mathrm{d}t, \tag{8}$$

then the metric of the BTZ black hole becomes

$$\mathrm{d}s^2 = -\Delta(r)\,\mathrm{d}t^2 + \frac{1}{\Delta(r)}\,\mathrm{d}r^2. \tag{9}$$

This metric can be Euclideanized by the transformation $t \to -\mathrm{i}\tau$ and be given by

$$\mathrm{d}s^2 = \Delta(r)\,\mathrm{d}\tau^2 + \frac{1}{\Delta(r)}\,\mathrm{d}r^2. \tag{10}$$

Now let us discuss the adiabatic covariant action for the rotating black holes. Jiang and Han[2] argued that the adiabatic invariant quantity should be of the covariant form $I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i$ rather than $I_{\mathrm{adia}} = \int p_i \mathrm{d}q_i$. There, they found that the black hole spectroscopy in the Schwarzschild frame is twice that in the Painlevé coordinate if one uses the action $\int p_i \mathrm{d}q_i$ for an adiabatic invariant quantity. That is to say, $I_{\mathrm{adia}} = \int p_i \mathrm{d}q_i$ is physically questionable since it is not invariant under the coordinate transformation, and its resulting quantum of the area spectrum apparently depends on the type of coordinate. The reason for such a discrepancy has also been explained in detain in Ref. [2]. Using the covariant form $I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i$, Jiang and Han investigated the spherically symmetric Schwarzschild black hole and found that the area quantum is universal independent of the choice of coordinate. Here, based on their work, we attempt to generalize the adiabatic covariant action in a spherically symmetric spacetime to the case of a rotating BTZ black hole. Generally speaking, the interesting quantum phenomena, such as tunneling radiation,[9–45] quantization of black hole entropy and area,[1,2,5,6,46–58] take place at the event horizon. So, we can work in the near horizon regime. For the Schwarzschild black hole, near the horizon, the closed contour integral[2] can be found by considering a closed path that goes from $q_i = q_i^{\mathrm{out}}$ (outside the horizon) to $q_i = q_i^{\mathrm{in}}$ (inside the horizon). For the rotating black hole, we still adopt this form of the closed contour integral, that is, the adiabatic covariant action is written as

$$I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i = \int_{q_i^{\mathrm{in}}}^{q_i^{\mathrm{out}}} p_i^{\mathrm{out}} \mathrm{d}q_i + \int_{q_i^{\mathrm{out}}}^{q_i^{\mathrm{in}}} p_i^{\mathrm{in}} \mathrm{d}q_i, \tag{11}$$

where q_i^{out}'s (q_i^{in}'s) stand for the coordinates of the Euclideanized BTZ spacetime, and p_i^{out} (p_i^{in}) corresponds to the canonical momentum of coordinate q_i^{out} (q_i^{in}). In order to investigate the spectroscopy of the rotating case, we should first give the expression of the adiabatic covariant action in the dragging coordinate system. Now we focus our discussion on this issue. For the rotating black hole, the integral $\int_{q_i^{\mathrm{in}}}^{q_i^{\mathrm{out}}} p_i^{\mathrm{out}} \mathrm{d}q_i$ can be given by

$$\begin{aligned}\int_{q_i^{\mathrm{in}}}^{q_i^{\mathrm{out}}} p_i^{\mathrm{out}}\,\mathrm{d}q_i &= \int_{q_i^{\mathrm{in}}}^{q_i^{\mathrm{out}}} \int_0^{p_i^{\mathrm{out}}} \mathrm{d}p_i^{\mathrm{out}'}\mathrm{d}q_i \\ &= \int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}} \int_0^H \mathrm{d}H'\,\mathrm{d}\tau + \int_{q_j^{\mathrm{in}}}^{q_j^{\mathrm{out}}} \int_0^H \frac{\mathrm{d}H'}{\dot{q}_j}\,\mathrm{d}q_j,\end{aligned} \tag{12}$$

