Mesoscopic entangled coherent states implemented with a circuit quantum electrodynamics system*

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We show a scheme to generate entangled coherent states in a circuit quantum electrodynamics system, which consists of a nanomechanical resonator, a superconducting Cooper-pair box (CPB), and a superconducting transmission line resonator. In the system, the CPB plays the role of a nonlinear medium and can be conveniently controlled by a gate voltage including direct-current and alternating-current components. The scheme provides a powerful tool for preparing the multipartite mesoscopic entangled coherent states.

Keywords: entanglement production and manipulation, superconducting devices

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1. Introduction

Entanglement is one of the most fascinating phenomena in quantum mechanics and plays a central role in quantum information processing.[1,2] Up to now, various theoretical and experimental schemes exploring the entanglement of coherent states have been extensively studied due to their important applications in different aspects, such as exploring some fundamental problems in quantum mechanics,[3] improving the weak-force detection,[4] and applications in quantum information processing.[5–7] Many of those schemes have been explored in various systems,[8,9] such as cavity quantum electrodynamics systems[10,11] and superconducting oscillators.[12,13] However, only a few schemes exploring entangled coherent states have been investigated in a circuit quantum electrodynamics (QED) system,[10,11] which has a great significance in experiments. In the circuit QED system, the extreme strong coupling limit in the cavity QED can be experimentally implemented. Moreover, the spontaneous emission in this system is strongly suppressed, and then the lifetime of qubits can be enhanced greatly, which allows high-fidelity quantum nondemonlition measurements of the multiple-qubit states.[13,14] So, the circuit QED[15,16] has been intensively studied and become a leading technology for solid-state-based quantum computation, and its performance is approaching the level realized in the trapped ion system[17] and all optical systems.[18] Considerable progress has been made in recent circuit QED schemes involving ultrastrong coupling, qubit gates, etc.[13–16] It is also attractive and significant to realize the mesoscopic entanglement among different types of states in this system.

In this paper, we explore the mesoscopic entanglement between two coherent states of resonators, and then extend the scheme to the multipartite case via considering the interaction among nanomechanical resonators (NAMRs),[14] superconducting Cooper-pair boxes (CPBs), and a superconducting transmission line resonator (STLR). The superconducting CPB of the system is controlled by a gate voltage with direct-current (DC) and alternating-current (AC) components. Our scheme has the following advantages. (i) The entangled state of the system can be conveniently prepared by just adjusting the parameters of the gate voltage. (ii) Both magnitude and frequency of the nonlinear coupling between the solid-state resonators can be easily controlled. (iii) The scheme can also be extended to study the multipartite entangled coherent states by adding an AC voltage, and it does not need qubit operations, so it avoids the restriction set by the decoherence time of the qubit. The paper is organized as follows. In Section 2, we introduce the model and the total Hamiltonian of the circuit QED system. In Section 3, we study how to produce entangled coherent states in the system, and then extend our scheme to the multipartite case in Section 4. Finally, a discussion and conclusion are presented in Section 5.

2. Model and Hamiltonian

We consider the circuit QED system shown in Fig. 1(b). In the system, an NAMR and an STLR are placed close to a CPB, which is coupled to a superconductor reservoir by two identical Josephson junctions with Josephson energy $E_{\rm J}$ and capacitance $C_{\rm J}$. The state of the CPB can be controlled by gate voltage $V_{\rm g}$ and gate capacitance $C_{\rm g}$. This obviously forms a superconducting quantum interference device (SQUID).[19,20] For the superconducting charge qubit, the capacitance $C_{\rm g}$ is much smaller than $C_{\rm J}$. In this regime, a convenient basis is

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formed by the charge states and characterized by the number of Cooper pairs in the CPB. In the neighborhood of $n_g = \frac{1}{2}$, only two charge states $|N\rangle$ and $|N+1\rangle$ are important, all the other charge states with higher energy can be ignored.[21] With an external flux Φ_e, the Hamiltonian of the CPB reads $H_q = -\frac{1}{2}E_c\left(1-2n_g\right)\rho_z - E_J\cos\left(\pi\Phi_e/\Phi_0\right)\rho_x$, where $E_c = 2e^2/C_\Sigma$ is the charging energy with $C_\Sigma = 2C_J + C_g$, $n_g = C_gV_g/(2e)$ is the gate charge, $\rho_z = |N\rangle\langle N| - |N+1\rangle\langle N+1|$, and $\rho_x = |N+1\rangle\langle N| + |N\rangle\langle N+1|$. At low temperatures, only one mode of the STLR couples to the CPB, and the quantum voltage applied on the gate capacitance will be $V_0(a+a^+)$, then the Hamiltonian of the combined system of STLR and CPB reads

