# Correlation dynamics of two-parameter qubit–qutrit states under decoherence*

Yuan Hao(袁 浩)[a] and Wei Lian-Fu(韦联福)[a)b)†]

[a] Laboratory of Quantum Optoelectronics, School of Physics and Technology, Southwest Jiaotong University, Chengdu 610031, China
[b] State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, China

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We investigate the dynamics of correlations for two-parameter qubit–qutrit states under various local decoherence channels including depahsing, phase-flip, bit- and trit-flip, bit- and trit-phase-flip, and depolarizing channels. We find that, under certain conditions, the classical correlations may not be affected by the noise or decay monotonically. The quantum correlations measured by measurement-induced disturbance (MID) show three types of dynamical behaviors: (i) monotonic decay to zero, (ii) monotonic decay to a nonzero steady value, (iii) increase from zero and then decrease to zero in a monotonic way. Consequently, we find that, differing from the dynamics of entanglement, the present classical and quantum correlations do not reveal sudden death behavior.

**Keywords:** classical correlations, measurement-induced disturbance, qubit–qutrit states, decoherence

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## 1. Introduction

Entanglement[1] is a unique property of quantum mechanics and plays a key role in quantum information processing. Previously, entanglement was considered as the quantumness of correlations in a quantum state. However, recent studies showed that entanglement cannot capture all the quantumness of correlations. A typical example is the separable mixed state that has vanishing entanglement but nonvanishing quantum correlations.[2] Such states can be utilized as a resource to implement deterministic quantum computation with one qubit (DQC1) both theoretically[2] and experimentally.[3] In fact, almost all quantum states have nonclassical correlations.[4] In this sense, quantum correlations are more fundamental than entanglement. Consequently, in recent years, much attention has been focused on how to quantify quantum correlations contained in a bipartite quantum state. So far, there are many methods to quantify quantumness of correlations, such as quantum discord (QD),[5–9] geometric measure of quantum discord (GMQD),[10,11] and measurement-induced disturbance (MID).[12]

The quantum correlations in qubit–qubit systems and their dynamical behaviors have been extensively investigated. It has been shown that three types of dynamical behaviors exist for the QD: monotonic decay,[13–15] a sudden change in the decay,[13] and stability for a finite time interval followed by a monotonic decay.[16,17] Aside from the above three types of dynamical behaviors,[17,19] the dynamical evolution of the GMQD exhibits another feature, i.e., two sudden change points exist in the relevant monotonic decay.[18]

As a natural generalization of the qubit–qubit correlations, some authors have discussed how to define the quantum correlations for the qubit–qudit system. For instance, Ali[20] introduced the QD for the qubit–qudit systems with the projective measurements on the qubit subsystem, and derived its analytical expressions for a certain class of states. Luo and Fu[11] obtained a bound to the GMQD for an arbitrary bipartite state in $(m \times n)$-dimensional quantum systems. Vinjanampathy and Rau[21] presented two analytical formulae to calculate the QD and GMQD for the extended $X$ states in qubit–qudit systems. The MID proposed by Luo[12] can also quantify the quantum correlations for an arbitrary bipartite state in $(m \times n)$-dimensional quantum systems. Moreover, the dynamics of classical and quantum correlations (QD and GMQD) for two special one-parameter qubit–qutrit states in independent common classical dephasing environments have been studied.[22] In this paper, we investigate the dynamical behaviors of classical and quantum correlations for a more general class of qubit–qutrit states, i.e., two-parameter states, undergoing various multi-local decoherence channels, i.e., dephasing, phase-flip, bit- and trit-flip, bit- and trit-phase-flip, and depolarizing channels. The amount of quantum correlations is measured by the MID.[12] We will show that the amount of classical correlations is unaffected under the dephasing and phase-flip channels, while decays monotonically under the other decoherence channels. The MID has three types of dynamical behaviors: (i) monotonic decay to zero under dephasing and phase-flip chan-

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†Corresponding author. E-mail: weilianfu@gmail.com

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nels, (ii) monotonic decay to a nonzero steady value under bit- and trit-flip, bit- and trit-phase-flip channels, and (iii) increasing from zero followed by decreasing to zero in the monotonic way under the depolarizing channels. One can also find that there is no sudden death of classical and quantum correlations, compared with the dynamics of entanglement measured by negativity. This indicates that quantum correlations are more robust than entanglement that can be utilized as a resource for quantum information processing.

