Several teleportation schemes of an arbitrary unknown multi-particle state via different quantum channels*

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We first provide four new schemes for two-party quantum teleportation of an arbitrary unknown multi-particle state by using three-, four-, and five-particle states as the quantum channel, respectively. The successful probability and fidelity of the four schemes reach 1. In the first two schemes, the receiver can only apply one of the unitary transformations to reconstruct the original state, making it easier for these two schemes to be directly realized. In the third and fourth schemes, the sender can preform Bell-state measurements instead of multipartite entanglement measurements of the existing similar schemes, which makes real experiments more suitable. It is found that the last three schemes may become tripartite controlled teleportation schemes of teleporting an arbitrary multi-particle state after a simple modification. Finally, we present a new scheme for three-party sharing an arbitrary unknown multi-particle state. In this scheme, the sender first shares three three-particle GHZ states with two agents. After setting up the secure quantum channel, an arbitrary unknown multi-particle state can be perfectly teleported if the sender performs three Bell-state measurements, and either of two receivers operates an appropriate unitary transformation to obtain the original state with the help of other receiver's three single-particle measurements. The successful probability and fidelity of this scheme also reach 1. It is demonstrated that this scheme can be generalized easily to the case of sharing an arbitrary unknown multi-particle state among several agents.

Keywords: quantum information, quantum teleportation, quantum state sharing, multi-particle state

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1. Introduction

Quantum teleportation, the disembodied transport of a quantum state based on the nonlocal properties of an entangled state resource, has been demonstrated to be one of the most peculiar and fascinating aspects of quantum information theory. After Bennett *et al*.[1] proposed the teleportation theory, quantum teleportation has been playing an important role in the rapidly evolving fields of quantum information and quantum communications. It may have applications in quantum cryptography,[2–4] quantum computing,[5,6] and quantum dense coding.[7] Moving beyond the teleportation of one-particle or two-particle unknown states, some previous studies have focused on the teleportation of a multi-particle state,[8–11] probabilistic teleportation,[12,13] and controlled teleportation.[14–17] In these studies, using the quantum entangled state as a quantum channel is the key physical resource in quantum teleportation, such as GHZ states,[18,19] W states,[20,21] graph states,[22] and non-maximally entangled states.[23,24] Recently, many papers have focused on the properties of multi-particle entangled states as quantum channels,[25–29] especially cluster states such as four-qubit states,[26] genuine five-qubit states,[15,27,28] and six-qubit states.[15–29] It is known that the n-qubit ($n > 3$) cluster state is maximally connected with better persistency than the GHZ state. Additionally, cluster states are robust against decoherence. Xiu *et al*.[30] proposed two schemes for teleporting an unknown one-particle state by a general W state as the quantum channel. In their first scheme, the receiver needs to introduce an auxiliary particle and perform two Von Neumann measurements. Very recently, explicit protocols for quantum information splitting of single- and two-qubit states using four- and five-partite cluster states have been illustrated by Muralidharan and Panigrahi.[31] In these schemes, the sender must perform four-particle measurements, which is difficult in current laboratories. Jiang *et al*. presented the protocols for teleportation of an unknown multi-particle state involving lots of spatially remote agents by using EPR channels or probabilistic channels.[32,33] However, results have not been reported about two- or three-partite teleportation of an arbitrary unknown multi-particle state by using a three-particle state, a cluster state, or a mixed state as the quantum channel. In this paper, we first propose four schemes to faithfully teleport an unknown multi-particle state using three-, four-, and five-particle entangled states as a quantum channel. In our schemes of teleportation using three-particle entangled state as a quantum channel, unlike Ref. [30], the receiver does not need to introduce an auxiliary particle, and applies only one of the unitary transformations to recover the original state. In our other schemes, it is shown that the three-particle state, as well as four- and five-particle cluster states, can be used to realize the teleportation of an unknown multi-particle state,

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where the sender needs to perform Bell-state measurements or single-particle measurements, making real experiments more suitable. Finally, as an example of multi-party teleportation of an arbitrary unknown multi-particle state, we present a new scheme for three-party teleportation of an arbitrary unknown n-particle state. In this scheme, the sender first shares three three-particle GHZ states with two agents, then performs three Bell-state measurements. Each of the two agents can recover successfully the original arbitrary unknown n-particle state with the help of the other agent's three single-particle measurements.