$$\begin{aligned} H_1 &= -\frac{1}{2}E_c\left(1-2n_g\right)\rho_z - E_J\cos\left(\frac{\pi\Phi_e}{\Phi_0}\right)\rho_x \\ &\quad + \frac{eC_gV_0}{C_\Sigma}\left(a+a^+\right)\rho_z + \hbar\omega_a a^+a, \end{aligned} \tag{1}$$

where a (a^+) is the annihilation (creation) operator of the STLR. Further, we consider the coupling between the CPB and the NAMR, which is fabricated as a part of the SQUID. Due to the small displacement of the NAMR, the flux bias of the SQUID becomes $\Phi_e = \Phi_e^0 + BWx$, where Φ_e^0 is the flux bias corresponding to the equilibrium position of the NAMR, and $x = \sqrt{\hbar/(2M\omega_b)}\left(b+b^\dagger\right)$ is the displacement operator with M, ω_b being the mass and the frequency of the NAMR, respectively. Thus, the effective Josephson coupling energy of the SQUID will be $E_J(x) = -E_J\cos(\pi\Phi_e^0/\Phi_0 + \pi BWx/\Phi_0)$, and then the Hamiltonian of the total system including STLR, NAMR, and CPB is written as[19]

$$\begin{aligned} H &= -\frac{1}{2}E_c(1-2n_g)\rho_z - E_J\cos\left(\pi\frac{\Phi_e^0}{\Phi_0} + \frac{\pi BWx}{\Phi_0}\right)\rho_x \\ &\quad + \frac{eC_gV_0}{C_\Sigma}(a+a^+)\rho_z + \hbar\omega_a a^+a + \hbar\omega_b b^+b, \end{aligned} \tag{2}$$

where b (b^+) is the annihilation (creation) operator of the NAMR, ω_a and ω_b are the frequencies of the STLR and the NAMR, respectively.

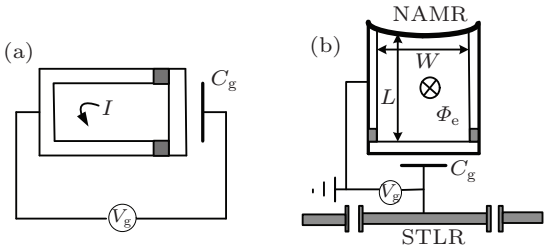


Fig. 1. (a) single CPB: a small superconducting island connected to a bulk superconductor via a tunnel junction, the island potential is controlled by the gate voltage V_g. (b) The circuit QED system consists of a NAMR, a superconducting CPB which can be manipulated by controlling the applied gate voltage $V_g = V_g' + V_{ac}$, and a STLR. The NAMR as a part of the SQUID is shown at the top through a magnetic field. The STLR is at the bottom.

3. Generation of entangled coherent states

In the case $\pi BWx/\Phi_0 \sim \pi\times10^{-4}$, the Josephson coupling energy $E_J(x) = -E_J\cos(\pi\Phi_e^0/\Phi_0 + \pi BWx/\Phi_0)$ in Eq. (2) can be expanded to the first order in x as $E_J(x) \approx -E_J\left[\cos(\pi\Phi_e^0/\Phi_0) - \pi BWx/\Phi_0\sin(\pi\Phi_e^0/\Phi_0)\right]$. By applying a gate voltage $V_g = V_g' + V_0\cos\omega_L t$ with DC component V_g' and AC component $V_{ac} = V_0\cos\omega_L t$, Hamiltonian (2) becomes

$$\begin{aligned} H &= \hbar\omega_a a^+a + \hbar\omega_b b^+b - \frac{1}{2}E_c(1-2n_g')\rho_z \\ &\quad + \frac{1}{2}E_c'V_0(e^{i\omega_L t} + e^{-i\omega_L t})\rho_z - E_J\left[\cos(\frac{\pi\Phi_e^0}{\Phi_0})\right]\rho_x \\ &\quad + \frac{E_J\pi BW}{\Phi_0}\sin\left(\frac{\pi\Phi_e^0}{\Phi_0}\right)\sqrt{\frac{\hbar}{2M\omega_b}}(b^+ + b)\rho_x \\ &\quad + \frac{eC_gV_0}{C_\Sigma}(a+a^+)\rho_z. \end{aligned} \tag{3}$$