where q_j's represent the spacelike coordinates with $j = 1, 2$, $q_1 = r$, $q_2 = \phi$, H is the Hamiltonian of the system, and $\int_{q_j^{\mathrm{in}}}^{q_j^{\mathrm{out}}} \int_0^H \frac{\mathrm{d}H'}{\dot{q}_j}\,\mathrm{d}q_j$ corresponds to the classical action I_{C}, which satisfies $I_{\mathrm{C}} = \int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}} L\,\mathrm{d}\tau$, with L being the Lagrangian function. As mentioned above, because there exists the dragging effect of the coordinate system in the rotating spacetime, the matter field in the ergosphere near the horizon must be described in the dragging coordinate system. Thus, it is noted that the coordinate ϕ does not appear in the dragged metric (10). Namely, ϕ is an ignorable coordinate in the Lagrangian function. In order to eliminate this degree of freedom completely, the classical action should be written as $I_{\mathrm{C}} = \int (L - p_\phi \dot{\phi})\,\mathrm{d}\tau$. Therefore, $\int_{q_j^{\mathrm{in}}}^{q_j^{\mathrm{out}}} \int_0^H \frac{\mathrm{d}H'}{\dot{q}_j}\,\mathrm{d}q_j$ is modified to

$$\begin{aligned}&\int_{q_j^{\mathrm{in}}}^{q_j^{\mathrm{out}}} \int_0^H \frac{\mathrm{d}H'}{\dot{q}_j}\mathrm{d}q_j = \int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}} (L - p_\phi\dot{\phi})\mathrm{d}\tau \\ &= \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} \int_0^{p_r^{\mathrm{out}}} \mathrm{d}p_r^{\mathrm{out}'}\mathrm{d}r - \int_{\phi_{\mathrm{in}}}^{\phi_{\mathrm{out}}} \int_0^{p_\phi^{\mathrm{out}}} \mathrm{d}p_\phi^{\mathrm{out}'}\mathrm{d}\phi \\ &= \int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}} \int_0^H \mathrm{d}H'|_{(r;\phi,p_\phi^{\mathrm{out}})}\mathrm{d}\tau - \int_{\tau_{ub}}^{\tau_{\mathrm{out}}} \int \mathrm{d}H'|_{(\phi;r,p_r^{\mathrm{out}})}\mathrm{d}\tau \\ &= \int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}} \int_0^H \mathrm{d}H'\mathrm{d}\tau,\end{aligned} \tag{13}$$

where the Hamilton canonical equations can be derived as

$$\dot{r} = \left.\frac{\mathrm{d}H}{\mathrm{d}p_r^{\mathrm{out}'}}\right|_{(r;\phi,P_\phi^{\mathrm{out}})}, \quad \mathrm{d}H|_{(r;\phi,P_\phi^{\mathrm{out}})} = \mathrm{d}M,$$

$$\dot{\phi} = \left.\frac{\mathrm{d}H}{\mathrm{d}p_{\phi}^{\mathrm{out}'}}\right|_{(\phi;r,P_r^{\mathrm{out}})}, \quad \mathrm{d}H|_{(\phi;r,P_r^{\mathrm{out}})} = \Omega_+ \mathrm{d}J. \tag{14}$$

By incorporating Eqs. (13) and (14), Eq. (12) can be further simplified as

$$\begin{aligned}\int_{q_i^{\mathrm{in}}}^{q_i^{\mathrm{out}}} p_i^{\mathrm{out}} \mathrm{d}q_i &= 2\int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}}\int_0^H \mathrm{d}H'\mathrm{d}\tau \\ &= 2\int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\dot{r}_{\mathrm{out}}}\mathrm{d}r, \end{aligned} \tag{15}$$

where $\dot{r}_{\mathrm{out}} \equiv \mathrm{d}r/\mathrm{d}\tau$ stands for the radial outgoing path when a particle tunnels out, r_{in} and r_{out} are locations of the outer horizons before and after the particle tunnels out. Repeating the same kind of analysis as above, we can get