This paper is organized as follows. In Section 2, measures of correlations in qubit–qutrit systems are introduced. In Section 3, we investigate the dynamics of classical and quantum correlations for a class of two-parameter states in qubit–qutrit systems subject to multi-local dephasing, phase-flip, bit- and trit-flip, bit- and trit-phase-flip, and depolarizing channels. A discussion and conclusion are presented in Section 4.

## 2. Measures of correlations in qubit–qutrit systems

In this paper, we adopt the MID[12] to quantify the quantum correlations. For any bipartite quantum state $\rho_{\rm AB}$, we perform the local projective measurement $\{\Pi_k\}$ satisfying $\Pi_k\Pi_{k'}=\delta_{kk'}\Pi_k$ and $\sum_k \Pi_k = 1$. Here, $\Pi_k = \Pi_i^{\rm A}\otimes\Pi_j^{\rm B}$, with $\Pi_i^{\rm A}$ and $\Pi_j^{\rm B}$ being complete projective measurements consisting of one-dimensional orthogonal projections for parties A and B, respectively. After the measurement, the state $\Pi(\rho_{\rm AB}) = \sum_{ij}(\Pi_i^{\rm A}\otimes\Pi_j^{\rm B})\rho_{\rm AB}(\Pi_i^{\rm A}\otimes\Pi_j^{\rm B})$ is obtained. If the measurement is induced by the spectral resolutions of the reduced states $\rho^{\rm A}=\sum_i p_i^{\rm A}\Pi_i^{\rm A}$ and $\rho^{\rm B}=\sum_j p_j^{\rm B}\Pi_j^{\rm B}$, the measurement keeps the marginal information invariant and is the least disturbing. Actually, $\Pi(\rho_{\rm AB})$ is a classical state that is closest to the initial state $\rho_{\rm AB}$ since this kind of measurement can keep the reduced states invariant. Thus, quantum correlations quantified by the MID are given by

$$Q(\rho_{\rm AB}) = I(\rho_{\rm AB}) - I(\Pi(\rho_{\rm AB})), \tag{1}$$

where $I(\rho_{\rm AB}) = S(\rho^{\rm A}) + S(\rho^{\rm B}) - S(\rho_{\rm AB})$ is the quantum mutual information which quantifies the total correlations in a bipartite state $\rho_{\rm AB}$. The amount of classical correlations in $\rho_{\rm AB}$ is quantified by

$$\begin{aligned} I(\Pi(\rho_{\rm AB})) &= S([\Pi(\rho_{\rm AB})]^{\rm A}) + S([\Pi(\rho_{\rm AB})]^{\rm B}) \\ &\quad - S(\Pi(\rho_{\rm AB})), \end{aligned} \tag{2}$$

with $[\Pi(\rho_{\rm AB})]^{\rm A} = {\rm Tr}_{\rm B}(\Pi(\rho_{\rm AB}))$ and $[\Pi(\rho_{\rm AB})]^{\rm B} = {\rm Tr}_{\rm A}(\Pi(\rho_{\rm AB}))$, since $\Pi(\rho_{\rm AB})$ is a classical state. Here $S(\rho) = -{\rm tr}\rho\log\rho$ is the Von Neumann entropy, and the base of the logarithm is 2.

