The stability of a shearing viscous star with an electromagnetic field

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We analyze the role of the electromagnetic field for the stability of a shearing viscous star with spherical symmetry. Matching conditions are given for the interior and the exterior metrics. We use a perturbation scheme to construct the collapse equation. The range of instability is explored in Newtonian and post Newtonian (pN) limits. We conclude that the electromagnetic field diminishes the effects of the shearing viscosity in the instability range and makes the system more unstable in both Newtonian and post Newtonian approximations.

Keywords: gravitational collapse, electromagnetic field, instability

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1. Introduction

Charged self-gravitating objects have the tendency of undergoing many phases during gravitational collapse, resulting in charged black holes or naked singularities.[1–3] The stability of these exact solutions is an interesting subject under the perturbation scheme. It is believed that these stars with huge charge cannot be stable.[4,5] However, the electric charge is greatly relevant to the structural formation and the evolution of astrophysical objects.[6–13]

A stellar model may be stable in one phase and later becomes unstable in another phase. The stability of the model is subjected against any disturbance. The dynamical instability of self-gravitating objects is interrelated with their structures as well as their evolutions. In this scenario, Chandrasekhar[14] investigated the problem of dynamical instability for the isotropic perfect fluid of a pulsating system and found the instability range in terms of adiabatic index $\Gamma < 4/3$.

It is well discussed in literature that the instability range of a fluid will be decreased or increased through different physical properties of the fluid. In this context, the dynamical instabilities of adiabatic, non-adiabatic, anisotropic, and shearing viscous fluids have been explored.[15,16,18,19] Chan[20] found that both pressure anisotropy and effective adiabatic index are increased by the shearing viscosity in a collapsing radiating star. Horvat *et al.*[21] have used the quasi-local equation of state[22,23] to explore the stability of anisotropic stars under radial perturbations. Sharif and Kausar[24] investigated the stability of an expansion-free fluid in $f(R)$ gravity and found that the stability of the fluid is constrained by energy density inhomogeneity, pressure anisotropy, and the $f(R)$ model.

The study of charged self-gravitating objects in the context of coupled Einstein–Maxwell field equations leads to the evolution of a black hole.[25–27] The physical aspects of the electromagnetic field have a significant role in general relativity. Stettner[28] investigated the stability of a pulsating sphere with constant surface charge. Glazer[29] generalized that result by taking an arbitrary distribution of charge and found that the Bonner's charged dust model is dynamically unstable. The stability limit for charged spheres has been proposed by many people[30–32,34] starting from the Buchdahl[35] work for neutral spheres. Recently, we have investigated the problem of dynamical instability of cylindrical and spherical systems with electromagnetic fields in Newtonian and post Newtonian (pN) regimes.[36]

Here we explore the role of electromagnetic field in the stability of collapsing fluid undergoing dissipation with shearing viscosity. Darmois matching conditions[37] are formulated for the continuity of the interior general spherically symmetric solution to the exterior vacuum Reissner–Nordström solution. The paper is organized as follows. In Section 2, we discuss some basic properties of the viscous fluid, the Einstein–Maxwell equations, and the junction conditions. Section 3 provides a perturbation scheme to form the collapse equation. In Section 4, we explore the collapse equations in Newtonian and pN regimes. Finally, we discuss our conclusion in Section 5.

2. Fluid distribution, field equations, and junction conditions

We consider a time-like three-space spherical surface Σ, which separates the four-dimensional (4D) geometries into two regions, interior M^- and exterior M^+. The M^- is given by the general spherically symmetric space–time in the comoving

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coordinates

$$\begin{aligned} \mathrm{d}s_-^2 &= -A^2(t,r)\mathrm{d}t^2 + B^2(t,r)\mathrm{d}r^2 \\ &\quad + R^2(t,r)(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \end{aligned} \tag{1}$$

For the exterior metric, we take the Reissner–Nordström metric describing the radiation field around a charged spherically symmetric source of the gravitational field

$$\begin{aligned} \mathrm{d}s_+^2 &= -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r \\ &\quad + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \end{aligned} \tag{2}$$

where M and Q are the total mass and the charge, respectively. The fluid under consideration is locally dissipative in the form of shearing viscosity. The interior energy–momentum tensor of the charged dissipative fluid is given by

$$\begin{aligned} T_{\alpha\beta}^- &= (\rho + \hat{p} - \xi\Theta)u_\alpha u_\beta + (\hat{p} - \xi\Theta)g_{\alpha\beta} - \eta\sigma_{\alpha\beta} \\ &\quad + \frac{1}{4\pi}\left(F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4}g_{\alpha\beta}F^{\gamma\delta}F_{\gamma\delta}\right), \end{aligned} \tag{3}$$

where ρ, $\hat{p}$, ξ, and u_α are the energy density, isotropic pressure, coefficient of bulk viscosity, and four-velocity associated with the fluid, respectively, and $F_{\alpha\beta}$ is the electromagnetic field tensor. We can write the above equation with $p = \hat{p} - \xi\Theta$ as

$$\begin{aligned} T_{\alpha\beta}^- &= (\rho + p)u_\alpha u_\beta + pg_{\alpha\beta} - \eta\sigma_{\alpha\beta} \\ &\quad + \frac{1}{4\pi}\left(F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4}g_{\alpha\beta}F^{\gamma\delta}F_{\gamma\delta}\right). \end{aligned} \tag{4}$$