By using $\alpha = \frac{1}{2}E_c(2n_g' - 1)$, $\beta = \frac{1}{2}E_c'V_0$, $\gamma = -E_J\cos(\pi\Phi_e^0/\Phi_0)$, $g_a = eC_gV_0/\hbar C_\Sigma$, and $g_b = E_J\pi BW/\hbar\Phi_0\sin(\pi\Phi_e^0/\Phi_0)\sqrt{\hbar/2M\omega_b}$, the total Hamiltonian of the system reads

$$\begin{aligned} H &= \hbar\omega_a a^+a + \hbar\omega_b b^+b + \alpha\rho_z + \beta(e^{i\omega_L t} + e^{-i\omega_L t})\rho_z \\ &\quad + \gamma\rho_x + \hbar g_a(a+a^+)\rho_z + \hbar g_b(b^+ + b)\rho_x. \end{aligned} \tag{4}$$

Choosing the eigenenergy basis $|g\rangle = \sin(\theta/2)|N+1\rangle + \cos(\theta/2)|N\rangle$, $|e\rangle = \cos(\theta/2)|N+1\rangle - \sin(\theta/2)|N\rangle$, where

$$\theta = \tan^{-1}\left[\frac{2E_J\cos(\pi\Phi_e^0/\Phi_0)}{E_c(1-2n_g')}\right]$$

is the rotation angle, and taking the rotating-wave approximation, we can reduce Hamiltonian (4) to

$$\begin{aligned} H &= \hbar\omega_a a^+a + \hbar\omega_b b^+b + \frac{1}{2}\hbar\Omega\sigma_z \\ &\quad + \hbar\omega_0(e^{-i\omega_L t}\sigma^+ + e^{i\omega_L t}\sigma^-) \\ &\quad + \hbar\kappa_a(a\sigma^+ + a^+\sigma^-) + \hbar\kappa_b(b\sigma^+ + b^+\sigma^-), \end{aligned} \tag{5}$$

where $\Omega = \sqrt{\left[E_c(1-2n_g')\right]^2 + \left[2E_J\cos(\pi\Phi_e^0/\Phi_0)\right]^2}/\hbar$, $\omega_0 = \beta\sin\theta/\hbar$, $\sigma^+ = |e\rangle\langle g|$, $\sigma^- = |g\rangle\langle e|$, $\sigma_z = |g\rangle\langle g| - |e\rangle\langle e|$, ω_L is the frequency of the driving field, $\kappa_a = -g_a\sin\theta$ and $\kappa_b = g_b\cos\theta$ are the qubit-STLR and the qubit-NAMR coupling coefficients, respectively.

In order to obtain the effective Hamiltonian, we can rewrite Eq. (5) as

$$\begin{aligned} H^L &= \frac{1}{2}\hbar\Delta\sigma_z + \hbar\delta_a a^+a + \hbar\delta_b b^+b \\ &\quad + \hbar\Big[\frac{1}{2}\omega_L\sigma_z + \omega_L a^+a + \omega_L b^+b \\ &\quad + \omega_0\left(e^{-i\omega_L t}\sigma^+ + e^{i\omega_L t}\sigma^-\right) \\ &\quad + \kappa_a(a\sigma^+ + a^+\sigma^-) + \kappa_b(b\sigma^+ + b^+\sigma^-)\Big], \end{aligned} \tag{6}$$

with $\Delta=\Omega-\omega_{\rm L}$, $\delta_a=\omega_a-\omega_{\rm L}$, and $\delta_b=\omega$. Let $\Delta=0$ and define

$$\begin{aligned}H^{\rm L} &= H_0^{\rm L}+H_1^{\rm L},\\ H_0^{\rm L} &= \hbar\left(\frac{1}{2}\omega_{\rm L}S_z+\omega_{\rm L}a^+a+\omega_{\rm L}b^+b\right),\\ H_1^{\rm L} &= \hbar\big[\delta_a a^+a+\delta_b b^+b+\omega_0\big(\mathrm{e}^{-\mathrm{i}\omega_{\rm L}t}\sigma^++\mathrm{e}^{\mathrm{i}\omega_{\rm L}t}\sigma^-\big)\\ &\quad+\kappa_a\big(a\sigma^++a^+\sigma^-\big)+\kappa_b\big(b\sigma^++b^+\sigma^-\big)\big],\end{aligned}\tag{7}$$

the Hamiltonian in the interaction picture will be

$$\begin{aligned}H^{\rm L_1} &= \hbar\big[\delta_a a^+a+\delta_b b^+b+\omega_0\left(\sigma^++\sigma^-\right)\\ &\quad+\kappa_a\left(a\sigma^++a^+\sigma^-\right)+\kappa_b\left(b\sigma^++b^+\sigma^-\right)\big].\end{aligned}\tag{8}$$