2. Two perfect teleportation schemes of an arbitrary unknown multi-particle state via three-particle state channels

Suppose that there is an arbitrary unknown state of particles $1,2,\ldots,n$ which has the following form:

$$|\varphi\rangle_{12\cdots n}=\sum_{m_n=0}^{1}\cdots\sum_{m_2=0}^{1}\sum_{m_1=0}^{1}k_{m_1m_2\cdots m_n}|m_1\rangle_1|m_2\rangle_2\cdots|m_n\rangle_n,$$

where all $k_{m_1m_2\cdots m_n}$ are complex coefficients, and $\sum|k_{m_1m_2\cdots m_n}|^2=1$. Now we can write it as

$$\begin{aligned}|\varphi\rangle_{12\cdots n}=&\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_n\\&+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_n,\end{aligned}\tag{1}$$

where $\sum|a_{\{i\}}|^2+\sum|b_{\{i\}}|^2=1$, and $|\{i\}\rangle_{12\cdots(n-1)}(i\in\{0,1\})$ is the computational basis of particles $1,2,\ldots,n-1$. This arbitrary unknown quantum state $|\varphi\rangle_{12\cdots n}$ can also be expressed as

$$\begin{aligned}|\varphi\rangle_{12\cdots n}=&\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|00\rangle_{(n-1)n}\\&+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|01\rangle_{(n-1)n}\\&+\left(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|10\rangle_{(n-1)n}\\&+\left(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|11\rangle_{(n-1)n},\end{aligned}\tag{2}$$

with $\sum|a_{\{i\}}|^2+\sum|b_{\{i\}}|^2+\sum|c_{\{i\}}|^2+\sum|d_{\{i\}}|^2=1$, where $|\{i\}\rangle_{12\cdots(n-2)}$ $(i\in\{0,1\})$ is the computational basis of particles $1,2,\ldots,n-2$.

The arbitrary unknown quantum state $|\varphi\rangle_{12\cdots n}$ can be rewritten as follows:

$$\begin{aligned}|\varphi\rangle_{12\cdots n}=&\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|000\rangle_{(n-2)(n-1)n}\\&+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|001\rangle_{(n-2)(n-1)n}\\&+\left(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|010\rangle_{(n-2)(n-1)n}\\&+\left(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|011\rangle_{(n-2)(n-1)n}\\&+\left(\sum e_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|100\rangle_{(n-2)(n-1)n}\\&+\left(\sum f_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|101\rangle_{(n-2)(n-1)n}\\&+\left(\sum g_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|110\rangle_{(n-2)(n-1)n}\\&+\left(\sum h_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|111\rangle_{(n-2)(n-1)n},\end{aligned}\tag{3}$$

where

$$\begin{aligned}&\sum|a_{\{i\}}|^2+\sum|b_{\{i\}}|^2+\sum|c_{\{i\}}|^2+\sum|d_{\{i\}}|^2\\&+\sum|e_{\{i\}}|^2+\sum|f_{\{i\}}|^2+\sum|g_{\{i\}}|^2=1,\end{aligned}$$

and $|\{i\}\rangle_{12\cdots(n-3)}$ $(i\in\{0,1\})$ is the computational basis of particles $1,2,\ldots,n-3$.

We seek to teleport this multi-particle quantum state $|\varphi\rangle_{12\cdots n}$ of the particles from Alice's place to Bob's. Firstly, we assume that Alice and Bob share one three-particle general W state[34] as the quantum channel

$$|W\rangle_{\mathrm{BAA'}}=\left(\alpha|001\rangle+\beta|010\rangle+\frac{\sqrt{2}}{2}|100\rangle\right)_{\mathrm{BAA'}},\tag{4}$$

with $|\alpha|^2+|\beta|^2=1/2$, where particles A and A$'$ belong to the sender Alice, while particle B belongs to the receiver Bob. The initial quantum state of the whole system reads

$$\begin{aligned}|T\rangle_{\mathrm{t}}=&\,|\varphi\rangle_{12\cdots n}\otimes|W\rangle_{\mathrm{BAA'}}\\=&\left[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_n\right.\\&\left.+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_n\right]\\&\otimes(\alpha|001\rangle+\beta|010\rangle+\frac{\sqrt{2}}{2}|100\rangle)_{\mathrm{BAA'}}.\end{aligned}\tag{5}$$

In order to teleport the state $|\varphi\rangle_{12\cdots n}$, Alice makes a Von Neumann-type measurement on particles n, A and A$'$ using the states $\{|\lambda_j^{\pm}\rangle|j=1,2\}$, which are given by

$$\begin{aligned}|\lambda_1^{\pm}\rangle&=\alpha|001\rangle+\beta|010\rangle\pm\frac{\sqrt{2}}{2}|100\rangle,\\|\lambda_2^{\pm}\rangle&=\alpha|101\rangle+\beta|110\rangle\pm\frac{\sqrt{2}}{2}|000\rangle.\end{aligned}$$