For comparison with the dynamics of entanglement, we adopt the negativity[23] to quantify entanglement. For any bipartite quantum state $\rho_{\rm AB}$, its negativity is defined as

$$N(\rho_{\rm AB}) = \sum_l |\lambda_l|, \tag{3}$$

where $\lambda_l$ is the negative eigenvalue of $\rho^{T_{\rm A}}$, and $\rho^{T_{\rm A}}$ denotes the partial transpose with respect to the subsystem A. The negativity is related to the trace norm of $\rho^{T_{\rm A}}$

$$N(\rho_{\rm AB}) = \frac{\|\rho^{T_{\rm A}}\| - 1}{2}, \tag{4}$$

where the trace norm of $\rho^{T_{\rm A}}$ is equal to the sum of the absolute values of the eigenvalues of $\rho^{T_{\rm A}}$. If $N(\rho_{\rm AB}) > 0$, the state $\rho_{\rm AB}$ is entangled, otherwise separated.

## 3. Correlation dynamics for a two-parameter class of states in qubit–qutrit systems

The initial state that we consider is a two-parameter class of states in qubit–qutrit systems, which is described as[24]

$$\begin{aligned} \rho_{\rm AB} &= a(|02\rangle\langle 02| + |12\rangle\langle 12|) + b(|\Phi^+\rangle\langle\Phi^+| \\ &\quad + |\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+|) \\ &\quad + c|\Psi^-\rangle\langle\Psi^-|, \end{aligned} \tag{5}$$

where

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),$$

and $a$, $b$, and $c$ are three real parameters satisfying the normalized condition $2a+3b+c=1$. $\{|ij\rangle : i=0,1; j=0,1,2\}$ is an orthonormal basis in qubit–qutrit systems. This two-parameter class of states can be reduced from an arbitrary bipartite state in qubit–qutrit systems by local operation and classical communication (LOCC). It includes the Bell states, the Werner states, and so forth.

For simplicity, we consider the case of multi-local decoherence channels. Generally, the evolution of the state in Eq. (5) under decoherence can be described by the following operator-sum form:[25]

$$\rho_{\rm AB}(t) = \sum_{i=0}^{1}\sum_{j=0}^{2} E_i^{\rm A} F_j^{\rm B} \rho_{\rm AB} (E_i^{\rm A} F_j^{\rm B})^\dagger, \tag{6}$$

where $E_i^{\rm A}$ and $F_j^{\rm B}$ are time-dependent Kraus operators which are used to describe the noise channels acting on qubits A and B, respectively. They satisfy the conditions of $E_i^{{\rm A}^\dagger} E_i^{\rm A} = I$ and $E_j^{{\rm B}^\dagger} E_j^{\rm B} = I$ for all $t$. With these generic evolutions, we now analyze the dynamics of correlations under multi-local depahsing, phase-flip, bit- and trit-flip, bit-phase- and trit-phase-flip, and depolarizing channels.

### 3.1. Dephasing channels

The Kraus operators for qubit A and qutrit B for the dephasing channels are specifically given by

$$\boldsymbol{E}_0^{\mathrm{A}} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p_{\mathrm{A}}} \end{pmatrix}, \quad \boldsymbol{E}_1^{\mathrm{A}} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p_{\mathrm{A}}} \end{pmatrix},$$
$$\boldsymbol{F}_0^{\mathrm{B}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-p_{\mathrm{B}}} & 0 \\ 0 & 0 & \sqrt{1-p_{\mathrm{B}}} \end{pmatrix}, \quad \boldsymbol{F}_1^{\mathrm{B}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{p_{\mathrm{B}}} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\boldsymbol{F}_2^{\mathrm{B}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{p_{\mathrm{B}}} \end{pmatrix}.$$