Now we choose basis $|+\rangle=\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)$, $|-\rangle=\frac{1}{\sqrt{2}}(|g\rangle-|e\rangle)$, then $H^{\rm L_1}$ can be written as

$$\begin{aligned}H^{\rm L_1} &= H_0^{\rm L_1}+H_1^{\rm L_1},\\ H_0^{\rm L_1} &= \hbar\left(\delta_a a^+a+\delta_b b^+b+\omega_0 S_z\right),\\ H_1^{\rm L_1} &= \frac{\hbar\kappa_a}{2}\left[S_z\left(a+a^+\right)+\left(\sigma^+-\sigma^-\right)\left(a-a^+\right)\right]\\ &\quad+\frac{\hbar\kappa_b}{2}\left[S_z\left(b+b^+\right)+\left(\sigma^+-\sigma^-\right)\left(b-b^+\right)\right],\end{aligned}\tag{9}$$

where $S_z=|+\rangle\langle+|-|-\rangle\langle-|$, $S^+=|+\rangle\langle-|$, and $S^-=|-\rangle\langle+|$. By using the unitary evolution operator $U(t)=\mathrm{e}^{-\mathrm{i}H_0^{\rm L_1}t}$, we can obtain the effective Hamiltonian

$$\begin{aligned}H_{\rm eff} &= \frac{\hbar}{2}\big[\big(S_z+\mathrm{e}^{2\mathrm{i}\omega_0 t}\sigma^+-\mathrm{e}^{-2\mathrm{i}\omega_0 t}\sigma^-\big)\\ &\quad\times\left(\kappa_a a\,\mathrm{e}^{-\mathrm{i}\delta_a t}+\kappa_b b\,\mathrm{e}^{-\mathrm{i}\delta_b t}\right)+\mathrm{H.c}\big].\end{aligned}\tag{10}$$

By assuming $\omega_0\gg\{\kappa_a,\kappa_b,\delta_a,\delta_b\}$, the terms that oscillate fast can be neglected. Then equation (10) reduces to

$$\begin{aligned}H_{\rm eff} &= \frac{\hbar}{2}S_z\big[\kappa_a\left(a\mathrm{e}^{-\mathrm{i}\delta_a t}+a^+\mathrm{e}^{\mathrm{i}\delta_a t}\right)\\ &\quad+\kappa_b\left(b\mathrm{e}^{-\mathrm{i}\delta_b t}+b^+\mathrm{e}^{\mathrm{i}\delta_b t}\right)\big]=H_{\rm eff}^1+H_{\rm eff}^2,\end{aligned}\tag{11}$$

$$H_{\rm eff}^1 = \frac{\hbar}{2}S_z\kappa_a\left(a\mathrm{e}^{-\mathrm{i}\delta_a t}+a^+\mathrm{e}^{\mathrm{i}\delta_a t}\right),\tag{12}$$

$$H_{\rm eff}^2 = \frac{\hbar}{2}S_z\kappa_b\left(b\mathrm{e}^{-\mathrm{i}\delta_b t}+b^+\mathrm{e}^{\mathrm{i}\delta_b t}\right).\tag{13}$$