Then, the state of particles $1,2,\ldots,n-1$ and B will collapse into the following states:

$$\begin{aligned}\langle\lambda_1^{\pm}|_{n\mathrm{AA'}}T\rangle_{\mathrm{t}}=&\,\frac{1}{2}\left[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\mathrm{B}}\right.\\&\left.\pm\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\mathrm{B}}\right],\\\langle\lambda_2^{\pm}|_{n\mathrm{AA'}}T\rangle_{\mathrm{t}}=&\,\frac{1}{2}\left[\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\mathrm{B}}\right.\end{aligned}$$

$$\pm\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}\Big].$$

After that, Alice informs Bob of the measurement results through a classical channel. Based on the received outcomes, Bob respectively applies the corresponding unitary operators $I_{\rm B}$, $Z_{\rm B}$, $X_{\rm B}$, or $iY_{\rm B}$ onto the particle B, where I is the identity, and X, Y, and Z are Pauli operators. As a consequence, by ignoring the global factor, the states of particles $1,2,\ldots,n-1$ and B become

$$\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\rm B}+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}. \quad (6)$$

Comparing state (6) with state (1), we clearly see that particle n held by Alice in the initial entangled system is now restored by particle B held by Bob. Thus, the unknown multi-particle quantum state can be faithfully transferred by only one qubit unitary operation on particles composed of the corresponding W channel, respectively. The successful probability and fidelity of this scheme both reach 1, i.e., it is perfect.

Now, we consider the other scheme where Alice transmits the state $|\varphi\rangle_{12\cdots n}$ to Bob using the following state as the quantum channel:

$$|\varphi'\rangle_{\rm ABA'}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{\rm AB}\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{\rm A'}, \quad (7)$$

where particles A and A$'$ belong to Alice, while B belong to Bob. From Eq. (1), the initial quantum state of the joint system can be described as

$$\begin{aligned}|T'\rangle_{\rm t} &= |\varphi\rangle_{12\cdots n}\otimes|\varphi'\rangle_{\rm ABA'}=\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_n\\ &+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_n\Big]\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{\rm AB}\\ &\otimes\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{\rm A'}. \quad (8)\end{aligned}$$

Alice performs a Bell-state measurement on particle pair $(n,{\rm A})$ with the basis set $\{|\phi^+\rangle,|\phi^-\rangle,|\psi^+\rangle,|\psi^-\rangle\}$, which forms a complete orthonormal basis of the combined Hilbert space of the three spin-1/2 particles

$$|\phi^\pm\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle),|\psi^\pm\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle).$$

and then operates a single measurement with the basis $\{|0\rangle,|1\rangle\}$ onto her own qubit A$'$, a collapsed sate of the retaining particles is one of the following states:

$$\begin{aligned}\langle 0|_{\rm A'}\langle\phi^\pm|_{n{\rm A}}T'\rangle_{\rm t} &= \frac{1}{2\sqrt{2}}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\rm B}\\ &\pm\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}\Big],\\ \langle 0|_{\rm A'}\langle\psi^\pm|_{n{\rm A}}T'\rangle_{\rm t} &= \frac{1}{2\sqrt{2}}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}\\ &\pm\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\rm B}\Big],\\ \langle 1|_{\rm A'}\langle\phi^\pm|_{n{\rm A}}T'\rangle_{\rm t} &= \frac{1}{2\sqrt{2}}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\rm B}\\ &\pm\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}\Big],\\ \langle 1|_{\rm A'}\langle\psi^\pm|_{n{\rm A}}T'\rangle_{\rm t} &= \frac{1}{2\sqrt{2}}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|1\rangle_{\rm B}\\ &\pm\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-1)}\right)|0\rangle_{\rm B}\Big].\end{aligned}$$

Alice then sends the outcomes of her measurements via classical channel to Bob who can apply one of the unitary transformations $\{I_{\rm B},Z_{\rm B},X_{\rm B},{\rm i}Y_{\rm B}\}$ to convert the state of his particle B to that of particle n. With this scheme, the unknown multi-particle entangled state can also be transmitted perfectly, and the successful probability and fidelity of this scheme both reach 1.

Remark 1 In this scheme, if particles A, B, and A$'$ respectively belong to Alice, Bob, and Charlie, Alice and Bob are still respectively sender and receiver, while Charlie is a controller. Alice performs a Bell-state measurement on particle pair $(n,{\rm A})$, and then Charlie makes a single measurement with the basis $\{|0\rangle,|1\rangle\}$ onto his own qubit A$'$. Based on the outcomes received, Bob can determine the state of particle B exactly by unitary transformation operators. Therefore, this scheme is changed into three-party quantum teleportation of an unknown multi-particle state.

