Scheme for implementing an economical phase-covariant quantum cloning machine of distant atomic qubits with single-photon interference

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By means of cavity-assisted photon interference, a simple scheme is proposed to implement a symmetric economical phase-covariant quantum cloning machine of two remote qubits, with each in a separate cavity. With our present scheme, a high-fidelity cloning machine is realized. Our scheme may be quite useful in terms of distributed quantum information processing.

Keywords: economical phase-covariant quantum cloning machine, distant atomic qubits, single-photon interference

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1. Introduction

The no-cloning theorem is one of the most fundamental differences between classical and quantum information. It claims that a completely unknown quantum state cannot be cloned exactly by any quantum process.[1] For a state which is selected randomly from a set of quantum states, Yuen[2] found that with merely unitary operations a device exists which would exactly clone the state if and only if those quantum states are mutually orthogonal; Duan and Guo[3,4] found that if a cloning machine performs measurements and unitary operations, the state which is selected randomly from a set of quantum states can be cloned exactly with certain probabilities even if those quantum states are mutually non-orthogonal. The phrase "certain probabilities" here implies that if the outcome is "successful", we gain the exact copies, and if the outcome "fails", we cannot gain the copies. Approximate cloning can be regarded as another orientation of quantum cloning development, since we cannot exactly clone a state which is selected randomly from a set of mutually non-orthogonal quantum states with merely unitary operations. Hence, in approximate cloning, we permit the difference between the input states and the output states, and expect the difference between them to be as small as possible. For approximate cloning, there are usually three types of optimal symmetric (copies obtained with equal fidelity) quantum cloning machines used to clone different states, including the universal quantum cloning machine (UQCM),[5] the real state cloning machine,[6] and the phase-covariant quantum cloning machine (PCCM).[7] In addition, in 2003, Fiurášek proposed an optimal economical phase-covariant quantum cloning machine (EPCCM)[8] for the input states on the southern and northern hemispheres of the Bloch sphere, which is a kind of approximate cloning machine and works without any ancilla.[9,10] Moreover, its fidelity is higher than that of the PCCM for an input state on the southern and northern hemispheres, while equal to that of the PCCM for an input state on the equator of the Bloch sphere. Some schemes have been proposed to realize those kinds of quantum cloning.[11–13]

At present, cavity quantum electrodynamics (QED) has already drawn much attention because it can provide strong coherent interactions between atoms and photons. The atomic systems are qualified to act as storing information qubits, and the photons are more suitable for use as flying qubits and can store quantum information for a very long time. Based on those advantages of atoms and photons, many schemes have been proposed to resolve the problems of quantum information.[14–17] In 2011, Fang *et al.* proposed a scheme to realize the UQCM and PCCM in separate cavities.[14] In 2012, they continued their work to propose a scheme for realizing the EPCCM in separate cavities.[18] Enlightened by their work, in this paper, we propose a scheme to implement an economical phase-covariant quantum cloning machine of distant atomic qubits with single-photon interference. Compared with the scheme for realizing EPCCM in separate cavities of Fang *et al.*, our scheme uses fewer atoms and photons. Qubits are a precious resource in practice, and any saving is a great relief for existing technologies.

2. The symmetric economical phase-covariant quantum cloning machine

In order to explain our scheme better, let us briefly review the EPCCM[8] proposed by Fiurášek. The input state to be