Here, $p_{\mathrm{A(B)}} = 1 - \mathrm{e}^{-t\Gamma_{\mathrm{A(B)}}}$ is the decoherence parameter of subsystem A (B), $p_{\mathrm{A(B)}} \in [0,1]$, and $\Gamma_{\mathrm{A(B)}}$ denotes the decay rate of the subsystem A(B), hereafter the same. Because the $p_{\mathrm{A(B)}}$ increases with increasing $t\Gamma_{\mathrm{A(B)}}$, it is general to study the dynamics of entanglement or correlations as a function of $p_{\mathrm{A(B)}}$. According to Eq. (6), the nonzero matrix elements of the time-evolved density matrix $\rho_{\mathrm{AB}}(t)$ in the basis state $\{|ij\rangle : i = 0,1; j = 0,1,2\}$ are given by

$$\begin{aligned} \rho_{11} &= \rho_{55} = b, \\ \rho_{22} &= \rho_{44} = \frac{b+c}{2}, \\ \rho_{24} &= \rho_{42} = \frac{b-c}{2}\sqrt{(1-p_{\mathrm{A}})(1-p_{\mathrm{B}})}, \\ \rho_{33} &= \rho_{66} = \frac{1-3b-c}{2}. \end{aligned} \tag{7}$$

For simplicity, we assume that the decoherence parameters of two independent decoherence channels are the same, i.e., $p_{\mathrm{A}} = p_{\mathrm{B}} = p$ (hereafter the same). We can calculate the MID, classical correlations, and negativity for the evolved state $\rho_{\mathrm{AB}}(t)$ according to Eqs. (1), (2), and (4). With the initial state parameters $b = 0.03$ and $c = 0.4$, we plot the classical correlations, MID, and negativity for the evolved state $\rho_{\mathrm{AB}}(t)$ as a function of the decoherence parameter $p$ in Fig. 1, from which, we can see that the classical correlations keeps unchanged at 0.227. This indicates that the classical correlations are unaffected by the noise environment. The MID decays in the monotonic way. The negativity decays monotonically and suffers a sudden death at $p = 0.838$. One can find that no sudden death occurs for the classical correlation and MID. This means that the classical correlations and MID are more robust than entanglement.

### 3.2. Phase flip channels

The Kraus operators for qubit A and qutrit B for the phase-flip channels are expressed by

$$\boldsymbol{E}_0^{\mathrm{A}} = \sqrt{1-\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{E}_1^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
$$\boldsymbol{F}_0^{\mathrm{B}} = \sqrt{1-\frac{2p_{\mathrm{A}}}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$\boldsymbol{F}_1^{\mathrm{B}} = \sqrt{\frac{p_{\mathrm{A}}}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}2\pi/3} & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}4\pi/3} \end{pmatrix},$$
$$\boldsymbol{F}_2^{\mathrm{B}} = \sqrt{\frac{p_{\mathrm{A}}}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}4\pi/3} & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}2\pi/3} \end{pmatrix}.$$

According to Eq. (6), the nonzero matrix elements of the time-evolved density matrix $\rho_{\mathrm{AB}}(t)$ in the basis state $\{|ij\rangle : i = 0,1; j = 0,1,2\}$ are given by

$$\begin{aligned} \rho_{11} &= \rho_{55} = b, \\ \rho_{22} &= \rho_{44} = \frac{b+c}{2}, \\ \rho_{24} &= \rho_{42} = \frac{b-c}{2}(1-p_{\mathrm{A}})(1-p_{\mathrm{B}}), \\ \rho_{33} &= \rho_{66} = \frac{1-3b-c}{2}. \end{aligned} \tag{8}$$

We can find that the matrix elements in Eq. (8) are the same as that in Eq. (7) for dephasing channels except the non-diagonal elements $\rho_{24}$ and $\rho_{42}$. With the same initial state parameters, the similar dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ is plotted in Fig. 2. The classical correlations keep unchanged at

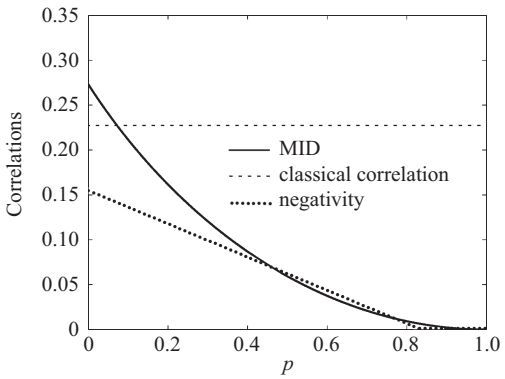



**Fig. 1.** Dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ under multi-local depahsing channels. The initial state parameters in Eq. (5) are chosen as $b = 0.03$ and $c = 0.4$.