3. Faithful teleportation of an arbitrary unknown multi-particle state by four-qubit cluster state

Now consider the following situation: Alice wants to teleport an unknown state $|\varphi\rangle_{12\cdots n}$ of entangled multi-particle state (given by Eq. (1)) to Bob using the following four-qubit cluster state as the quantum channel:

$$\begin{aligned}|\varphi\rangle_{\rm ABA'B'} &= |\phi^+\rangle_{\rm AB}\otimes|\phi^+\rangle_{\rm A'B'}\\ &= \frac{1}{2}(|0000\rangle+|0011\rangle+|1100\rangle+|1111\rangle)_{\rm ABA'B'}, \quad (9)\end{aligned}$$

where the particle pairs $({\rm A},{\rm A}')$ and $({\rm B},{\rm B}')$ belong to Alice and Bob, respectively. From Eq. (2), the initial state of the joint system can be described as

$$\begin{aligned}|\Phi\rangle_{\rm t} &= |\varphi\rangle_{12\cdots n}\otimes|\varphi\rangle_{\rm AA'BB'}\\ &= \Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|00\rangle_{(n-1)n}\\ &+\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|01\rangle_{(n-1)n}\\ &+\left(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|10\rangle_{(n-1)n}\\ &+\left(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\right)|11\rangle_{(n-1)n}\Big]\end{aligned}$$

$$\otimes\frac{1}{2}(|0000\rangle+|0011\rangle+|1100\rangle+|1111\rangle)_{\mathrm{ABA'B'}}.$$

In order to send the state, Alice performs two Bell-state measurements on particles $(n-1,\mathrm{A})$ and (n,A'), respectively. Then, the whole system's state collapses to one of the following states:

$$\begin{aligned}
\langle\phi^+|_{n\mathrm{A}'}\langle\phi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\Big],\\
\langle\phi^-|_{n\mathrm{A}'}\langle\phi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&-\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&\mp\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\Big],\\
\langle\psi^+|_{n\mathrm{A}'}\langle\phi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\Big],\\
\langle\psi^-|_{n\mathrm{A}'}\langle\phi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&-\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&\mp\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\Big],\\
\langle\phi^+|_{n\mathrm{A}'}\langle\psi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\Big],\\
\langle\phi^-|_{n\mathrm{A}'}\langle\psi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&-\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&\mp\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\Big],\\
\langle\psi^+|_{n\mathrm{A}'}\langle\psi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\Big],\\
\langle\psi^-|_{n\mathrm{A}'}\langle\psi^\pm|_{(n-1)\mathrm{A}}\Phi\rangle_{\mathrm{t}} &= \frac{1}{4}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&-\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&\pm\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}\\
&\mp\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\Big].
\end{aligned}$$

Afterwards, Alice informs Bob of the measurement outcomes through the classical channel. According to the results, Bob performs corresponding unitary transformations on his own particles B and B′. For example, if the outcome of Alice is $|\phi^-\rangle_{(n-1)\mathrm{A}}|\phi^+\rangle_{n\mathrm{A}'}$, Bob operates Z_{B} on particle B, the collapsed state and the retaining particles will be transformed into

$$\begin{aligned}
&\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|10\rangle_{\mathrm{BB}'}\\
&+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|11\rangle_{\mathrm{BB}'}\\
&+\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|00\rangle_{\mathrm{BB}'}\\
&+\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Big)|01\rangle_{\mathrm{BB}'}. \qquad (10)
\end{aligned}$$

Note that the states (2) and (10) of different particles have the same form. Compared with states (2) and (10), the position of the particle pair $(n-1,n)$ in the initial joint system is totally replaced by the particle pair (B,B') possessed by the remote agent Bob. Thus, the particles B and B′ take the same positions as the particles $n-1$ and n take. It means that Alice and Bob accomplish the teleportation successfully. Both the successful probability and fidelity of this scheme are one. The following corresponding relationship between the measurement outcomes and the unitary operators is shown in Table 1.

Remark 2 In this scheme, if particles A, B and B′, and A′ belong to Alice, Bob, and Charlie, respectively, the sender Alice wants Bob to restore the original state with Charlie's help. Alice and Charlie perform Bell-state measurements on particle pairs $(n-1,\mathrm{A})$ and (n,A'), respectively, and publish their measurement results. According to the outcomes received,

Bob can determine the state of particles B and B′ exactly by unitary transformation operators. Therefore, this scheme is also changed into the scheme for controlled quantum teleportation of an unknown multi-particle state.

Table 1. Relationship between Alice's measurements and the corresponding unitary operations.

Alice's results	Unitary operation by Bob
$\|\phi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\phi^{+}\rangle_{n\mathrm{A}'}$	I or Z_{B}
$\|\phi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\phi^{-}\rangle_{n\mathrm{A}'}$	$Z_{\mathrm{B}'}$ or $Z_{\mathrm{B}}Z_{\mathrm{B}'}$
$\|\phi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\psi^{+}\rangle_{n\mathrm{A}'}$	$X_{\mathrm{B}'}$ or $Z_{\mathrm{B}}X_{\mathrm{B}'}$
$\|\phi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\psi^{-}\rangle_{n\mathrm{A}'}$	$Z_{\mathrm{B}'}X_{\mathrm{B}'}$ or $Z_{\mathrm{B}}Z_{\mathrm{B}'}X_{\mathrm{B}'}$
$\|\psi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\phi^{+}\rangle_{n\mathrm{A}'}$	X_{B} or $Z_{\mathrm{B}}X_{\mathrm{B}}$
$\|\psi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\phi^{-}\rangle_{n\mathrm{A}'}$	$Z_{\mathrm{B}'}X_{\mathrm{B}}$ or $Z_{\mathrm{B}'}Z_{\mathrm{B}}X_{\mathrm{B}}$
$\|\psi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\psi^{+}\rangle_{n\mathrm{A}'}$	$X_{\mathrm{B}}X_{\mathrm{B}'}$ or $Z_{\mathrm{B}}X_{\mathrm{B}}X_{\mathrm{B}'}$
$\|\psi^{\pm}\rangle_{(n-1)\mathrm{A}}\|\psi^{-}\rangle_{n\mathrm{A}'}$	$Z_{\mathrm{B}'}X_{\mathrm{B}}X_{\mathrm{B}'}$ or $Z_{\mathrm{B}}Z_{\mathrm{B}'}X_{\mathrm{B}}X_{\mathrm{B}'}$