The evolution operator for Hamiltonian $H_{\rm eff}$ can be written as $U(t)=\mathrm{e}^{-\mathrm{i}H_{\rm eff}t/\hbar}=U_a(t)U_b(t)$, with $U_a(t)=\mathrm{e}^{-\mathrm{i}H_{\rm eff}^1t/\hbar}$ and $U_b(t)=\mathrm{e}^{-\mathrm{i}H_{\rm eff}^2t/\hbar}$. Obviously, we have $U_a(t)|+\rangle|0\rangle_a=\exp\left[-\mathrm{i}\frac{\kappa_a t}{2}\left(a\mathrm{e}^{-\mathrm{i}\delta_a t}+a^+\mathrm{e}^{\mathrm{i}\delta_a t}\right)\right]|+\rangle|0\rangle_a=|+\rangle\left|-\mathrm{i}\frac{\kappa_a t}{2}\mathrm{e}^{\mathrm{i}\delta_a t}\right\rangle_a$, similarly, $U_a(t)|-\rangle|0\rangle_a=|-\rangle\left|\mathrm{i}\frac{\kappa_a t}{2}\mathrm{e}^{\mathrm{i}\delta_a t}\right\rangle_a$, $U_b(t)|+\rangle|0\rangle_b=|+\rangle\left|-\mathrm{i}\frac{\kappa_b t}{2}\mathrm{e}^{\mathrm{i}\delta_b t}\right\rangle_b$, and $U_b(t)|-\rangle|0\rangle_b=|-\rangle\left|\mathrm{i}\frac{\kappa_b t}{2}\mathrm{e}^{\mathrm{i}\delta_b t}\right\rangle_b$. If at $t=0$, with $\delta_a=\delta_b=0$, the total system is in the state $|g\rangle|0\rangle_a|0\rangle_b=\frac{1}{\sqrt{2}}(|+\rangle+|-\rangle)|0\rangle_a|0\rangle_b$, then the evolved state at time t will be

$$\begin{aligned}|\psi(t)\rangle &= U(t)|g\rangle|0\rangle_a|0\rangle_b\\ &= \frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_a|\beta\rangle_b+|-\rangle|-\alpha\rangle_a|-\beta\rangle_b),\end{aligned}\tag{14}$$

where $\alpha=-\mathrm{i}\kappa_a t/2$ and $\beta=-\mathrm{i}\kappa_b t/2$. The $|\alpha\rangle_a$ and $|\beta\rangle_b$ are the coherent states of the STLR and the NAMR resulted from the displacement operators, respectively. Obviously, the prepared state in Eq. (14) is an elaborate tripartite entangled state which consists of one microscopic component and two mesoscopic components. Returning to the Schrödinger picture and detecting the microscopic qubit with the basis $\{|g\rangle,|e\rangle\}$ and consuming $\omega_a=\omega_b=\omega$, we obtain the following bigger entangled coherent state:

$$\begin{aligned}|\psi\rangle_{\rm mBell} &= N_{a,b}^{\pm}\big(\mathrm{e}^{-\mathrm{i}2\omega_0 t}|\alpha\,\mathrm{e}^{-\mathrm{i}\omega t}\rangle_a|\beta\,\mathrm{e}^{-\mathrm{i}\omega t}\rangle_b\\ &\quad\pm\mathrm{e}^{\mathrm{i}2\omega_0 t}|-\alpha\,\mathrm{e}^{-\mathrm{i}\omega t}\rangle_a|-\beta\,\mathrm{e}^{-\mathrm{i}\omega t}\rangle_b\big),\end{aligned}\tag{15}$$

where $N_{a,b}^{\pm}$ are the normalization constants.

Moreover, if we set $\delta_a=\delta_b=\pm2\omega_0$ and $\kappa_a=\kappa_b=k$, then from Eq. (9) we can obtain the effective Hamiltonian of two-photon JC interactions in the dressed basis $|\pm\rangle$

$$H_{\rm TJC}=\frac{\hbar k}{2}\left[|+\rangle\langle-|(a+b)+|-\rangle\langle+|\left(a^++b^+\right)\right].$$

This Hamiltonian enables us to produce other kinds of interesting entangled states. For instance, if the system is initially in the state $|+\rangle|0\rangle_a|0\rangle_b$, then, after time interval t, the evolved state will be

$$\cos(\sqrt{2}kt)|+\rangle|0\rangle_a|0\rangle_b-\mathrm{i}\sin(\sqrt{2}kt)|-\rangle\frac{|0\rangle_a|1\rangle_b+|1\rangle_a|0\rangle_b}{\sqrt{2}}.$$

If the interaction time $\tau=\sqrt{2}\pi/4k$ is taken, the final state of the STLR and the NAMR is

$$\frac{1}{\sqrt{2}}(|0\rangle_a|1\rangle_b+|1\rangle_a|0\rangle_b),$$

where $|0\rangle$ and $|1\rangle$ are the Fock states. Obviously, it is a maximally entangled microscopic qubit state.