$$\int_{q_i^{\mathrm{out}}}^{q_i^{\mathrm{in}}} p_i^{\mathrm{in}} \mathrm{d}q_i = 2\int_{r_{\mathrm{out}}}^{r_{\mathrm{in}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\dot{r}_{\mathrm{in}}}\mathrm{d}r, \tag{16}$$

where $\dot{r}$ stands for the radial ingoing path during the particle's tunneling process. Therefore, the adiabatic covariant action takes the form

$$\begin{aligned} I_{\mathrm{adia}} &= \oint p_i \mathrm{d}q_i \\ &= 2\int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\dot{r}_{\mathrm{out}}}\mathrm{d}r \\ &\quad + 2\int_{r_{\mathrm{out}}}^{r_{\mathrm{in}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\dot{r}_{\mathrm{in}}}\mathrm{d}r. \end{aligned} \tag{17}$$

Through the adiabatic covariant action, we can derive the spectroscopy of the rotating BTZ black hole in different coordinates.

3. Spectroscopy of rotating BTZ black hole in the dragged BTZ coordinate

In this section, via adiabatic covariant action $I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i$ in expression (17), using the Bohr–Sommerfeld quantization rule and the laws of the black hole thermodynamics, we investigate the area and entropy spectra of the rotating BTZ black hole in the dragged BTZ coordinate system. To obtain $\dot{r}$, we should consider the radial null geodesics. According to Eq. (10), the radial null paths now are written as

$$\dot{r} \equiv \frac{\mathrm{d}r}{\mathrm{d}\tau} = \pm \mathrm{i}\Delta(r), \tag{18}$$

where the positive (negative) sign stands for the outgoing (ingoing) radial null paths. That is

$$\dot{r}_{\mathrm{out}} = \mathrm{i}\Delta(r), \tag{19}$$

$$\dot{r}_{\mathrm{in}} = -\mathrm{i}\Delta(r). \tag{20}$$

From Eqs. (17), (19), and (20), the adiabatic covariant action connected to the parameters of the BTZ black hole can be expressed as

$$I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i = 4\int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\mathrm{i}\Delta(r)}\mathrm{d}r. \tag{21}$$

Note that there is a pole at the out horizon r_+, taking a contour integral over a small half-loop going above the pole from right to left, in this way, we finish the r integral and obtain

$$\begin{aligned} I_{\mathrm{adia}} &= \oint p_i \mathrm{d}q_i \\ &= 4\pi\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\Delta_r(r_+)} \\ &= 2\pi\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\kappa}. \end{aligned} \tag{22}$$

It is well known that the black hole temperature is related to the surface gravity of the horizon by $T = \hbar k/2\pi$. From the first law of the black hole thermodynamics

$$\mathrm{d}M = T\mathrm{d}S + \Omega_+ \mathrm{d}J, \tag{23}$$

the adiabatic covariant action in terms of the entropy is simply

$$I_{\mathrm{adia}} = \oint p_i \mathrm{d}q_i = \hbar\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{T} = \hbar S. \tag{24}$$

Implementing the Bohr–Sommerfeld quantization rule

$$\oint p_i \mathrm{d}q_i = nh = 2\pi n\hbar, \qquad n = 1,2,3,\ldots \tag{25}$$

we derive the entropy spectrum of the rotating BTZ black hole

$$S = 2\pi n, \qquad n = 1,2,3,\ldots \tag{26}$$

and the spacing of the entropy spectrum

$$\Delta S = S_{n+1} - S_n = 2\pi. \tag{27}$$

It shows that the entropy is discrete and equidistant for the rotating black hole. Using the relation between entropy and horizon area

$$S = \frac{A}{4l_{\mathrm{P}}^2}, \tag{28}$$

and the spacing of the entropy spectrum given in Eq. (27), we can get the spacing of the horizon area