The four-velocity in the comoving coordinates is

$$u^\alpha = A^{-1}\delta_0^\alpha, \quad u^\alpha u_\alpha = -1. \tag{5}$$

Here $\eta > 0$ is the coefficient of the shearing viscosity, while the shear tensor $\sigma_{\alpha\beta}$ is defined as

$$\sigma_{\alpha\beta} = u_{(\alpha;b)} + a_\alpha u_\beta - \frac{1}{3}\Theta(g_{\alpha\beta} + u_\alpha u_\beta), \tag{6}$$

where $a_\alpha = u_{\alpha;\beta}u^\beta$ is the four-acceleration, and $\Theta = u^\alpha_{;\alpha}$ is the expansion scalar. The corresponding non-zero components are

$$a_1 = \frac{A'}{A}, \quad a^\alpha a_\alpha = \left(\frac{A'}{AB}\right)^2, \quad \Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{R}}{R}\right), \tag{7}$$

where $\cdot$ and $'$ represent differentiations with respect to t and r, respectively. The non-vanishing components of the shear tensor are

$$\sigma_{11} = 2B^2\sigma, \ \sin^2\theta\sigma_{22} = \sigma_{33} = -R^2\sigma, \ \sigma^{\alpha\beta}\sigma_{\alpha\beta} = 2\sigma^2, \tag{8}$$

where

$$\sigma = \frac{1}{3A}\left(\frac{\dot{R}}{R} - \frac{\dot{B}}{B}\right). \tag{9}$$

2.1. The Einstein–Maxwell field equations

In the four-vector formalism, the Maxwell equations are

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \ \ F^{\alpha\beta}_{;\beta} = 4\pi J^\alpha, \tag{10}$$

where ϕ_α and J^α are the four-potential and the current density vector, respectively. We consider the charge to be at rest, resulting in no magnetic field in this local coordinate system. Thus ϕ_α and J^α can be written as

$$\phi_\alpha = \Phi\delta_\alpha^0, \ \ J^\alpha = \zeta u^\alpha, \tag{11}$$

where $\zeta(t,r)$ and $\Phi(t,r)$ describe the charge density and the scalar potential, respectively. From Eqs. (10) and (11), the only non-vanishing component of the electromagnetic field tensor is

$$F_{10} = -F_{01} = \frac{\partial\Phi}{\partial r}. \tag{12}$$

The corresponding Maxwell equations are

$$\frac{\partial^2\Phi}{\partial r^2} - \left(\frac{A'}{A} + \frac{B'}{B} - 2\frac{R'}{R}\right)\frac{\partial\Phi}{\partial r} = 4\pi\zeta AB^2, \tag{13}$$

$$\frac{\partial^2\Phi}{\partial t\partial r} - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - 2\frac{\dot{R}}{R}\right)\frac{\partial\Phi}{\partial r} = 0. \tag{14}$$

Solving the above equations, we have

$$\frac{\partial\Phi}{\partial r} = \frac{qAB}{R^2}, \tag{15}$$

where $q(r)$ is the total amount of charge from the center to the boundary surface of the star

$$q(r) = 4\pi\int_0^r \zeta BR^2\,\mathrm{d}r. \tag{16}$$

The electric field intensity is the charge per unit surface area of the sphere

$$E(t,r) = \frac{q}{4\pi R^2}. \tag{17}$$

Using Eqs. (1) and (4), we have the following non-vanishing components of the Einstein–Maxwell field equations:

$$\begin{aligned} &8\pi A^2(\rho + 2\pi E^2) \\ &= \left(\frac{2\dot{B}}{B} + \frac{\dot{R}}{R}\right)\frac{\dot{R}}{R} - \left(\frac{A}{B}\right)^2 \\ &\quad \times \left[\frac{2R''}{R} + \left(\frac{R'}{R}\right)^2 - \frac{2B'R'}{BR} - \left(\frac{B}{R}\right)^2\right], \end{aligned} \tag{18}$$

$$0 = -2\left(\frac{\dot{R}'}{R} - \frac{\dot{R}A'}{RA} - \frac{\dot{B}R'}{BR}\right), \tag{19}$$

$$\begin{aligned} &8\pi B^2(p + 2\eta\sigma - 2\pi E^2) \\ &= -\left(\frac{B}{A}\right)^2\left[\frac{2\ddot{R}}{R} - \left(\frac{2\dot{A}}{A} - \frac{\dot{R}}{R}\right)\frac{\dot{R}}{R}\right] \\ &\quad + \left(\frac{2A'}{A} + \frac{R'}{R}\right)\frac{R'}{R} - \left(\frac{B}{R}\right)^2, \end{aligned} \tag{20}$$