4. Perfect teleportation of an arbitrary unknown multi-particle state by five-qubit cluster state

Suppose that sender Alice wants to send the state $|\varphi\rangle_{12\cdots n}$ in Eq. (1) to agent Bob. The quantum channel which is shared by Alice and Bob is five-qubit cluster state

$$|\varphi\rangle_{\mathrm{A_1B_1A_2B_2A_3}} = \frac{1}{2}(|00000\rangle+|00111\rangle+|11101\rangle +|11010\rangle)_{\mathrm{A_1B_1A_2B_2A_3}}, \tag{11}$$

where A_i belongs to Alice, and B_i belongs to Bob. From Eq. (2), the initial state of this joint system state can be written as

$$\begin{aligned}
&|\Psi\rangle_{\mathrm{t}} = |\varphi\rangle_{12\cdots n}\otimes|\varphi\rangle_{\mathrm{A_1B_1A_2B_2A_3}}\\
&= \Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Bigg)|00\rangle_{(n-1)n}\\
&+\Bigg(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Bigg)|01\rangle_{(n-1)n}\\
&+\Bigg(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Bigg)|10\rangle_{(n-1)n}\\
&+\Bigg(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-2)}\Bigg)|11\rangle_{(n-1)n}\Bigg]\\
&\otimes\frac{1}{2}(|00000\rangle+|00111\rangle+|11101\rangle+|11010\rangle)_{\mathrm{A_1B_1A_2B_2A_3}}.
\end{aligned}$$

In order to realize the teleportation, Alice performs two Bell-state measurements on the particle pairs $(n-1,\mathrm{A}_1)$ and (n,A_3), respectively, then operates a single-particle measurement on her particle A_2 in the basis $\{|+\rangle,|-\rangle\}$, and she tells Bob her measurement outcomes via the classical channel. Here,

$$|+\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle),|-\rangle=\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle).$$

Then, the state of the retained particles collapses into one of the following states:

$$\begin{aligned}
&\langle+|_{\mathrm{A}_2}\langle\phi^{+}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle+\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\pm\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle-|_{\mathrm{A}_2}\langle\phi^{+}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle-\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\mp\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle+|_{\mathrm{A}_2}\langle\phi^{-}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle-\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\mp\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle-|_{\mathrm{A}_2}\langle\phi^{-}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle+\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\pm\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle+|_{\mathrm{A}_2}\langle\psi^{+}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle+\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\pm\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle-|_{\mathrm{A}_2}\langle\psi^{+}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=-\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle-\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\mp\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle+|_{\mathrm{A}_2}\langle\psi^{-}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle-\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\mp\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle-|_{\mathrm{A}_2}\langle\psi^{-}|_{n\mathrm{A}_3}\langle\phi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=-\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle+\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\pm\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle+|_{\mathrm{A}_2}\langle\phi^{+}|_{n\mathrm{A}_3}\langle\psi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle+\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle\\
&\pm\Bigg(\sum c_{\{i\}}|\{i\}\rangle\Bigg)|00\rangle\pm\Bigg(\sum d_{\{i\}}|\{i\}\rangle\Bigg)|01\rangle\Bigg]_{12\cdots(n-2)\mathrm{B_1B_2}},\\
&\langle-|_{\mathrm{A}_2}\langle\phi^{+}|_{n\mathrm{A}_3}\langle\psi^{\pm}|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}\\
&=\frac{1}{4\sqrt{2}}\Bigg[\Bigg(\sum a_{\{i\}}|\{i\}\rangle\Bigg)|11\rangle-\Bigg(\sum b_{\{i\}}|\{i\}\rangle\Bigg)|10\rangle
\end{aligned}$$