4. Multipartite coherent state entanglement

Next, we extend the above scheme to the multipartite case. The considered system is depicted in Fig. 2. The Hamiltonian of the total system reads

$$\begin{aligned}H &= \hbar\omega_a a^+a+\hbar\omega_b\sum_{j=1}^{N}b_j^+b_j+\frac{1}{2}\hbar\Omega\sum_{j=1}^{N}\sigma_{z,j}\\ &\quad+\hbar\omega_0\sum_{j=1}^{N}\left(\mathrm{e}^{-\mathrm{i}\omega_{\rm L}t}\sigma_j^++\mathrm{e}^{\mathrm{i}\omega_{\rm L}t}\sigma_j^-\right)\\ &\quad+\sum_{j=1}^{N}\hbar\kappa_{a,j}\left(a^+\sigma_j^-+a\sigma_j^+\right)\\ &\quad+\sum_{j=1}^{N}\hbar\kappa_{bj}\left(b_j^+\sigma_j^-+b_j\sigma_j^+\right),\end{aligned}\tag{16}$$

where b_j^+ (b_j) and a^+ (a) are the creation (annihilation) operators of the j-th NAMR and the STLR, respectively, $\kappa_{a,j}$ is the coupling constant between the j-th CPB and the STLR,

κ_{bj} is the coupling constant between the j-th CPB and the j-th NAMR, and ω_0 is the frequency of the driving field. In the interaction picture, choosing $\kappa_{a,j} = \kappa_{bj} = k_j$, we can similarly derive

$$H_{\rm I} = \frac{\hbar}{2}\sum_{j=1}^{N} k_j\big[(|+\rangle_{jj}\langle +| - |-\rangle_{jj}\langle -| + {\rm e}^{2{\rm i}\omega_0 t}|+\rangle_{jj}\langle -| - {\rm e}^{-2{\rm i}\omega_0 t}|-\rangle j\, j\langle +|\big) \times \big(a{\rm e}^{-{\rm i}\delta_a t} + b_j{\rm e}^{-{\rm i}\delta_b t}\big) + {\rm H.c}\big]. \quad (17)$$

Setting $\omega_0 \gg \{k_j, \delta_a, \delta_b\}$, we can acquire the effective Hamiltonian

$$H_{\rm eff} = \frac{\hbar}{2}\sum_{j=1}^{N} S_{z,j}k_j[(a{\rm e}^{-{\rm i}\delta_a t} + a^{+}{\rm e}^{{\rm i}\delta_a t}) + (b_j{\rm e}^{-{\rm i}\delta_b t} + b_j^{+}{\rm e}^{{\rm i}\delta_b t})]. \quad (18)$$

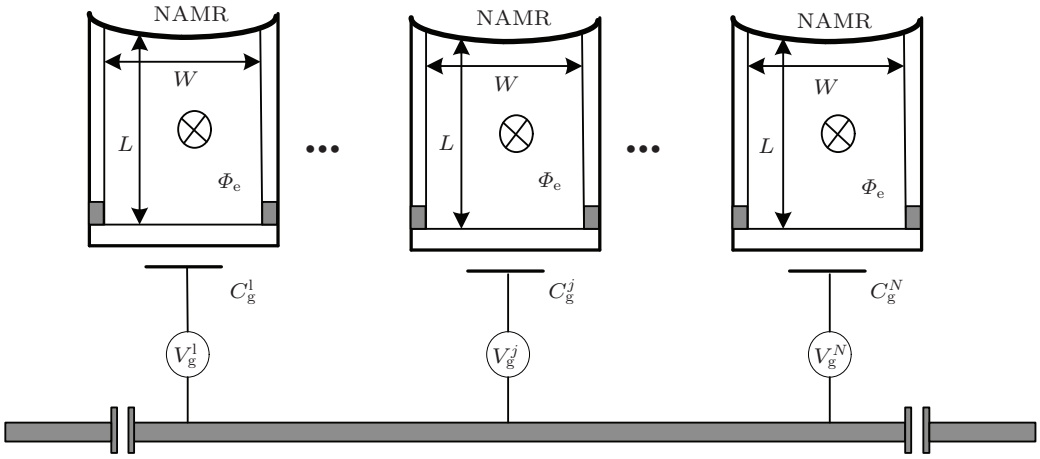


Fig. 2. The coupling system of N CPBs, N NAMRs (each of which is a part of a SQUID), and a STLR. The STLR shown at the bottom is coupled to each superconducting CPB through gate capacitance $C_{\rm g}^{j}$. Each CPB has an identical two-level energy structure, $|g\rangle$ and $|e\rangle$ are the ground and the exciting states, respectively.