$$
\begin{aligned}
&8\pi R^2(p-\eta\sigma+2\pi E^2)\\
&=8\pi R^2(p-\eta\sigma+2\pi E^2)\sin^{-2}\theta\\
&=-\left(\frac{R}{A}\right)^2\left[\frac{\ddot{B}}{B}+\frac{\ddot{R}}{R}-\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}+\frac{\dot{R}}{R}\right)+\frac{\dot{B}\dot{R}}{BR}\right]\\
&\quad+\left(\frac{R}{B}\right)^2\left[\frac{A''}{A}+\frac{R''}{R}-\frac{A'}{A}\left(\frac{B'}{B}-\frac{R'}{R}\right)-\frac{B'R'}{BR}\right]. \qquad (21)
\end{aligned}
$$

The Misner and Sharp mass function $m(t,r)$ with charge[31] is given by

$$
\begin{aligned}
m(t,r) &= \frac{R}{2}(1-g^{\alpha\beta}R_{,\alpha}R_{,\beta})\\
&= \frac{R}{2}\left(1+\frac{\dot{R}^2}{A^2}-\frac{R'^2}{B^2}\right)+\frac{q^2}{2R}. \qquad (22)
\end{aligned}
$$

The conservation equation, $(T^{-\alpha\beta})_{;\beta}=0$, yields

$$
\dot{\rho}+(\rho+p)\left(\frac{\dot{B}}{B}+2\frac{\dot{R}}{R}\right)+2\eta\sigma\left(\frac{\dot{B}}{B}-\frac{\dot{R}}{R}\right)=0, \qquad (23)
$$

$$
\begin{aligned}
&p'+2\eta\sigma'+(\rho+p)\frac{A'}{A}+2\eta\sigma\left(\frac{A'}{A}+3\frac{R'}{R}\right)\\
&-4\pi\frac{E}{R}(RE'+2R'E)=0. \qquad (24)
\end{aligned}
$$

2.2. Junction conditions

We connect M^- and M^+ metrics by considering the Darmois junction conditions. For the smooth matching of these geometries, it is required that the boundary is continuous and smooth. Thus the continuity of the first fundamental form of the metrics provides

$$
\left(\mathrm{d}s_-^2\right)_\Sigma=\left(\mathrm{d}s_+^2\right)_\Sigma=\left(\mathrm{d}s^2\right)_\Sigma, \qquad (25)
$$

and the continuity of the second fundamental form of the extrinsic curvature gives

$$
K_{ij}^+=K_{ij}^-,\quad (i,j=0,2,3). \qquad (26)
$$

Considering the interior and the exterior space–times with Eq. (25), we have

$$
\frac{\mathrm{d}t}{\mathrm{d}\tau}=A(t,r_\Sigma)^{-1},\quad R(t,r_\Sigma)=r_\Sigma(v), \qquad (27)
$$

$$
\left(\frac{\mathrm{d}v}{\mathrm{d}\tau}\right)^{-2}=1-\frac{2M}{r_\Sigma}+\frac{Q^2}{r_\Sigma^2}+2\frac{\mathrm{d}r_\Sigma}{\mathrm{d}v}. \qquad (28)
$$

From Eq. (26), the non-null components of the extrinsic curvature turn out to be

$$
K_{00}^-=-\left[\frac{A'}{AB}\right]_\Sigma,\quad K_{22}^-=\left[\frac{RR'}{B}\right]_\Sigma,\quad K_{33}^-=K_{22}^-\sin^2\theta, \qquad (29)
$$

$$
K_{00}^+=\left[\left(\frac{\mathrm{d}^2v}{\mathrm{d}\tau^2}\right)\left(\frac{\mathrm{d}v}{\mathrm{d}\tau}\right)^{-1}-\left(\frac{\mathrm{d}v}{\mathrm{d}\tau}\right)\left(\frac{M}{r^2}-\frac{Q^2}{r^3}\right)\right]_\Sigma, \qquad (30)
$$

$$
K_{22}^+=\left[\left(\frac{\mathrm{d}v}{\mathrm{d}\tau}\right)\left(1-\frac{2M}{r}-\frac{Q^2}{r^2}\right)r+\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)r\right]_\Sigma, \qquad (31)
$$

$$
K_{33}^+=K_{22}^+\sin^2\theta. \qquad (32)
$$

Using Eqs. (27)–(31) and the field equations, we obtain

$$
m\overset{\Sigma}{=}M,\quad p+2\eta\sigma\overset{\Sigma}{=}0, \qquad (33)
$$

where $q(r)=Q$ has been used. The above equation shows that across the boundary Σ, the masses of interior and exterior space–times are matched and the momentum flux is conserved.