$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|00\rangle\mp\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|01\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle+|_{\mathrm{A}_2}\langle\phi^-|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|11\rangle-\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|10\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|00\rangle\mp\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|01\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle-|_{\mathrm{A}_2}\langle\phi^-|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|11\rangle+\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|10\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|00\rangle\pm\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|01\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle+|_{\mathrm{A}_2}\langle\psi^+|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|10\rangle+\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|11\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|01\rangle\pm\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|00\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle-|_{\mathrm{A}_2}\langle\psi^+|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=-\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|10\rangle-\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|11\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|01\rangle\mp\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|00\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle+|_{\mathrm{A}_2}\langle\psi^-|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|10\rangle-\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|11\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|01\rangle\mp\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|00\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2},$$

$$\langle-|_{\mathrm{A}_2}\langle\psi^-|_{n\mathrm{A}_3}\langle\psi^\pm|_{(n-1)\mathrm{A}_1}\Psi\rangle_{\mathrm{t}}$$

$$=-\frac{1}{4\sqrt{2}}\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle\Big)|10\rangle+\Big(\sum b_{\{i\}}|\{i\}\rangle\Big)|11\rangle$$
$$\pm\Big(\sum c_{\{i\}}|\{i\}\rangle\Big)|01\rangle\pm\Big(\sum d_{\{i\}}|\{i\}\rangle\Big)|00\rangle\Big]_{12\cdots(n-2)\mathrm{B}_1\mathrm{B}_2}.$$

After receiving Alice's classical messages, Bob performs corresponding unitary transformations on his particles B and B′. For example, if Alice's measurement result is $|\psi^-\rangle_{(n-1)\mathrm{A}_1}|\psi^-\rangle_{n\mathrm{A}_3}|-\rangle_{\mathrm{A}_2}$, Bob does the unitary operation $Z_{\mathrm{B}_1}X_{\mathrm{B}_1}$ on his particle B_1, and then performs a Control-Not (C) gate operation $C_{\mathrm{B}_1\mathrm{B}_2}$ on his particle B_1 as the control qubit and B_2 as the target qubit. Hence, Bob obtains the original quantum information successfully, and both the successful probability and fidelity of this quantum teleportation scheme are one. The relations between the measurement outcomes and the unitary operators is shown in Table 2.

Unlike Ref. [31], the sender Alice need not perform the four-particle measurement. Specially, the classical communication cost does not increase in our scheme.

Remark 3 When particles A_1 and A_2, B_1 and B_2, and A_3 belong to Alice, Bob, and Charlie, respectively, the sender Alice wants Bob to restore the original state when Bob obtains the help of Charlie. Alice performs Bell-state measurements on particle pairs $(n-1,\mathrm{A}_1)$ and (n,A_2), respectively, and then Charlie makes a single-particle measurement on the particle A_3. According to the outcomes received, Bob can determine the state of particles B_1 and B_2 exactly by unitary transformation operators. Thus, this scheme is also changed into the scheme of tripartite controlled teleportation of an unknown multi-particle state.

Table 2. Relationships between measurement outcomes and the corresponding unitary operations.

Alice's measuement outcomes	Unitary operation by Bob
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^+\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}I$ or $C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^+\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_2}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^-\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_2}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^-\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}I$ or $C_{\mathrm{B}_1\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^+\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^+\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_2}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^-\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_2}$
$\|\phi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^-\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^+\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^+\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^-\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\phi^-\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}X_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}X_{\mathrm{B}_2}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^+\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_1}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^+\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_2}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^-\rangle_{n\mathrm{A}_3}\|+\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_1}Z_{\mathrm{B}_2}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_2}$
$\|\psi^\pm\rangle_{(n-1)\mathrm{A}_1}\|\psi^-\rangle_{n\mathrm{A}_3}\|-\rangle_{\mathrm{A}_2}$	$C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}$ or $C_{\mathrm{B}_1\mathrm{B}_2}X_{\mathrm{B}_1}Z_{\mathrm{B}_1}$

5. Three-party teleportation of an arbitrary unknown multi-particle state via three GHZ states

Now we consider that the sender Alice wants to teleport the state $|\varphi\rangle_{12\cdots n}$ in Eq. (1) to her two agents Bob and Charlie using three three-qubit GHZ states

$$|\varphi\rangle_{\mathrm{ABC}}=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{ABC}},\tag{12}$$

$$|\varphi\rangle_{\mathrm{DEF}}=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{DEF}},\tag{13}$$

$$|\varphi\rangle_{\mathrm{GHI}}=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{GHI}},\tag{14}$$

where particles A, D, and G belong to Alice, while particles (B, E, H) and (C, F, I) belong to Bob and Charlie, respectively. From Eq. (15), the state of the joint system can be expressed as

$$\begin{aligned}|\Omega\rangle&=|\varphi\rangle_{12\cdots n}\otimes|\varphi\rangle_{\mathrm{ABC}}\otimes|\varphi\rangle_{\mathrm{DEF}}\otimes|\varphi\rangle_{\mathrm{GHI}}\\&=\Big[\Big(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\Big)|000\rangle_{(n-2)(n-1)n}\\&\quad+\Big(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\Big)|001\rangle_{(n-2)(n-1)n}\\&\quad+\Big(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\Big)|010\rangle_{(n-2)(n-1)n}\\&\quad+\Big(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\Big)|011\rangle_{(n-2)(n-1)n}\end{aligned}$$