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cloned is written as

$$|\psi\rangle_{\rm in} = \cos\frac{\theta}{2}|0\rangle + \exp(i\varphi)\sin\frac{\theta}{2}|1\rangle, \quad (1)$$

where $\theta \in [0,\pi]$ and $\varphi \in [0,2\pi]$. The transformation of the symmetric EPCCM for the state on the northern hemisphere ($\theta \in [0,\pi/2]$) is defined as

$$\begin{aligned} &|00\rangle_{\rm AB} \to |00\rangle_{\rm AB}, \\ &|10\rangle_{\rm AB} \to \frac{1}{\sqrt{2}}(|10\rangle_{\rm AB} + |01\rangle_{\rm AB}), \end{aligned} \quad (2)$$

where A denotes the initial qubit to be cloned, and B the blank copy. The corresponding fidelity is

$$\begin{aligned} F^{(\rm N)} &= \frac{1}{2}\sin^2\frac{\theta}{2} + \cos^4\frac{\theta}{2} + \frac{\sqrt{2}}{4}\sin^2\theta \\ &= 0.854 + 0.25\cos\theta - 0.104\cos^2\theta. \end{aligned} \quad (3)$$

The EPCCM for the input state on the southern hemisphere ($\theta \in [\pi/2,\pi]$) is obtained by interchanging the states $|0\rangle$ and $|1\rangle$

$$\begin{aligned} &|00\rangle_{\rm AB} \to \frac{1}{\sqrt{2}}(|10\rangle_{\rm AB} + |01\rangle_{\rm AB}), \\ &|10\rangle_{\rm AB} \to |00\rangle_{\rm AB}. \end{aligned} \quad (4)$$

The fidelity is

$$\begin{aligned} F^{(\rm S)} &= \frac{1}{2}\cos^2\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{\sqrt{2}}{4}\sin^2\theta \\ &= 0.854 - 0.25\cos\theta - 0.104\cos^2\theta. \end{aligned} \quad (5)$$

It can be seen that both fidelities $F^{(\rm N)}$ and $F^{(\rm S)}$ are simply dependent on the parameter θ, and their values are higher than that of the PCCM for the inputs on the northern and southern hemispheres ($\theta \neq \pi/2$), while equal to that of the PCCM for an input on the equator ($\theta \neq \pi/2$).

3. Implementation of a symmetric economical phase-covariant quantum cloning machine of distant atomic qubits

The local controlled phase gate between two atoms is used in our scheme, So it is necessary to review the main idea for realizing the controlled phase gate.[19–21] Two atoms are confined in a cavity, each of which has a three-level configuration, as shown in Fig. 1(b). The levels $|0\rangle_j$ and $|e\rangle_j$ of the j-th atom are resonantly coupled to a cavity mode with horizontal polarization, while level $|1\rangle_j$ and $|e\rangle_j$ are decoupled for large detuning. In the case where the incoming photon is horizontal polarization and the atom is in the state $|1\rangle_j$, the photon will resonantly interact with the cavity with its pulse shape almost unchanged, but its phase is added by π. If the atom is in the state $|0\rangle_j$, the interaction between atom and cavity will shift the frequency of the cavity mode. Hence, the pulse will be reflected by an off-resonant cavity with both its shape and phase unchanged. When the incoming photon is vertical polarization, it will be reflected directly by the polarization beam splitter (PBS) and the mirror, so the controlled phase gate $U_{\rm phase} = \exp(i\pi|1\rangle\langle 1|h\rangle\langle h|)$ is obtained.

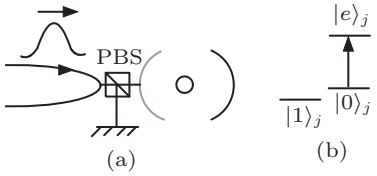


Fig. 1. (a) Schematic setup for implementing a two-atom controlled phase gate. (b) Level configuration of atom j.