3. Perturbation scheme and collapse equation

Here we construct the collapse equation. For this purpose, we expand the field equations, the dynamical equations, and the mass function to the first order in ε by using the perturbation scheme.[24,36] We assume that initially all the physical functions and the metric coefficients depend on r, i.e., the fluid is unperturbed. Afterwards, all these quantities depend on the time coordinate, which are given by

$$
\begin{aligned}
A(t,r) &= A_0(r)+\varepsilon T(t)a(r), \qquad (34)\\
B(t,r) &= B_0(r)+\varepsilon T(t)b(r), \qquad (35)\\
R(t,r) &= rB(t,r)[1+\varepsilon T(t)\bar{c}(r)], \qquad (36)\\
E(t,r) &= E_0(r)+\varepsilon T(t)e(r), \qquad (37)\\
\rho(t,r) &= \rho_0(r)+\varepsilon\bar{\rho}(t,r), \qquad (38)\\
p(t,r) &= p_0(r)+\varepsilon\bar{p}(t,r), \qquad (39)\\
\sigma(t,r) &= \varepsilon\bar{\sigma}(t,r), \qquad (40)\\
m(t,r) &= m_0(r)+\varepsilon\bar{m}(t,r), \qquad (41)
\end{aligned}
$$

where $0<\varepsilon\ll 1$. For $\bar{c}=0$, we have a shear-free metric. Using this scheme, the static configuration of Eqs. (18)–(21) is written as

$$
\begin{aligned}
&8\pi\left(\rho_0+2\pi E_0^2\right)\\
&=-\frac{1}{B_0^2}\left[2\frac{B_0''}{B_0}-\left(\frac{B_0'}{B_0}\right)^2+\frac{4}{r}\frac{B_0'}{B_0}\right], \qquad (42)\\
&8\pi\left(p_0-2\pi E_0^2\right)\\
&=\frac{1}{B_0^2}\left[\left(\frac{B_0'}{B_0}\right)^2+\frac{2}{r}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)+2\frac{A_0'}{A_0}\frac{B_0'}{B_0}\right], \qquad (43)\\
&8\pi\left(p_0+2\pi E_0^2\right)\\
&=\frac{1}{B_0^2}\left[\frac{A_0''}{A_0}+\frac{B_0''}{B_0}+\frac{1}{r}\left(\frac{A_0'}{A_0}+\frac{B_0'}{B_0}\right)-\left(\frac{B_0'}{B_0}\right)^2\right]. \qquad (44)
\end{aligned}
$$

The corresponding perturbed quantities up to the first order in ε with Eqs. (34)–(40) become

$$
\begin{aligned}
&8\pi\bar{\rho}+32\pi^2E_0Te\\
&=-\frac{T}{B_0^3}\left\{b\left(\frac{B_0'}{B_0}\right)^2-2b'\left(\frac{B_0'}{B_0}-\frac{2}{r}\right)+2b''+2B_0\right.\\
&\quad\left.\times\left[\bar{c}''+\bar{c}'\left(2\frac{B_0'}{B_0}+\frac{3}{r}\right)+\left(\frac{\bar{c}}{r^2}\right)\right]\right\}\\
&\quad-24\pi\frac{Tb}{B_0}\left(\rho_0+2\pi E_0^2\right), \qquad (45)
\end{aligned}
$$

$$0 = 2\left[\left(\frac{b}{A_0B_0}\right)' + \left(\frac{\bar{c}}{A_0}\right)' + \left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{\bar{c}}{A_0}\right)\right]\dot{T}, \quad (46)$$

$$\begin{aligned} &8\pi(\bar{p}+2\eta\bar{\sigma}) - 32\pi^2E_0Te \\ &= -\frac{2b}{A_0^2B_0}\ddot{T} + \frac{2T}{B_0^2}\left[\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{a}{B_0}\right)'\right. \\ &+ \left(\frac{b}{B_0}+\bar{c}\right)'\left(\frac{A_0'}{A_0}+\frac{1}{r}+\frac{B_0'}{B_0}\right)+\frac{\bar{c}}{r^2} \\ &\left. - \bar{c}\left(\frac{B_0}{A_0}\right)^2\frac{\ddot{T}}{T}\right] - 16\pi\frac{Tb}{B_0}\left(p_0-2\pi E_0^2\right), \end{aligned} \quad (47)$$

$$\begin{aligned} &8\pi(\bar{p}-\eta\bar{\sigma}) + 32\pi^2E_0Te \\ &= -\frac{\ddot{T}}{A_0^2}\left(2\frac{b}{B_0}+\bar{c}\right) + \frac{T}{B_0^2}\left[\frac{a''}{A_0}+\frac{b''}{B_0}+\bar{c}''\right. \\ &+ \frac{A_0''}{A_0}\left(\frac{a}{A_0}-\frac{b}{B_0}\right)+\frac{b'}{B_0}\left(\frac{1}{r}-\frac{B_0'}{B_0}\right) \\ &+ \frac{B_0'}{B_0}\left(\bar{c}-\frac{b}{B_0}\right)'+\frac{A_0'}{A_0}\left(\frac{b}{rB_0}+\bar{c}'\right) \\ &\left.+ \frac{1}{r}\left(\frac{a}{A_0}+2\bar{c}\right)'\right] - 24\pi\frac{Tb}{B_0}\left(p_0+2\pi E_0^2\right). \end{aligned} \quad (48)$$

