Simulation optimization for train movement on a single-track railway*

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Optimizing train movement has a great significance for railway traffic. In this paper, based on the optimal velocity car-following model, we propose a new simulation model for optimizing train movement in railway traffic. Here a kind of single-track railway is considered. Our aim is to reduce the energy consumption of train movement and ensure the train being on time by controlling the velocity curve of train movement. The simulation results indicate that the proposed model is effective for optimizing train movement. In addition, some major characteristics of train movement can be well captured. This method provides a new way to optimize train movement in railway traffic.

Keywords: train movement, simulation optimization, car-following model

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1. Introduction

On single-track section, optimizing train movement is realized by controlling the velocity curve of the train movement. Usually, the optimal objectives are to reduce energy consumption, enhance its comfort, make it on run on schedule, etc. In the past decades, a number of theoretical methods and technical schemes have been proposed to complete these optimal objectives, such as utilizing regenerative braking technology, employing direct-current (DC) traction motors, and using optimal control strategies.[1–3] However, with the development of railway traffic, the velocity of trains increases continuously, and the control system becomes more and more complex. These factors bring great difficulties to optimizing train movement, so it has not been solved well.

Optimizing the velocity curve of train movement is one of the basic methods for reducing a train's energy consumption, enhancing its comfort, and ensuring that the train can arrive at a station on time. Here a key step is to determine the optimal velocity curve with which the train can arrive at its stations on schedule. Theoretically, two types of methods have been discussed, i.e., the mathematical model and the simulation analysis approach.[4–10] The mathematical model has some computation difficulties, and it is not easy to find an analytical solution. The simulation analysis approach can avoid falling into those difficulties, but we cannot obtain an accurate solution. Despite all these, the simulation analysis approach can be used to solve many reality problems with the help of computer simulation, so it attracts more and more attention.

The car-following model is one such important simulation model, it has been widely used to study and discuss the evolution characteristics of traffic flow under various conditions, including the early car-following models[11–13] and some later improvements.[14–17] The main advantage of the car-following model is that realistic driver behavior and detailed vehicle characteristics can be included. This implies that the car-following model is suitable for simulating the vehicle's movement under various complicate conditions. Recently, the car-following model has been used to analyze and discuss a vehicle's energy consumption. For example, Shi *et al.* studied the energy consumption in several typical car-following traffic models where the effects of many-neighbor, non-locality, and relative velocity were considered.[18] A microscopic traffic simulation model, in which the car-following model was integrated, was used to analyze the impact of roadway grades on the vehicle fuel consumption.[19] Based on the optimal velocity models, Mo *et al.* proposed an extended model to investigate the energy consumption of cars in road traffic.[20]

So far, most car-following models have focused on analyzing and discussing some road traffic problems, but few of them study railway traffic. In the present work, a new optimal velocity car-following model is proposed to solve the problem of the optimal velocity curve in railway traffic. Here the optimization of the velocity curve is carried out by controlling the cruise velocity of the train. This paper is organized as follows. In Section 2, the optimal velocity car-following model is introduced; the fundamental principle of optimizing train movement is outlined in Section 3; our new model is proposed in Section 4; and the simulation results are shown in Section 5. Finally, a conclusion is drawn in Section 6.

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2. Optimal velocity car-following model

In the classic car-following model,[11] the velocity of the n-th vehicle is affected by the velocity of the $(n+1)$-th vehicle. When the velocity of the leader $(n+1)$-th vehicle changes, the velocity of the following n-th vehicle also changes. The n-th vehicle tends to move with the same velocity as the $(n+1)$-th vehicle. The dynamic equation can be written as

$$\dot{v}_n(t) = \left(\frac{1}{\tau}\right)[v_{n+1}(t) - v_n(t)], \tag{1}$$

where $v_n(t)$ is the velocity of the n-th vehicle at time t, and τ is a parameter related to the driver's response.

As we all know, for two successive vehicles, whether one vehicle will accelerate or decelerate is directly related to the distance between them. This means that in order to avoid collision between two successive vehicles, the minimum safety distance must be kept. From Eq. (1), Pipes derived a new equation[11]

$$\Delta x_n(t) = x_{n+1}(t) - x_n(t) = \Delta x_{\text{safe}} + \tau v_n(t), \tag{2}$$

where Δx_{safe} is a safe distance. According to Eq. (2), the larger the distance between two successive vehicles is, the faster the following vehicle is.