Next, we report realization of the optimal EPCCM transformation for the input states on the northern and southern hemispheres of the Bloch sphere, respectively. In order to realize our scheme, both 22.5° and 45° tilted half-wave plates are needed, and we distinguish them by labeling them with open and solid rectangles, respectively. The action of the 22.5° tilted half-wave plate is given by $|h\rangle \to \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)$ and $|v\rangle \to \frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)$. The 45° tilted half-wave plate rotates the photon polarization as $|h\rangle \leftrightarrow |v\rangle$. The schematic setup for realizing the EPCCM on the northern hemisphere is shown in Fig. 2. The initial state of the logical qubits in cavities 1 and 2 is

$$\left(\cos\frac{\theta}{2}|0\rangle + \exp(i\varphi)\sin\frac{\theta}{2}|1\rangle\right)_1 |0\rangle_2, \quad (6)$$

where $\theta \in [0,\pi/2]$ and $\varphi \in [0,2\pi]$. A single-photon pulse in path 1 with horizontal polarization passes through a 22.5° tilted half-wave plate H_1 and a polarization beam splitter PBS_1. Then the photon will have two possible paths, i.e., path 2 and path 3, each with $1/2$ probability. After that, the state of the system is

$$\frac{1}{\sqrt{2}}\left[\cos\frac{\theta}{2}|0\rangle + \exp(i\varphi)\sin\frac{\theta}{2}|1\rangle\right]_1 |0\rangle_2(|h\rangle_2 + |v\rangle_3). \quad (7)$$

The photon in path 2 is reflected by the cavity after the controlled phase gate $U_{\rm phase}$ is realized with the help of a single-photon pulse with horizontal polarization, and then we obtain

$$\begin{aligned} &\frac{1}{\sqrt{2}}\left[\cos\frac{\theta}{2}|00\rangle_{12}|h\rangle_2 - \exp(i\varphi)\sin\frac{\theta}{2}|10\rangle_{12}|h\rangle_2\right. \\ &\left. + \cos\frac{\theta}{2}|00\rangle_{12}|v\rangle_3 + \exp(i\varphi)\sin\frac{\theta}{2}|10\rangle_{12}|v\rangle_3\right]. \end{aligned} \quad (8)$$

We let the photon in paths 2 and 3 be mixed at the PBS_2, and the photon in path 4 goes through H_2 and H_3. Then, we have

$$\cos\frac{\theta}{2}|00\rangle_{12}|h\rangle_4 + \exp(i\varphi)\sin\frac{\theta}{2}|10\rangle_{12}|v\rangle_4. \quad (9)$$

Actually, the above process realizes a controlled NOT gate with the qubit in cavity 1 as the control qubit, and with the

initial state $|h\rangle$ of the photon as the target qubit. During the above process, the photon may take two possible paths from PBS_1 to PBS_2. We have to make sure that the optical lengths of the two possible paths are equal, and we cannot distinguish which path the photon comes from.

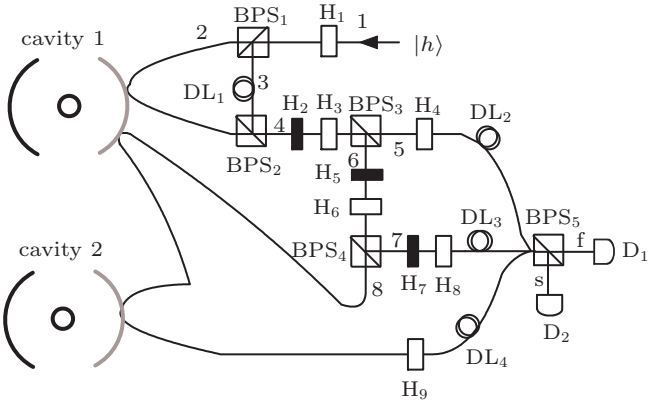


Fig. 2. Schematic setup for realizing the EPCCM on the northern hemisphere. H_1, H_3, H_4, H_6, H_8, and H_9 (open rectangles) represent 22.5° tilted half-wave plates. H_2, H_5, and H_7 (solid rectangles) represent 45° tilted half-wave plates. DL_1, DL_2, DL_3, and DL_4 denote the delay lines. D_1 and D_2 denote detectors.