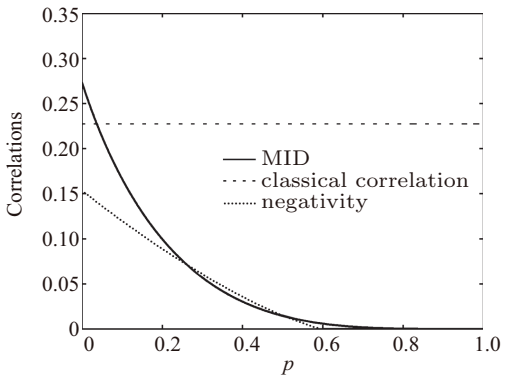



**Fig. 2.** Dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ under multi-local phase-flip channels. The initial state parameters in Eq. (5) are chosen as $b = 0.03$ and $c = 0.4$.

the same value 0.227. The MID decays in the monotonicallt without the sudden death. The negativity first decays monotonically and then sudden death occurs at $p = 0.6$.

### 3.3. Bit- and trit-flip channels

The Kraus operators for qubit A and qutrit B for bit-flip and trit-flip channels are described by

$$\begin{aligned}
\boldsymbol{E}_0^{\mathrm{A}} &= \sqrt{1-\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix}, \quad \boldsymbol{E}_1^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix}0&1\\1&0\end{pmatrix},\\
\boldsymbol{F}_0^{\mathrm{B}} &= \sqrt{1-\frac{2p_{\mathrm{A}}}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \quad \boldsymbol{F}_1^{\mathrm{B}} = \sqrt{\frac{p_{\mathrm{A}}}{3}}\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\\
\boldsymbol{F}_2^{\mathrm{B}} &= \sqrt{\frac{p_{\mathrm{A}}}{3}}\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}. \qquad (9)
\end{aligned}$$

According to Eq. (6), the nonzero matrix elements of the evolved state $\rho_{\mathrm{AB}}(t)$ in the basis state $\{|ij\rangle : i = 0,1; j = 0,1,2\}$ under the multi-local bit- and trit-flip channels are given by

$$\begin{aligned}
\rho_{11} &= \rho_{55} = b + \frac{1}{4}(b-c)p_{\mathrm{A}}(p_{\mathrm{B}}-1) + \left(\frac{1}{6}-b\right)p_{\mathrm{B}},\\
\rho_{22} &= \rho_{44} = \frac{1}{2}(b+c) - \frac{1}{4}(b-c)p_{\mathrm{A}}(p_{\mathrm{B}}-1)\\
&\quad + \frac{1}{6}(1-3b-3c)p_{\mathrm{B}},\\
\rho_{33} &= \rho_{66} = \frac{1}{6}[3(1-3b-c) + (9b+3c-2)p_{\mathrm{B}}],\\
\rho_{15} &= \rho_{51} = \frac{1}{12}(b-c)p_{\mathrm{A}}(3-2p_{\mathrm{B}}),\\
\rho_{16} &= \rho_{61} = \rho_{35} = \rho_{53} = \frac{1}{12}(b-c)(2-p_{\mathrm{A}})p_{\mathrm{B}},\\
\rho_{24} &= \rho_{42} = \frac{1}{12}(b-c)(2-p_{\mathrm{A}})(3-2p_{\mathrm{B}}),\\
\rho_{26} &= \rho_{62} = \rho_{34} = \rho_{43} = \frac{1}{12}(b-c)p_{\mathrm{A}}p_{\mathrm{B}}. \qquad (10)
\end{aligned}$$

With the chosen initial state parameters $b = 0.05$ and $c = 0.3$, we display the dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ in Fig. 3. It is shown that the classical correlations decay in the monotonically. The MID decays monotonically and finally to a steady value 0.0227. The negativity decays in the monotonically and suffers a sudden death at $p = 0.254$.