For more realistic reason, Newell proposed the following optimal velocity car-following model:[13]

$$\dot{x}_n(t+T) = V^{\text{opt}}(\Delta x_n(t)), \tag{3}$$

where function V^{opt} is called the optimal velocity, $V^{\text{opt}}(\Delta x_n(t)) = v_0\{1 - \exp[-(\Delta x_n - \Delta x_{\text{safe}})/(v_0 T_s)]\}$; v_0 is the desired velocity; T_s is the safe time interval characterizing the car-following in dense traffic; and T is the length of iteration time step.

In 1995, Bando *et al.* proposed a famous optimal velocity car-following model[14]

$$\ddot{x}_n(t) = \left(\frac{1}{\tau}\right)[V^{\text{opt}}(\Delta x_n(t)) - \dot{x}_n(t)], \tag{4}$$

where

$$V^{\text{opt}}(\Delta x_n(t)) = (v_{\max}/2)\{\tanh[\Delta x_n(t) - \Delta x_{\text{safe}}] + \tanh(\Delta x_{\text{safe}})\}.$$

In general, as $\Delta x \to 0$, $V^{\text{opt}}(\Delta x_n(t)) \to 0$, and as $\Delta x \to \infty$, $V^{\text{opt}}(\Delta x_n(t))$ is bounded. This model using the position of the n-th vehicle as an input is formulated mathematically in terms of the second-order differential equations. According to the driving strategy described in formula (4), the n-th vehicle tends to maintain a safe velocity that depends on the relative position. Lately, based on such a model, many works have been done, which extend formula (4) to simulate various traffic phenomena.

3. The principle of optimizing train movement

In railway traffic, when the locomotive, railway, and train schedule are specified, train energy conservation can be implemented by optimizing the velocity curve of the train movement. In general, the train movement is divided into three stages: starting acceleration, moving, and decelerating brake. As one optimizes the velocity curve of the train movement, discussing and analyzing the train's movement are the major work.

When the railway is straight and has no gradient, it is well known that the train movement with a constant velocity (called cruise velocity) is the ideal option for reducing a train's energy consumption. This is because a train moving with the cruise velocity can overcome the minimum basic resistance. In the processes of a train's acceleration and braking, the maximum traction and the maximum braking can reduce the train's energy consumption. In these cases, the train firstly accelerates and then moves with a cruise velocity, such a velocity is kept before the train brakes. Figure 1 illustrates the principle of optimizing the train movement. According to such a principle, we can obtain the optimal velocity curve of the train movement.

In Fig. 1, A is the start site of train movement, and B is the destination of train movement. We introduce parameter E to describe the energy consumption of train movement at site x, $E = (F(x) + f(x))S$, $F(x) = Ma(x)$, $f(x) = M(r_0 + r_1 v(x) + r_2 v(x)^2)$, where M is the mass of train, and $f(x)$ is the basic resistance, r_0, r_1, r_2 are called the Davis coefficients, and S represents the train's moving distance from an initial velocity v_0 to the final velocity v_1. The energy consumption per unit mass is

$$E = (a + r_0 + r_2 v_0^2)\Delta S + r_2 a \Delta S + \frac{r_1[(v_0^2 + 2a\Delta S)^{3/2} - r_1 v_0^3]}{3a}. \tag{5}$$

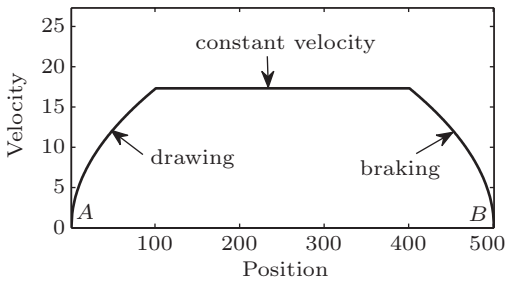


Fig. 1. Velocity curve of train movement.