The static configuration for dynamical Eq. (23) is trivially satisfied, while Eq. (24) implies

$$\begin{aligned} &p_0' + (\rho_0+p_0)\frac{A_0'}{A_0} \\ &- 4\pi E_0\left[E_0' + 2E_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\right] = 0, \end{aligned} \quad (49)$$

which yields

$$\frac{A_0'}{A_0} = -\frac{1}{\rho_0+p_0}\left\{p_0' - 4\pi E_0\left[E_0' - 2E_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\right]\right\}. \quad (50)$$

The perturbed part of Eq. (23) becomes

$$\dot{\bar{p}} + (\rho_0+p_{r0})\left(3\frac{b}{B_0}+2\bar{c}\right)\dot{T} = 0, \quad (51)$$

which through integration leads to

$$\bar{p} = -(\rho_0+p_0)\left(3\frac{b}{B_0}+2\bar{c}\right)T. \quad (52)$$

For the second dynamical equation, we have

$$\begin{aligned} &\bar{p}' + 2\eta\bar{\sigma}' + (\bar{p}+\bar{p})\frac{A_0'}{A_0} + (\rho_0+p_{r0})T\left(\frac{a}{A_0}\right)' \\ &+ 2\eta\sigma\left(\frac{A_0'}{A_0}+\frac{3}{r}+\frac{3B_0'}{B_0}\right) \\ &- 4\pi T\left[(eE_0)' + 4eE_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\right. \\ &\left.+ 2E_0^2\left(\frac{b}{B_0}\right)' + 2E_0^2\bar{c}'\right] = 0. \end{aligned} \quad (53)$$

The unperturbed and the perturbed configurations of the electromagnetic field are given by

$$E_0(r) = \frac{q}{r^2B_0^2}, \quad e(r) = -\left(\frac{2b}{B_0}+\bar{c}\right)E_0. \quad (54)$$

This shows that $e(r)$ depends on the static configuration of the electromagnetic field. Similarly, we can write the static and the perturbed configurations for Eq. (22) as

$$m_0 = -\frac{rB_0}{2}\left(2r\frac{B_0'}{B_0}+\left(r\frac{B_0'}{B_0}\right)^2\right) + 8\pi^2r^3E_0^2B_0^3, \quad (55)$$

$$\begin{aligned} \bar{m} = &-\left[r^2b' + r^3\frac{B_0'^2}{2B_0}\left(2\frac{b'}{B_0'}-\frac{b}{B_0}\right)\right]T \\ &- \bar{c}'r^3B_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)T \\ &- \bar{c}r^2B_0\left[\frac{3}{2}r\left(\frac{B_0'}{B_0}\right)^2+3\frac{B_0'}{B_0}+\frac{1}{r}\right]T \\ &+ 24\pi^2r^3B^2E_0^2\left(b+B_0\bar{c}\right)T. \end{aligned} \quad (56)$$

From Eq. (55), we have

$$\frac{B_0'}{B_0} = -\frac{1}{r}+\frac{1}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}. \quad (57)$$

The perturbed configuration for the shear scalar is written as

$$\bar{\sigma} = \frac{1}{3A_0}\bar{c}\dot{T}. \quad (58)$$

The junction condition (33) with Eqs. (39) and (40) leads to

$$p_0 \overset{\Sigma}{=} 0, \quad \bar{p}+2\eta\bar{\sigma} \overset{\Sigma}{=} 0. \quad (59)$$

Substituting the above relations in Eq. (47), we get the temporal equation

$$\ddot{T}(t) - \psi(r)T(t) \overset{\Sigma}{=} 0, \quad (60)$$

where

$$\begin{aligned} \psi(r) \overset{\Sigma}{=} &\left(\frac{A_0}{B_0}\right)^2\left(\frac{b}{B_0}+\bar{c}\right)^{-1}\left[\left(\frac{A_0'}{A_0}+\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{b}{B_0}+\bar{c}\right)'\right. \\ &+ \left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{a}{A_0}\right)' + \frac{\bar{c}}{r^2} \\ &\left.+ 16\pi^2E_0B_0(eB_0+bE_0)\right]. \end{aligned} \quad (61)$$