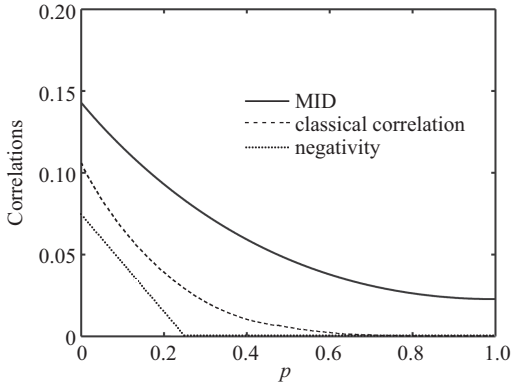



**Fig. 3.** Dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ under local bit-(trit-) flip channels. The initial state parameters in Eq. (5) are chosen as $b = 0.05$, $c = 0.3$.

### 3.4. Bit-(Trit) phase-flip channels

The Kraus operators for qubit $A$ and qutrit $B$ for bit-(trit-) phase-flip channels are given by

$$\begin{aligned}
\boldsymbol{E}_0^{\mathrm{A}} &= \sqrt{1-\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix}, \quad \boldsymbol{E}_1^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{2}}\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\\
\boldsymbol{F}_0^{\mathrm{B}} &= \sqrt{1-\frac{2p_{\mathrm{B}}}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\\
\boldsymbol{F}_1^{\mathrm{B}} &= \sqrt{\frac{p_{\mathrm{B}}}{6}}\begin{pmatrix}0&0&\mathrm{e}^{\mathrm{i}2\pi/3}\\1&0&0\\0&\mathrm{e}^{\mathrm{i}4\pi/3}&0\end{pmatrix},\\
\boldsymbol{F}_2^{\mathrm{B}} &= \sqrt{\frac{p_{\mathrm{B}}}{6}}\begin{pmatrix}0&0&\mathrm{e}^{\mathrm{i}4\pi/3}\\1&0&0\\0&\mathrm{e}^{\mathrm{i}2\pi/3}&0\end{pmatrix},\\
\boldsymbol{F}_3^{\mathrm{B}} &= \sqrt{\frac{p_{\mathrm{B}}}{6}}\begin{pmatrix}0&\mathrm{e}^{\mathrm{i}4\pi/3}&0\\0&0&\mathrm{e}^{\mathrm{i}2\pi/3}\\1&0&0\end{pmatrix},\\
\boldsymbol{F}_4^{\mathrm{B}} &= \sqrt{\frac{p_{\mathrm{B}}}{6}}\begin{pmatrix}0&\mathrm{e}^{\mathrm{i}2\pi/3}&0\\0&0&\mathrm{e}^{\mathrm{i}4\pi/3}\\1&0&0\end{pmatrix}.
\end{aligned}$$

According to Eq. (6), the nonzero matrix elements of the evolved state $\rho_{\mathrm{AB}}(t)$ in the basis state $\{|ij\rangle : i = 0,1; j = 0,1,2\}$ under multi-local bit-(trit-) phase-flip channels are given by