4. The proposed model

4.1. Optimal cruise velocity

In Fig. 1, L is the length from A to B, t_1, t_2, and t_3 represent the travel time of acceleration, moving, and deceleration,

respectively. Here t_1, t_2, and t_3 are given as follows:

$$t_1 = \frac{v_{cr}}{a_{dr}}, \tag{6}$$

$$t_2 = \frac{L - v_{cr}^2/2a_{dr} - v_{cr}^2/2a_{br}}{v_{cr}} = \frac{L}{v_{cr}} - \frac{v_{cr}}{2a_{dr}} - \frac{v_{cr}}{2a_{br}}, \tag{7}$$

$$t_3 = \frac{v_{cr}}{a_{br}}, \tag{8}$$

where parameters a_{dr}, a_{br}, and v_{cr} represent the maximum acceleration rate, the maximum deceleration rate, and the cruise velocity, respectively. Usually, from A to B, the train movement is under a time interval constraint which is given in advance. Let T_{co} represent such a time interval (called planned time), from Eqs. (6)–(8), T_{co} can be calculated as

$$T_{co} = t_1 + t_2 + t_3 = \frac{v_{cr}}{2a_{dr}} + \frac{v_{cr}}{2a_{br}} + \frac{L}{v_{cr}}. \tag{9}$$

Equation (9) can also be written as

$$\left(\frac{1}{2a_{dr}} + \frac{1}{2a_{br}}\right) v_{cr}^2 - T_{co} v_{cr} + L = 0. \tag{10}$$

According to the specific traffic situation, T_{co} is given in advance. From Eq. (10), we obtain the cruise velocity of the train movement

$$v_{cr} = \frac{T_{co} \pm \sqrt{T_{co}^2 - 4\left(\frac{1}{2a_{dr}} + \frac{1}{2a_{br}}\right)L}}{\frac{1}{a_{dr}} + \frac{1}{a_{br}}}. \tag{11}$$

In reality, railway traffic, with the planned time T_{co} increasing, v_{cr} should become smaller. So from Eq. (11), v_{cr} should be chosen as

$$v_{cr} = \frac{T_{co} - \sqrt{T_{co}^2 - 4\left(\frac{1}{2a_{dr}} + \frac{1}{2a_{br}}\right)L}}{\frac{1}{a_{dr}} + \frac{1}{a_{br}}}. \tag{12}$$

In Eq. (12), if v_{cr} has a real solution, the following condition must be met:

$$T_{co}^2 - 4\left(\frac{1}{2a_{dr}} + \frac{1}{2a_{br}}\right)L \geq 0, \tag{13}$$

$$T_{co}^2 \geq 2\left(\frac{a_{dr} + a_{br}}{a_{dr}a_{br}}\right)L. \tag{14}$$

From inequality (13), it is concluded that the time interval T_{co} has a minimum value. In other words, if the given time interval is smaller than such a minimum value, the optimal velocity curve cannot be obtained. The minimum value of T_{co} is

$$T_{co}^{min} = \left[2\left(\frac{a_{dr} + a_{br}}{a_{dr}a_{br}}\right)L\right]^{1/2}. \tag{15}$$

4.2. Improved car-following model

Using the simulation analysis approach, theoretically, optimizing the velocity curve of train movement needs to be done with the help of an accurate simulation model. However, because of the complex environment of train movement, establishing the simulation model and designing the corresponding solving algorithm are the key steps. In our study, we improve the optimal velocity car-following model, and use it to simulate the train movement with optimal velocity curve. Here train energy conservation is realized by following steps: (i) during the train's acceleration or deceleration, the train tracts or brakes with the maximum acceleration or deceleration rate, (ii) as the train moves, its velocity is kept constant. From Eq. (11), the train's optimal velocity is determined by the planned time T_{co}, this ensures that the train can arrive at its destination on time.