Considering the case of $N = 2n+1$ with the initial state $\prod_j^{2n+1}|{\rm g}\rangle_j|0\rangle_a|0\rangle_{bj}$, in the schrödinger picture, we can obtain the macroscopic superposition state of $2(n+1)$-mesoscopic coherent state without Fock state $|0\rangle$ as

$$|\psi\rangle_{2n+1s} = N_{2n+1}\sum_{j=-n+1}^{n} C_{2n+1}^{n-j}{\rm e}^{-(2j+1)\omega_0 t}|(2j+1)\alpha{\rm e}^{{\rm i}\omega_a t}\rangle_a \times|(2j+1)\beta{\rm e}^{-{\rm i}\omega_b t}\rangle_{bj}, \quad (19)$$

where $C_{2n+1}^{n-j} = \frac{(2n+1)!}{(n-j)!(n+j+1)!}$. Then, we consider the case of $N = 2n$ with the initial state $\prod_j^{2n}|{\rm g}\rangle_j|0\rangle_a|0\rangle_{bj}$, another macroscopic superposition state with $2n$ mesoscopic component coherent states can also be acquired as

$$|\psi\rangle_{2ns} = N_{2n}\sum_{j=-n}^{n} C_{2n}^{n-j}{\rm e}^{-2j\omega_0 t}|2j\alpha{\rm e}^{{\rm i}\omega_a t}\rangle_a|2j\beta{\rm e}^{-{\rm i}\omega_b t}\rangle_{bj}, \quad (20)$$

where $C_{2n}^{n-j} = \frac{(2n)!}{(n-j)!(n+j)!}$. When N=1, we can naturally obtain the entangled coherent state

$$|\psi\rangle = N_{a,b}^{\pm}({\rm e}^{-{\rm i}2\omega_0 t}|\alpha{\rm e}^{-i\omega_a t}\rangle_a|\beta{\rm e}^{-{\rm i}\omega_b t}\rangle_b \pm {\rm e}^{{\rm i}2\omega_0 t}|-\alpha{\rm e}^{-{\rm i}\omega_a t}\rangle_a|-\beta{\rm e}^{-{\rm i}\omega_b t}\rangle_b).$$

5. Discussion and conclusion

It is notable that to generate entangled coherent states via a process with sufficiently strong coupling, the Josephson coupling energy is expanded to the first order in x as $E_{\rm J}(x) = -E_{\rm J}\left[\cos(\pi\Phi_{\rm e}^0/\Phi_0) - \sin(\pi\Phi_{\rm e}^0/\Phi_0)(\pi BWx/\Phi_0)\right]$. In order to estimate the corresponding coupling strength, we choose the typical parameters[13,16,22–24] in current solid-state circuit experiments as follows: $E_{\rm J}/\hbar = 2\pi\times 4$ GHz, $C_{\rm g}/C_\Sigma = 0.1$, $V_0 = 2$ μV, $W = 1$ μm, $B = 0.2$ T, $\omega_b = 2\pi\times 10$ GHz, the ratio $\Phi_{\rm e}^0/\Phi_0$ is about 10^{-5}, then the coupling strengths κ_a and κ_b are about $2\pi\times 48$ MHz and $2\pi\times 1.2$ MHz, respectively, which are workable in experiment.

Recently, the disadvantages of decoherence on coherent states have been shown in different systems, such as cavity QED. However, our scheme is different from the ones presented by some references such as Refs. [25]– [29]. In our scheme, no operations on the CPB are needed, and we do not use dissipation and measurement to generate the needed nonlinearity. We just need to adiabatically keep the CPB in its ground state. We get different coherent variables by controlling $n_{\rm g} > 1/2$ (or $n_{\rm g} < 1/2$) and the displacement operator respectively.

In summary, we propose a scheme to generate entangled coherent states via the circuit QED system including CPB, NAMR, and STLR. By controlling the external biased flux and the gate voltages of the CPB, we can get a nonlinear interaction between the CPB and the nanomechanical resonator (or the STLR). The scheme given here is significant for generat-

ing mesoscopic coherent entanglement,[30–34] which may be useful for future solid-state quantum computation and communication.

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