$$\begin{aligned}
\rho_{11} &= \rho_{55} = b + \frac{1}{4}(b-c)p_{\mathrm{A}}(p_{\mathrm{B}}-1) + \left(\frac{1}{6}-b\right)p_{\mathrm{B}},\\
\rho_{22} &= \rho_{44} = \frac{1}{2}(b+c) - \frac{1}{4}(b-c)p_{\mathrm{A}}(p_{\mathrm{B}}-1)\\
&\quad + \frac{1}{6}(1-3b-3c)p_{\mathrm{B}},\\
\rho_{33} &= \rho_{66} = \frac{1}{2}(1-3b-c) + \frac{1}{6}(9b+3c-2)p_{\mathrm{B}},\\
\rho_{15} &= \rho_{51} = \frac{1}{12}(c-b)p_{\mathrm{A}}(3-2p_{\mathrm{B}}),\\
\rho_{16} &= \rho_{61} = \rho_{35} = \rho_{53} = \frac{1}{24}(b-c)(p_{\mathrm{A}}-2)p_{\mathrm{B}},\\
\rho_{24} &= \rho_{42} = \frac{1}{12}(b-c)(2-p_{\mathrm{A}})(3-2p_{\mathrm{B}}),\\
\rho_{26} &= \rho_{62} = \rho_{34} = \rho_{43} = \frac{1}{12}(b-c)p_{\mathrm{A}}p_{\mathrm{B}}. \qquad (11)
\end{aligned}$$

The similar dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ is shown in Fig. 4 with the same initial state parameters as that in bit- and trit-flip channels. We can see that the classical correlations decay monotonically. The MID decays monotonically and finally approaches a steady value of 0.017. The negativity also decays monotonically and suffers sudden death at $p = 0.252$.

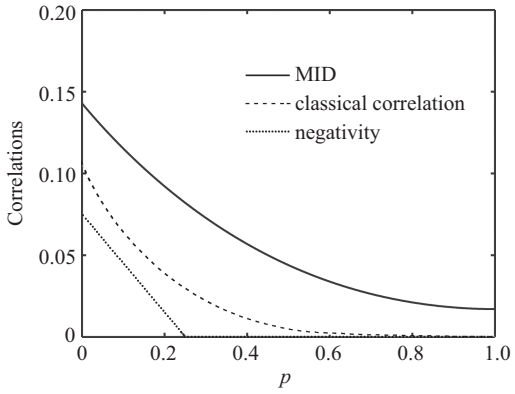



**Fig. 4.** Dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ under local bit-phase- and trit-phase-flip channels. The initial state parameters in Eq. (5) are chosen as $b = 0.03$ and $c = 0.5$.

### 3.5. Depolarizing channels

The Kraus operators for qubit A for the depolarizing channel are given by

$$E_0^{\mathrm{A}} = \sqrt{1-\frac{3p_{\mathrm{A}}}{4}}I_2,\ E_1^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{4}}\sigma_x,$$
$$E_2^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{4}}\sigma_y,\ E_3^{\mathrm{A}} = \sqrt{\frac{p_{\mathrm{A}}}{4}}\sigma_z,$$

where $\sigma_i (i = 1,2,3)$ are the three Pauli matrices. The Kraus operator for qutrit B for the depolarizing channel are given by

$$\begin{aligned}
F_0^{\mathrm{B}} &= \sqrt{1-\frac{8p_{\mathrm{B}}}{9}}I_3,\ F_1^{\mathrm{B}} = \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y},\\
F_2^{\mathrm{B}} &= \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Z},\ F_3^{\mathrm{B}} = \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y}^2,\\
F_4^{\mathrm{B}} &= \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y}Z,\ F_5^{\mathrm{B}} = \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y}^2\boldsymbol{Z},\\
F_6^{\mathrm{B}} &= \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y}Z^2,\ F_7^{\mathrm{B}} = \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Y}^2\boldsymbol{Z}^2,\\
F_8^{\mathrm{B}} &= \frac{\sqrt{p_{\mathrm{B}}}}{3}\boldsymbol{Z}^2,
\end{aligned}$$

where

$$Y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}2\pi/3} & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}2\pi/3} \end{pmatrix}.$$