The general solution of Eq. (60) yields

$$T(t) = c_1\exp(\sqrt{\psi_\Sigma}t) + c_2\exp(-\sqrt{\psi_\Sigma}t), \quad (62)$$

where c_1 and c_2 are arbitrary constants. This provides two independent solutions. Here we take ψ_Σ to be positive for physically meaningful results, i.e., when the system is in a static position, it starts collapsing at $t=-\infty$ when $T(-\infty)=0$ and goes on collapsing, diminishing its areal radius with the increase of t. The corresponding solution is found for $c_2=0$, too, we choose $c_1=-1$, hence

$$T(t) = -\exp(\sqrt{\psi_\Sigma}t). \quad (63)$$

Next, we are interested to find the instability range of the collapsing fluid in terms of adiabatic index Γ. Chan *et al.*[18]

found a relationship between $\bar{p}$ and $\bar{\rho}$ with an equation of state of the Harrison–Wheeler type[38] for the static configuration

$$\bar{p}=\Gamma\frac{p_0}{\rho_0+p_0}\bar{\rho}, \tag{64}$$

where Γ describes the change in pressure for a given change in density (taken to be constant in the whole fluid distribution). The above equation with Eq. (52) leads to

$$\bar{p}=-p_0\Gamma\left(\frac{3b}{B_0}+2\bar{c}\right)T. \tag{65}$$

Using the above equation and Eq. (59) in Eq. (47), we have

$$\begin{aligned}\left(\frac{a}{A_0}\right)'=&B_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)^{-1}\left[-4\pi\left(\frac{3b}{B_0}+2\bar{c}\right)\Gamma p_0B_0\right.\\&+8\pi p_0b-\frac{\bar{c}}{r^2B_0}-16\pi^2E_0(eB_0+bE_0)\\&-\frac{1}{B_0}\left(\frac{A_0'}{A_0}+\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{b}{B_0}+\bar{c}\right)'\\&\left.+\frac{B_0}{A_0^2}\left(\frac{b}{B_0}+\bar{c}\right)\frac{\ddot{T}}{T}+8\pi\eta B_0\frac{\bar{\sigma}}{T}\right].\end{aligned} \tag{66}$$

We can write the collapse equation by substituting Eqs. (52), (58), (65), and (66) in the perturbed configuration of the second dynamical equation

$$\begin{aligned}&-\left[p_0\Gamma\left(\frac{3b}{B_0}+2\bar{c}\right)\right]'T+\frac{2}{3A_0}\left(\eta\bar{c}'+\eta\bar{c}\frac{A_0'}{A_0}\right)\dot{T}\\&-\left(\frac{3b}{B_0}+2\bar{c}\right)T\left(\rho_0+p_0(1+\Gamma)\right)\frac{A_0'}{A_0}\\&+(\rho_0+p_0)TB_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)^{-1}\left[\frac{B_0}{A_0^2}\left(\frac{b}{B_0}+\bar{c}\right)\frac{\ddot{T}}{T}\right.\\&+8\pi p_0b+8\pi\eta B_0\frac{\bar{c}\dot{T}}{3A_0T}-16\pi^2E_0(eB_0+bE_0)\\&-4\pi\left(\frac{3b}{B_0}+2\bar{c}\right)\Gamma p_0B_0-\frac{1}{B_0}\left(\frac{A_0'}{A_0}+\frac{1}{r}+\frac{B_0'}{B_0}\right)\\&\left.\times\left(\frac{b}{B_0}+\bar{c}\right)'-\frac{\bar{c}}{r^2B_0}\right]+\frac{2}{3A_0}\left(\frac{A_0'}{A_0}+\frac{1}{r}+\frac{B_0'}{B_0}\right)\\&\times\eta\bar{c}\dot{T}-T\left[2E_0^2\left(\frac{b}{B_0}\right)'+(eE_0)'+2E_0^2\bar{c}'\right.\\&\left.+4\pi E_0\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\right]=0.\end{aligned} \tag{67}$$

4. Dynamical instability of charged viscous perturbation

This section deals with the dynamical instability of the charged viscous fluid in the Newtonian and the pN regimes. We can see from Eq. (36) that the effects of shear explicitly appear only on metric function $R(t,r)$. Using this fact, we can split Eq. (46) into two types of terms, one with shearing viscosity and the other without it. Thus, the first three terms in Eq. (46) are taken identically to be zero

$$\left(\frac{b}{A_0B_0}\right)'=0,\quad\left(\frac{\bar{c}}{A_0}\right)'+\left(\frac{1}{r}+\frac{B_0'}{B_0}\right)\left(\frac{\bar{c}}{A_0}\right)=0. \tag{68}$$

The first of the above equations provides

$$b=A_0B_0, \tag{69}$$

while the second one leads to

$$\begin{aligned}\bar{c}'=&-\bar{c}\left\{\left[\frac{p_0'}{\rho_0+p_0}+\frac{4\pi E_0}{\rho_0+p_0}\right.\right.\\&\left.\times\left(E_0'\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]\\&\left.+\frac{1}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right\}.\end{aligned} \tag{70}$$