$$
\begin{aligned}
&+\left(\sum e_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|100\rangle_{(n-2)(n-1)n}\\
&+\left(\sum f_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|101\rangle_{(n-2)(n-1)n}\\
&+\left(\sum h_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|110\rangle_{(n-2)(n-1)n}\\
&+\left(\sum g_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|111\rangle_{(n-2)(n-1)n}\Big]\\
&\otimes\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{ABC}}\otimes\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{DEF}}\\
&\otimes\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{\mathrm{GHI}}. \qquad (15)
\end{aligned}
$$

Subsequently, Alice performs three Bell-state measurements on three particle pairs $(n-2,\mathrm{A}),(n-1,\mathrm{D})$ and (n,G), respectively. Here, we assume that the outcomes obtained by Alice are $|\phi^+\rangle_{(n-2)\mathrm{A}}$, $|\phi^-\rangle_{(n-1)\mathrm{D}}$, and $|\phi^-\rangle_{n\mathrm{G}}$. Then, the collapsed state of $|\Omega\rangle$ can be described as

$$
\begin{aligned}
&\langle\phi^-|_{n\mathrm{G}}\langle\phi^-|_{(n-1)\mathrm{D}}\langle\phi^+|_{(n-2)\mathrm{A}}\Omega\rangle\\
&=\frac{1}{8}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|000000\rangle_{\mathrm{BCEFHI}}\\
&-\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|000011\rangle_{\mathrm{BCEFHI}}\\
&-\left(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|001100\rangle_{\mathrm{BCEFHI}}\\
&+\left(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|001111\rangle_{\mathrm{BCEFHI}}\\
&+\left(\sum e_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|110000\rangle_{\mathrm{BCEFHI}}\\
&-\left(\sum f_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|110011\rangle_{\mathrm{BCEFHI}}\\
&-\left(\sum h_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|111100\rangle_{\mathrm{BCEFHI}}\\
&+\left(\sum g_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|111111\rangle_{\mathrm{BCEFHI}}\Big].
\end{aligned}
$$

Furthermore, Alice informs Bob and Charlie of her measurement outcomes via a classical channel. Obviously, either Bob or Charlie has the chance to gain the original state $|\varphi\rangle_{12\cdots n}$ in Eq. (1). Without loss of generality, Bob is assigned to recover the original state, and Charlie is asked to make three single-qubit projective measurements on his particles C, F and I using the basis $\{|+\rangle,|-\rangle\}$. Here, we only consider that Charlie's measurement results are $|-\rangle_{\mathrm{C}},|-\rangle_{\mathrm{F}}$, and $|+\rangle_{\mathrm{I}}$. The other cases can be deduced similarly. Then,

$$
\begin{aligned}
&\langle+|_I\langle-|_F\langle-|_C\langle\phi^-|_{n\mathrm{G}}\langle\phi^-|_{(n-1)\mathrm{D}}\langle\phi^+|_{(n-2)\mathrm{A}}\Omega\rangle\\
&=\frac{1}{16\sqrt{2}}\Big[\left(\sum a_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|000\rangle_{\mathrm{BEH}}\\
&-\left(\sum b_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|001\rangle_{\mathrm{BEH}}\\
&+\left(\sum c_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|010\rangle_{\mathrm{BEH}}\\
&-\left(\sum d_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|011\rangle_{\mathrm{BEH}}\\
&-\left(\sum e_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|100\rangle_{\mathrm{BEH}}\\
&+\left(\sum f_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|101\rangle_{\mathrm{BEH}}\\
&-\left(\sum h_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|110\rangle_{\mathrm{BEH}}\\
&+\left(\sum g_{\{i\}}|\{i\}\rangle_{12\cdots(n-3)}\right)|111\rangle_{\mathrm{BEH}}\Big].
\end{aligned}
$$

After the measurements, Charlie tells his outcomes to Bob via a classical channel. To recover the arbitrary unknown original state (3), Bob only needs to perform a joint unitary transformation $Z_{\mathrm{B}}Z_{\mathrm{H}}$ on his particles B and H.

By the similar method, Bob can make the teleportation successfully in other outcomes of Alice's Bell-state measurements. For the sake of saving space, we will not write them out. The successful probability and fidelity of this scheme both reach 1.