The photon in path 4 passes through PBS_3, the photon in path 6 passes through H_5, H_6, and PBS_4, and the state of the system becomes

$$\cos\frac{\theta}{2}|00\rangle_{12}|h\rangle_5+\frac{1}{\sqrt{2}}\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|10\rangle_{12}(|h\rangle_8+|v\rangle_7). \quad (10)$$

The photon in path 8 enters cavities 1 and 2 one after another. Before the photon in path 8 enters the cavities 1 and 2, an operation H should be performed on atoms in cavities 1 and 2 by a laser illuminating atoms 1 and 2, respectively. After the interaction between the photon in path 8 with cavities 1 and 2, an operation H is performed on atoms 1 and 2 again. Actually, the above process realizes a local controlled NOT operation with the photon in path 8 as the control qubit and the atoms in cavities 1 and 2 as the target qubits. Then, we obtain

$$\begin{aligned}&\cos\frac{\theta}{2}|00\rangle_{12}|h\rangle_5\\&+\frac{1}{\sqrt{2}}\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}(|10\rangle_{12}|v\rangle_7+|01\rangle_{12}|h\rangle_8). \quad (11)\end{aligned}$$

Finally, the photon in path 5 is rotated by H_4, the photon in path 7 is rotated by H_7 and H_8, and the photon in path 8 is rotated by H_9. After that, as shown in Fig. 2, we mix paths 5, 7, and 8 at PBS_5.[19] The state of the system is written as

$$\begin{aligned}&\frac{1}{\sqrt{2}}\left[\cos\frac{\theta}{2}|00\rangle_{12}+\frac{1}{\sqrt{2}}\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}(|10\rangle_{12}+|01\rangle_{12}\right]|h\rangle_{\mathrm{f}}\\&+\frac{1}{\sqrt{2}}\left[\cos\frac{\theta}{2}|00\rangle_{12}\right.\\&\left.+\frac{1}{\sqrt{2}}\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}(|10\rangle_{12}+|01\rangle_{12}\right]|v\rangle_{\mathrm{s}}. \quad (12)\end{aligned}$$

During the above process, the photon may take three possible paths. The role of DL_2, DL_3, and DL_4 is to make sure that the photon from the three possible paths achieve PBS_5 simultaneously. At the output ports, we measure the photon state with photon detectors D_1 and D_2. No matter which detector detects a photon, the clone process is successful. The fidelity can be calculated easily using Eq. (3).

The optimal EPCCM transformation for the input state on the southern hemisphere of the Bloch sphere can be realized in the same way as the above process. The schematic setup for realizing the EPCCM on the southern hemisphere is shown in Fig. 3. The initial state of the qubits is

$$\left(\cos\frac{\theta}{2}|0\rangle+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|1\rangle\right)_1|0\rangle_2, \quad (13)$$

where $\theta\in[\pi/2,\pi]$ and $\varphi\in[0,2\pi]$. Before proceeding with the schematic, we first need to flip the state $|0\rangle_2$ to $|1\rangle_2$ of the qubit in cavity 2. It can be realized by performing σ_x on the atoms in cavity 2. Then, following the steps in Fig. 3, we have

$$\begin{aligned}&\mathrm{H_1},\ \mathrm{PBS_1},\ \mathrm{cavity1}(U_{\mathrm{phase}}),\ \mathrm{PBS_2},\ \mathrm{H_2},\ \mathrm{H_3}\rightarrow\\&\quad\cos\frac{\theta}{2}|01\rangle_{12}|h\rangle_4+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|11\rangle_{12}|v\rangle_4,\\&\mathrm{PBS_3},\ \mathrm{H_4},\ \mathrm{PBS_4}\rightarrow\\&\quad\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}(|01\rangle_{12}|h\rangle_7+|01\rangle_{12}|v\rangle_8)\\&\quad+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|11\rangle_{12}|v\rangle_6,\\&\mathrm{cavity1}(U_{\mathrm{flip}}),\ \mathrm{cavity2}(U_{\mathrm{flip}})\rightarrow\\&\quad\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}(|10\rangle_{12}|h\rangle_7+|01\rangle_{12}|v\rangle_8)\\&\quad+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|11\rangle_{12}|v\rangle_6,\\&\mathrm{H_5},\ \mathrm{H_6},\ \mathrm{H_7},\ \mathrm{H_8},\ \mathrm{H_9},\ \mathrm{PBS_5}\rightarrow\\&\quad\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}(|10\rangle_{12}+|01\rangle_{12})\right.\\&\quad\left.+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|11\rangle_{12}\right]|h\rangle_{\mathrm{f}}\\&\quad+\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}(|10\rangle_{12}+|01\rangle_{12})\right.\\&\quad\left.+\exp(\mathrm{i}\varphi)\sin\frac{\theta}{2}|11\rangle_{12}\right]|v\rangle_{\mathrm{s}}, \quad (14)\end{aligned}$$