Inserting Eqs. (50), (57), (63), (69), and (70) in the collapse equation, we have

$$\begin{aligned}&-(3A_0+2\bar{c})\Gamma p_0'+\Gamma p_0\left\{A_0\left[\frac{3p_0'}{\rho_0+p_0}\right.\right.\\&\left.-\frac{12\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}+\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]\\&+\frac{2p_0'\bar{c}}{\rho_0+p_0}+\frac{2\bar{c}}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\\&\left.-\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right\}\\&-(\rho_0+p_0+\Gamma p_0)(3A_0+2\bar{c})\left[\frac{p_0'}{\rho_0+p_0}\right.\\&\left.-\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]\\&+(\rho_0+p_0)B_0\left(\frac{1}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)^{-1}\\&\times\left\{\frac{8\pi}{3}\frac{B_0}{A_0}\eta\bar{c}\sqrt{\psi_\Sigma}-16\pi^2E_0(eB_0+bE_0)+8\pi p_0A_0B_0\right.\\&-12\pi\Gamma p_0A_0B_0+\left(\frac{B_0}{A_0}\right)\psi_\Sigma\\&+\left(\frac{A_0}{B_0}\right)\left[\frac{1}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}-\frac{p_0'}{\rho_0+p_0}\right.\\&\left.+\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]\\&\times\left[\frac{p_0'}{\rho_0+p_0}-\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\right.\right.\\&\left.\left.\times\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]+\frac{\bar{c}}{B_0}\left[\frac{p_0'}{\rho_0+p_0}\right.\\&\left.-\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]^2\\&\left.-\frac{\bar{c}}{B_0}\left(\frac{2m_0}{r^3B_0}-16\pi^2E_0^2B_0^2+8\pi B_0^2p_0\Gamma-\left(\frac{B_0}{A_0}\right)^2\psi_\Sigma\right)\right\}\\&+\frac{2}{3A_0r}\eta\bar{c}\sqrt{\psi_\Sigma}\left[2\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}-\frac{p_0'}{\rho_0+p_0}\right.\\&\left.+\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\right)\right]\\&-\left\{(eE_0)'+2(A_0+\bar{c})\left[-\frac{p_0'}{\rho_0+p_0}+\frac{4\pi E_0}{\rho_0+p_0}\left(E_0'\right.\right.\right.\end{aligned}$$

$$+\frac{2E_0}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\Bigg)\Bigg]E_0^2$$
$$+\frac{(4\pi E_0+2E_0^2\bar{c})}{r}\sqrt{1-\frac{2m_0}{rB_0}+16\pi^2r^2E_0^2B_0^2}\Bigg\}=0. \quad (71)$$

4.1. Newtonian limit

Now we explore the instability range by applying the Newtonian limit, i.e., $A_0=1$, $B_0=1$, $\rho_0\gg p_0$, and discarding the terms of the order m_0/r in Eq. (71). It follows

$$-3p_0'\Gamma+4p_0'+\frac{(1+8\pi^2r^2E_0^2)}{r}$$
$$\times\left(\frac{4}{3}\eta\bar{c}\sqrt{\psi_\Sigma}-32\pi E_0^2-4\pi E_0\right)$$
$$-16\pi E_0E_0'+(1-16\pi^2r^2E_0^2)r\rho_0\psi_\Sigma=0. \quad (72)$$

We can see from Eq. (58) that the perturbed configuration of the shear scalar depends on the velocity gradients. Also, it is known that the velocity of the particle for a self-gravitating star increases towards the center. This shows that $\bar{\sigma}$ is negative inside the body and zero on the boundary surface. It is worth noticing that for a collapsing body, $\dot{T}<0$ as $t\to-\infty$, thus Eq. (58) implies that $\bar{c}>0$. Considering the physical requirement, i.e., $p_0'<0$, and neglecting the relativistic terms like ρ_0/p_0, we have the instability condition (independent of the linear perturbation functions) for the charged viscous fluid

$$\Gamma<\frac{4}{3}-\frac{4}{9}\eta|\bar{c}|\frac{\sqrt{\psi_\Sigma}}{|p_0'|r}+\frac{16\pi E_0E_0'}{3|p_0'|}$$
$$+\frac{1+8\pi^2r^2E_0^2}{3|p_0'|r}(32\pi E_0^2+4\pi E_0). \quad (73)$$

Here the critical value $4/3$ corresponds to the spherical geometry and the Newtonian gravity. In fact, 4 in the numerator corresponds to the weight of the envelop in Newtonian mechanics varying as r^{-2}, which is distributed over the surface of a sphere yielding another r^{-2}. The denominator 3 corresponds to the volume of the sphere r^3. We note from the above equation that the electromagnetic field diminishes the impact of the shearing viscosity on the dynamical instability and makes the system unstable in the Newtonian approximation. We retain the Newtonian classical result, i.e., $\Gamma<4/3$, for the case of shear free or when the fluid is not charged viscous.