Remark 4 In this scheme, we may generalize the method discussed above to the case of sharing an arbitrary unknown n-particle state $|\varphi\rangle_{12\cdots n}$ in Eq. (3) with several agents as follows: there are three channels

$$
|\varphi\rangle_{\mathrm{AB_1B_2\cdots B_s}}=\frac{1}{\sqrt{2}}(|00\cdots0\rangle+|11\cdots1\rangle)_{\mathrm{AB_1B_2\cdots B_s}}, \qquad (16)
$$

$$
|\varphi\rangle_{\mathrm{DE_1E_2\cdots E_k}}=\frac{1}{\sqrt{2}}(|00\cdots0\rangle+|11\cdots1\rangle)_{\mathrm{DE_1E_2\cdots E_k}}, \qquad (17)
$$

$$
|\varphi\rangle_{\mathrm{GH_1H_2\cdots H_m}}=\frac{1}{\sqrt{2}}(|00\cdots0\rangle+|11\cdots1\rangle)_{\mathrm{GH_1H_2\cdots H_m}}, \qquad (18)
$$

where $s,k,m\geq 3$. Without loss of generality, suppose $3\leq s\leq k\leq m$. The sender Alice shares the above three states with her agents: Bob_1, Bob_2, …, Bob_m. Here, particles A, D, and G belong to Alice, particles B_i, E_i, and H_i belong to Bob_i when $i\leq s$; particles E_i and H_i belong to Bob_i when $s<i\leq k$; H_i belong to Bob_i when $k<i\leq m$.

In order to realize quantum state sharing for an arbitrary unknown n-particle state $|\varphi\rangle_{12\cdots n}$ in Eq. (3), the sender Alice performs three Bell-state measurements on her particles (A, D, G), and informs all agents of her measurement results via a classical channel. From Eqs. (16)–(18), any one of Bob_1, Bob_2, …, Bob_s has a chance to recover the original state $|\varphi\rangle_{12\cdots n}$ in Eq. (3) with the help of other agents, but each one of Bob_{s+1}, Bob_{s+2}, …, Bob_m cannot reconstruct the original state $|\varphi\rangle_{12\cdots n}$ in Eq. (3) by only using his own particles. Assume that Bob_1 is assigned to recover the original state, and the other Bobs agree cooperation. That is, the agent Bob_1 is the receiver and all other agents are the controllers. During the stage of recovering the arbitrary unknown state $|\varphi\rangle_{12\cdots n}$ in Eq. (3), each of the first $(s-1)$ controllers needs to perform three single-particle measurements on their respective particles with the basis $\{|+\rangle,|-\rangle\}$. Any one of the last

$(m-k)$ controllers needs to operate a single-particle measurement, while each of the remaining $(k-s)$ controllers needs to perform two single-particle measurements on their respective particles with the basis $\{|+\rangle, |-\rangle\}$. Then, all controllers inform Bob_1 of their measurement results via classical channels. Finally, according to the measurement outcomes of Alice and all controllers, Bob_1 can recover the arbitrary unknown n-particle state $|\varphi\rangle_{12\cdots n}$ in Eq. (3) by applying the corresponding local unitary operations on his particles.

6. Conclusion

Although the protocols for teleportation and controlled teleportation of an arbitrary unknown multi-particle state by using EPR channels or nonmaximally entangled channels[32,33] have been proposed, results have not been reported about two- or three-partite teleportation of an arbitrary unknown multi-particle state by using a three-particle state, a cluster state, or a mixed state as a quantum channel. In this paper, we first present four schemes for two-partite teleportation of an arbitrary unknown multi-particle state by using three-, four-, and five-particle states as the quantum channel, respectively. It can be seen that the successful probability and fidelity of these schemes both reach 1. In the first scheme, if the sender chooses appropriate measurement basis, different from Ref. [30], the receiver needs not to introduce an auxiliary particle and to obtain a simple original state by applying one of the unitary transformations on his particle, and only four states are given. In fact, for complete measurement basis, they must have eight states. We only give four measurement states because the transformation operators of other four measurement basis are zero operators. In the other schemes, the sender performs Bell-sate measurements, and the teleportation can be successfully realized with maximal probability, which makes real experiments more suitable. Especially, the fourth scheme is more simple than those with four-particle joint measurement.[31] We find that the last three schemes can also be used for tripartite controlled teleportation of an unknown multi-particle state with just a little modification. Finally, we present a new scheme for three-party teleportation of an arbitrary unknown n-particle state. The results show that by using three three-particle GHZ states as quantum channels, three-party teleportation of an arbitrary unknown multi-particle state can be realized successfully with a probability of one and a fidelity of one. In this scheme, the sender performs three Bell-state measurements, and then informs two agents. Applying an appropriate unitary transformation, each of the two agents has a chance to recover the original arbitrary unknown n-particle state with the help of the other agent's three single-particle measurements. In addition, we generalize this scheme to the case of sharing an arbitrary unknown n-particle state among m agents. Our quantum resources are well-known entanglements which can be easily generated with model physical techniques, such as linear optics. Moreover, all the transformations and measurements in this paper are small qubit systems, which can be decomposed into one or two qubit gates. Therefore, our schemes are easily implemented with linear optics or atoms.

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