where cavity $i(U_{\mathrm{flip}})$ is a controlled NOT gate with the photon as the control qubit, and the atom in cavity i as the target qubit, which can be realized similarly: before the photon enters the cavity i, an operation H should be performed on atoms in cavity i by a laser illuminating atom i. After the interaction between the photon with cavity i, an operation H is performed on atom i again. The detection stage is the same as before. The photon may take three possible paths, and we

also need to make sure that the photon from the three possible paths achieve PBS_5 simultaneously.[19] The fidelity can be calculated easily using Eq. (5). Thus, by detecting the photon, the optimal EPCCM transformations for the input states on the northern and southern hemispheres of the Bloch sphere are realized.

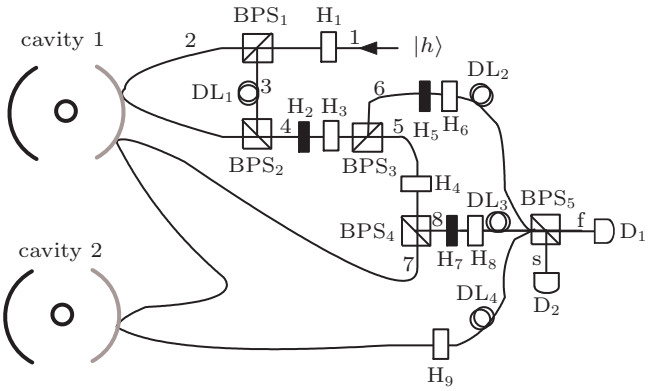


Fig. 3. Schematic setup for realizing the EPCCM on the southern hemisphere. H_1, H_3, H_4, H_6, H_8, and H_9 (open rectangles) are 22.5° tilted half-wave plates. H_2, H_5, and H_7 (solid rectangles) are 45° tilted half-wave plates. DL_1, DL_2, DL_3, and DL_4 denote delay lines. D_1 and D_2 denote detectors.

The basic building blocks U_{phase}[19–21] have been well studied previously by numerical simulations. The gate fidelity is almost unity and the spontaneous emission probability is quite small. In real experiments, there are other factors leading to photon loss in the middle stage, which would affect the subsequent interactions. However, the detectors at the end of the design can signal those photon loss event out. If the photon loss has happened, we have to restart the scheme. The photon detectors used in our scheme only need to distinguish the vacuum and non-vacuum states. This makes the scheme more realizable in experiments. On the other hand, the single-photon multi-path interference has been used, and a tremendous challenge is to maintain phase stability, which requires the path lengths to be subwavelength stable. Anyway, with the rapid development of single-photon experimental technology, our scheme may be realizable in the near future.

4. Summary

In summary, by means of the cavity-assisted photon scattering, we have proposed a simple scheme for the implementation of EPCCM between two remote logical qubits. With the existing linear optical schemes, we combine the advantages of photons and atoms, using the photon as the flying qubit to transmit the information between two separate nodes. Our scheme may have potential applications in distributed quantum information processing.

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