4.2. Post-Newtonian limit

In the pN limit, we consider $A_0=1-m_0/r$, $B_0=1+m_0/r$, and the relativistic corrections of the order m_0/r in Eq. (71), so we have

$$-(3+2\bar{c})p_0'\Gamma+(4+2\bar{c})p_0'+\frac{(1+8\pi^2r^2E_0^2)}{r}$$
$$\times\left[\frac{4}{3}\eta\bar{c}\sqrt{\psi_\Sigma}+4\pi E_0+2E_0^2\bar{c}-8\pi E_0^2(2+2\bar{c})\right]$$
$$+(1-16\pi^2r^2E_0^2)r\rho_0\Bigg[8\pi p_0+\psi_\Sigma-16\pi^2E_0(e+E_0)$$
$$+\frac{8\pi}{3}\eta\bar{c}\sqrt{\psi_\Sigma}\Bigg](4+2\bar{c})4\pi E_0E_0'-(eE_0)'$$
$$+16\pi^2r\bar{c}\rho_0E_0^2+\rho_0r\bar{c}\psi_\Sigma(1-8\pi^2r^2E_0^2)=0. \quad (74)$$

Here we apply the same procedure as that in the Newtonian limit and ignore terms with higher orders of m_0/r, we get the instability range at the pN limit as follows:

$$\Gamma<\frac{4}{3}-\frac{1}{3|p_0'|}\Bigg[\left(\frac{4}{3r}\eta|\bar{c}|\sqrt{\psi_\Sigma}+4\pi E_0+2E_0^2|\bar{c}|\right)$$
$$\times(1+8\pi^2r^2E_0^2)+16\pi^2E_0^2r\rho_0(|\bar{c}|+16\pi^2E_0r^2(e+E_0))$$
$$+2|\bar{c}|+\rho_0\psi_\Sigma r(1+|\bar{c}|)+\frac{8\pi}{3}\eta\rho_0r|\bar{c}|\sqrt{\psi_\Sigma}\Bigg]$$
$$+\frac{16\pi^2rE_0\rho_0}{3|p_0'|}\Bigg[\frac{8\pi}{3}\eta|\bar{c}|r^2E_0\sqrt{\psi_\Sigma}+e+E_0$$
$$+r^2E_0\psi_\Sigma\left(1+\frac{|\bar{c}|}{2}\right)\Bigg]+\frac{1}{3|p_0'|}\Bigg[8\pi\rho_0p_0r$$
$$+4\pi E_0E_0'(4+2|\bar{c}|)+(eE_0)'$$
$$+16\pi E_0^2(1+|\bar{c}|)\frac{(1+8\pi^2r^2E_0^2)}{r}\Bigg]. \quad (75)$$

This equation shows that the positive terms diminish the relativistic effects of the negative terms occurring from the shearing viscosity and the electromagnetic field at the pN approximation, making the fluid more unstable. It is mentioned here that the instability condition depends upon the static configuration of the system, as perturbed variables $e(r)$ and $b(r)$ depend on the static configurations given in Eqs. (54) and (69), respectively.

5. Conclusion

We have investigated the role of electromagnetic field on the instability conditions (73) and (75) in Newtonian and pN approximations. Pinheiro and Chan[39] found that the collapsing stars with huge amounts of charge ($Q\sim5.408\times10^{20}$ C) end up as Reissner–Nordström black holes. They concluded that the models with charge to mass ratio $Q/m_0\leq0.631$ would form black holes. Ernesto and Simeone[40] examined the stability of a charged thin shell and found that the Reissner–Nordström geometries for different values of charge either have an inner and outer event horizon or a naked singularity. This showed the relevance of charge to the evolution and the structural formation of astrophysical objects.

In general, the behavior of an electromagnetic field is always positive being the Coulomb's repulsion force. Sharif and Abbas[1] explored that due to the weak nature of the electromagnetic field, the end state of collapsing cylinder results in a charged black string. We can see from Eq. (73) that the shearing viscosity boosts the stability of the fluid, which is the consequence of the fact that the collapse with shear proceeds faster than that without shear. Whereas the electromagnetic field being a positive quantity diminishes the effects of the

shearing viscosity and makes the fluid unstable in the Newtonian approximation. This corresponds to the fact that charge delays the event horizon formation or even halts the complete contraction of the star.[12] The electromagnetic field has the same impact for Eq. (75) in the pN approximation. It is worth mentioning here that our results for the electromagnetic field are consistent with the results obtained in Ref. [36].

Finally, we would like to mention that we have made all the discussion on the hypersurface, where the areal radius is a constant. The solution of the temporal Eq. (60) includes oscillating and non-oscillating functions corresponding to stable and unstable systems. For the instability conditions, we have confined our interest in the non-oscillating ones. We have investigated the role of physical quantities in the onset of the dynamical instability of the fluid during the collapse, hence the instability conditions contain those terms that have radial dependences.

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