In the optimal velocity car-following model, i.e., Eq. (4), V_{opt} represents the driver's desired velocity, which depends on the headway distance of vehicles. As the optimal velocity car-following model is used to describe various aspects of traffic phenomena, one has to postulate a specific function form for V_{opt}. Here, we propose a new form of V_{opt}, which is based on the discussion in Subsection 4.1. Specifically, the new form of V_{opt} is as follows: (i) in the process of train acceleration and cruising, $V_{opt} = v_{cr}$; (ii) as the train brakes, $V_{opt} = [2a_{br}(x_{obj} - x_n(t))]^{1/2}$. Here x_{obj} is the objective point of the n-th train. The objective point can be the leader train or the station in front of the n-th train.

In the new form of V_{opt}, case (ii) is used to keep the minimum safety distance between two successive trains. As $x_n(t) \geqq x_{obj} - v_{cr}^2/(2a_{br})$, the n-th train must brake. In addition, if $v_{cr} > v_{max}$, the train movement would use more time than the planning time T_{co}. However, because the optimal velocity v_{cr} must be smaller than the maximum velocity v_{max}, so if $v_{cr} > v_{max}$, v_{cr} must be limited, i.e., $v_{cr} = v_{max}$. The executed process of the proposed model can be described as follows.

Step 1 Determining the planning time T_{co}, $T_{co} \geqq T_{co}^{min}$.

Step 2 According to Eq. (11), calculating the cruise velocity v_{cr}; if $v_{cr} > v_{max}$, $v_{cr} = v_{max}$; Else $v_{cr} = v_{cr}$.

Step 3 Determining the optimal velocity V_{opt}; if $x_n(t) \geqq x_{obj} - v_{cr}^2/(2a_{br})$, $V_{opt} = \sqrt{2a_{br}(x_{obj} - x_n(t))}$; else $V_{opt} = v_{cr}$.

Step 4 Calculating the acceleration $\ddot{x}_n(t)$

$$\ddot{x}_n(t) = \frac{V_{opt} - v_n(t)}{\tau}.$$

Step 5 Updating the site of the n-th train

$$v_n(t+1) = v_n(t) + \ddot{x}_n(t),$$
$$x_n(t+1) = x_n(t) + v_n(t+1).$$

Here v_{max} represents the maximum velocity of the train movement. The boundary condition used in this paper is open.

Considering a single-track section with length L, the boundary condition is as follows. When the section from site 1 to site L_s is empty, a train with velocity $v_n(t) = 0$ is created. Here L_s is called the minimum safety distance, $L_s = \tau v_{max} + v_{max}^2/(2a_{br})$. The newborn train immediately moves according to the proposed model. At site L, the train simply moves out of the track section.

5. Numerical simulation

We use the proposed optimal velocity car-following model to simulate the train movement under moving block condition. In this case, trains can move only when the minimum safety distance is kept among them. The length of the simulated track section is set to be $L = 2000$, and the maximum iteration time step is $T = 1000$. Some other parameters are taken as follows: $v_{max} = 15$, $a_{dr} = 1.5$, $\tau = 1$, and $a_{br} = 1$. A station is designed at the middle of the simulated system. All trains should stop at the station for time T_d and then leave, and T_d is called the station dwell time. The T_{co} represents the planned time that a train moves from $x = 1$ to $x = 1000$.

In order to verify our proposed model, firstly, we compare the simulation result and the theoretical calculation of the optimal velocity curve for $T_{co} = T_{co}^{min}$. The reason is that as the constraint time of train movement is taken as T_{co}^{min}, we can find some major characteristics of the railway traffic flow under high-velocity condition. The theoretical calculation is derived from Subsection 4.1. The compared result is shown in Fig. 2, which displays how the velocity varies with the train's position. In Fig. 2, the dotted line denotes the simulation values using the proposed model, and the solid line denotes the theoretical values. From Fig. 2, it is clear that the simulation values using the proposed model are close to the theoretical ones. Only small differences exist between them when the train brakes. This is because the theoretical values are derived using the maximum acceleration rate and the maximum deceleration rate, but the simulation results use some varied acceleration rates and decelerating rates. Despite these factors, the simulation results demonstrate that the proposed model can be successfully used for optimizing the train movement.