$$\Delta A = 8\pi l_{\mathrm{P}}^2. \tag{29}$$

The result is consistent with that obtained by the other methods.[50–52] Therefore, the equally spaced spectroscopy is presented and irrelevant to the parameters of the BTZ black hole. Here, in the dragged BTZ black coordinate system, if we use the adiabatic invariant $\int p_i \mathrm{d}q_i$, we have

$$\begin{aligned} I_{\mathrm{adia}} &= \int p_i \mathrm{d}q_i = 2\int_{\tau_{\mathrm{in}}}^{\tau_{\mathrm{out}}}\int_0^H \mathrm{d}H'\mathrm{d}\tau \\ &= 2\int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}}\int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\dot{r}_{\mathrm{out}}}\mathrm{d}r = \frac{\hbar}{2}S. \end{aligned} \tag{30}$$

This implies that the entropy and area spectra are

$$\Delta S = 4\pi, \tag{31}$$

$$\Delta A = 16\pi l_{\rm P}^2. \tag{32}$$

Obviously, via the adiabatic invariant action $\int p_i \mathrm{d}q_i$, the entropy and area spectra are twice the results of Eqs. (27) and (29) by using the covariant action $\int p_i \mathrm{d}q_i$, which are not in consistent with those obtained by Maggiore and those derived by Bekenstein.

4. Spectroscopy of rotating BTZ black hole in the dragged Painlevé coordinate

In this section, via the adiabatic covariant action $\oint p_i \mathrm{d}q_i$ in expression (17), we continue to quantize the horizon area and entropy of the BTZ black hole in the dragged Painlevé coordinate system. Introduce the Painlevé coordinate transformation[59] in metric (10)

$$\mathrm{d}\tau = \mathrm{d}\tau' + \sqrt{\frac{\Delta(r)-1}{\Delta^2(r)}}\,\mathrm{d}r. \tag{33}$$

Substituting Eq. (33) into Eq. (10), we can get the dragged Painlevé BTZ black hole metric

$$\mathrm{d}s^2 = \Delta(r)\,\mathrm{d}\tau'^2 + 2\sqrt{\Delta(r)-1}\,\mathrm{d}\tau'\mathrm{d}r + \mathrm{d}r^2. \tag{34}$$

From Eq. (34), the radial null geodesics is obtained as

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}\tau'} = \mathrm{i}\left(\pm 1 - \sqrt{1-\Delta(r)}\right), \tag{35}$$

where the positive (negative) sign stands for the outgoing (ingoing) radial null paths, namely

$$\dot{r}_{\rm out} = \mathrm{i}\left(1 - \sqrt{1-\Delta(r)}\right), \tag{36}$$

$$\dot{r}_{\rm in} = \mathrm{i}\left(-1 - \sqrt{1-\Delta(r)}\right). \tag{37}$$

Inserting Eqs. (36) and (37) into Eq. (17) yields

$$\begin{aligned} I_{\rm adia} &= \oint p_i \mathrm{d}q_i \\ &= -2\mathrm{i}\int_{r_{\rm in}}^{r_{\rm out}} \int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{1-\sqrt{1-\Delta(r)}}\mathrm{d}r \\ &\quad + 2\mathrm{i}\int_{r_{\rm out}}^{r_{\rm in}} \int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{1+\sqrt{1-\Delta(r)}}\mathrm{d}r. \end{aligned} \tag{38}$$

Note that in the right part of Eq. (38), the integral of the second term has no contribution in the dragged Painlevé coordinate. So the adiabatic covariant action reads

$$\begin{aligned} I_{\rm adia} &= \oint p_i \mathrm{d}q_i \\ &= -2\mathrm{i}\int_{r_{\rm in}}^{r_{\rm out}} \int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{1-\sqrt{1-\Delta(r)}}\mathrm{d}r \\ &= 2\pi \int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{\kappa}. \end{aligned} \tag{39}$$