In the basis $\{|ij\rangle : i = 0,1; j = 0,1,2\}$, the nonzero matrix elements of the evolved state $\rho_{\mathrm{AB}}(t)$ under multi-local depolarizing channels are given by

$$\begin{aligned}
\rho_{11} &= \rho_{55} = b + \frac{1}{4}(b-c)(p_{\mathrm{B}}-1)p_{\mathrm{A}} + \left(\frac{1}{6}-b\right)p_{\mathrm{B}},\\
\rho_{22} &= \rho_{44} = \frac{1}{2}(b+c) - \frac{1}{4}(b-c)(p_{\mathrm{B}}-1)p_{\mathrm{A}}\\
&\quad + \frac{1}{6}(1-3b-3c)p_{\mathrm{B}},\\
\rho_{33} &= \rho_{66} = \frac{1}{6}[3(1-3b-c) + (9b+3c-2)p_{\mathrm{B}}],\\
\rho_{24} &= \rho_{42} = \frac{1}{2}(c-b)(1-p_{\mathrm{A}})p_{\mathrm{B}},
\end{aligned}$$

With the initial state parameters $b = 0.12$ and $c = 0.8$, we plot the dynamics of classical correlations, MID, and negativity as a function of decoherence parameter $p$ in Fig. 5. It can be seen that the classical correlations decay monotonically. The MID first increases from zero and reaches the maximal value 0.0407 at $p = 0.562$ and then decreases to zero monotonically. This means that the MID is a convex function for the decoehrence parameter $p$ varying from 0 to 1. While, the negativity suffers sudden death at $p = 0.326$.

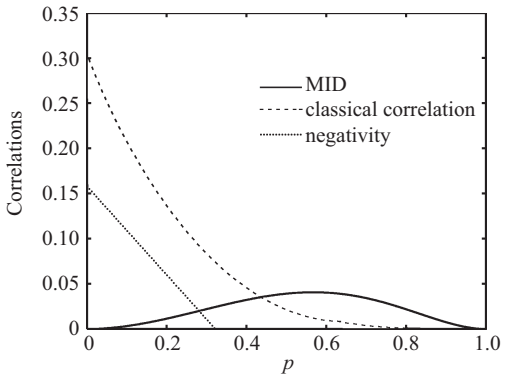



**Fig. 5.** Dynamics of classical correlations, MID, and negativity as a function of the decoherence parameter $p$ under multi-local depolarizing channels. The initial state parameters in Eq. (5) are chosen as $b = 0.12$ and $c = 0.8$.

## 4. Discussion and conclusion

In summary, we have investigated the dynamics of the classical and quantum correlations for a two-parameter class of states in qubit–qutrit systems undergoing multi-local dephasing, phase-flip, bit- and trit-flip, bit-phase- and trit-phase-flip, and depolarizing channels. It has been shown that the the amount of classical correlations are unaffected under the dephasing and phase-flip channels, and decays monotonically under the other decoherence channels. The quantum correlations measured by the MID decay in the monotonically to zero under the dephasing and phase-flip channels or even to a nonzero steady value under the bit- and trit-flip and bit-phase- and trit-phase-flip channels, or first increases monotonically from zero, and then decreases monotonically to zero under the depolarizing channels, which is not revealed in the previous studies. For the cases of only qubit/qutrit subject to the decoherence channel, the similar dynamics of classical correlations, MID, and entanglement can also be identified.

For this kind of two-parameter qubit–qutrit states, its dynamics of quantum correlation measured by GMQD has been studied by Wei *et al.*[26] They showed that the GMQD under multi-local decoherence channels decays monotonically to zero, which is the same as the dynamics of the MID under dephasing and phase-flip channels. When only qutrit undergoes local trit-flip or trit-phase flip channel, the GMQD decays

monotonically to a nonzero steady state, which is identical to the dynamics of the MID under multi-local (bit-) trit-flip and (bit-phase-) trit-phase-flip channels. The different dynamics between the MID and the GMQD is mainly due to different measures of quantum correlations, initial states, and intrinsic properties of the decoherent channels. On the other hand, one can also find that there is no sudden death of classical and quantum correlations other than entanglement.[22,27,28] This indicates that the classical and quantum correlations are more robust than entanglement that can be utilized as a resource for quantum information processing.

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