Applying the first law of the black hole thermodynamics $\mathrm{d}M = T\mathrm{d}S + \Omega_+ \mathrm{d}J$ and the Bohr–Sommerfeld quantization rule $\oint p_i \mathrm{d}q_i = nh$, we find the quantization of the entropy

$$\Delta S = 2\pi. \tag{40}$$

This implies that the equally space area spectrum is

$$\Delta A = 8\pi l_{\rm P}^2. \tag{41}$$

This result is in full consistence with that obtained in Section 3. Bekenstein's original result is recovered in the dragged BTZ coordinate and the dragged Painlevé coordinate. This is a physically desired result since the area quantum should be invariant under the coordinate transformations. Here, in the dragged Painlevé coordinate system, if we use the adiabatic invariant action $\int p_i \mathrm{d}q_i$, we arrive at

$$\begin{aligned} \int p_i \mathrm{d}q_i &= 2\mathrm{i}\int_{\tau_{\rm in}}^{\tau_{\rm out}} \int_0^H \mathrm{d}H' \mathrm{d}\tau \\ &= -2\mathrm{i}\int_{r_{\rm in}}^{r_{\rm out}} \int_{(0,0)}^{(M,J)} \frac{\mathrm{d}M' - \Omega_+ \mathrm{d}J'}{1-\sqrt{1-\Delta(r)}}\mathrm{d}r \\ &= \hbar S. \end{aligned} \tag{42}$$

Naturally, the entropy and area spectra are obtained as

$$\Delta S = 2\pi, \tag{43}$$

$$\Delta A = 8\pi l_{\rm P}^2. \tag{44}$$

It is evident that the results are in agreement with those obtained in Eqs. (40) and (41), but they are half the results of Eqs. (31) and (32). That is to say, via the adiabatic invariant action $\int p_i \mathrm{d}q_i$, the black hole spectroscopy is dependent on the choice of coordinate system. However, adopting the invariant form $\oint p_i \mathrm{d}q_i$, the black hole spectroscopy becomes coordinate independent.

5. Conclusion

During the last few years, in order to obtain the quantized area spectrum and the entropy spectrum, one had to consider the proposal about how quasinormal frequencies are related with black holes. If one wants to investigate the area and entropy spectra of more complicated background spacetimes from the viewpoint of quasinormal mode, there are some difficulties in finding the quasinormal mode frequency mathematically which can produce the area and entropy spectra. Without the quasinormal mode, the method using solely the adiabatic covariant action to quantize the horizon area is more convenient and simple.

In this paper, without the quasinormal mode, applying the modified adiabatic covariant action in the rotating black hole, we derive the area and entropy spectra for the rotating BTZ black hole. In order to investigate the spectroscopy of the rotating BTZ black hole, firstly, we perform a dragging coordinate transform. Due to $\oint p_i \mathrm{d}q_i$ being connected with the radial null geodesics during the tunneling process, a legitimate

physical picture should be described in the dragging coordinate system. Secondly, the Lagrangian function is modified in the dragging coordinate system and the expression of the adiabatic covariant action $\oint p_i \mathrm{d}q_i$ in the rotating case is presented. Finally, via the adiabatic covariant action, using the Bohr–Sommerfeld quantization rule and the laws of black hole thermodynamics, we have succeeded in quantizing the horizon area and entropy of the rotating BTZ black hole. It is noteworthy that the adiabatic covariant action $\oint p_i \mathrm{d}q_i$ gives the same result for the black hole spectroscopy in different coordinates, which strengthens the validity and power of the revised adiabatic invariant quantity. The result shows that the area spectrum is $\Delta A = 8\pi l_{\mathrm{P}}^2$, which confirms Bekenstein's initial proposal, and the result is consistent with that already obtained by means of other methods. This method is universal and can be directly extended to different-type rotating black holes.

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