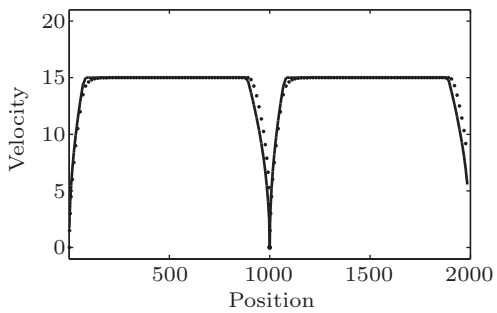


Fig. 2. The v versus x for $T_{co}^{min} = 57.74$.

In order to further verify the proposed model, we measure the headway distance $D_i(t)$ of one tracked train. Here $D_i(t)$ is the distance from the i-th train to the $(i+1)$-th train at time t. Figure 3 shows the distribution of the distance headway of one tracked train which departures at t=600. In Fig. 3, the solid line denotes the simulation result, and the dotted line denotes the minimum safety distance L_s. From Fig. 3, we can see that the simulation results are almost larger than the safety distance L_s. Only a few of them are slightly smaller than the safety distance L_s. In fact, when the headway distance of the tracked train is smaller than the safety distance L_s, the tracked train will decelerate quickly so that its headway distance can become larger. This further demonstrates that the proposed model is successful for simulating the train movement.

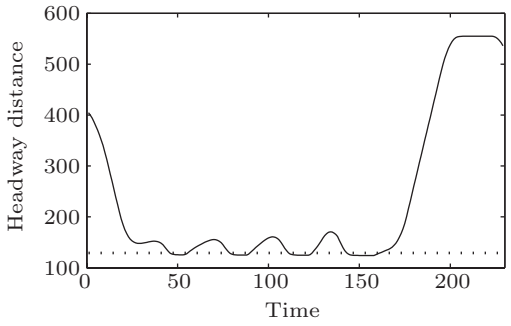


Fig. 3. Distribution of the distance headway of one tracked train for $T_{co}^{min} = 57.74$.

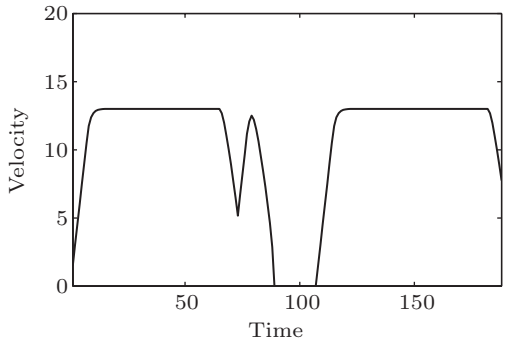


Fig. 4. Velocity versus time for $T_{co} = 87.74$.

During train movement, the leader train has a great effect on the following train, because the minimum safety distance must be kept between them. Only when the headway distance is larger enough, can the following train move according to the optimal velocity curve. In this case, the train can arrive at its destination at the planned time. Figure 4 displays the velocity and time of one tracked train, in which the horizontal lines denote that the train moves with a constant velocity. The tracked train departures from site $x = 1$ at time $t = 138$. In Fig. 4, it is obvious that in order to keep the minimum safety distance, the following train adjusts its velocity at any time. As the simulation time t is between 15 and 65, the tracked train moves with the optimal cruise velocity. The time that the tracked train

takes is larger than the planned time $T_{\rm co}$. However, a number of simulations indicate that when the departure time interval is larger enough, the time that all trains take is equal to the planned time $T_{\rm co}$. The result demonstrates that the proposed model is effective for train energy conservation.

Finally, we measure the energy consumption E of train movement under a different planned time. The E is calculated by Eq. (5), where the Davis coefficients r_0, r_1, r_2 are respectively set to 2.25, 0.019, 0.00032, which correspond to the SS8 electric locomotive in China. Figure 5 displays the measured results, which shows how the total energy and the electrical peak power vary with the optimal velocity respectively. From Fig. 5, it is clear that when the optimal velocity of train movement is smaller, the total energy consumption is smaller, and the electrical peak power is also smaller. These measured results are consistent with some empirical observations. Moreover, the data in Fig. 5(a) show a power law distribution, i.e., $E \propto T_{\rm co}^{-\gamma}$, where γ is an exponent, and that the relationship between total energy consumption and optimal velocity is linear.

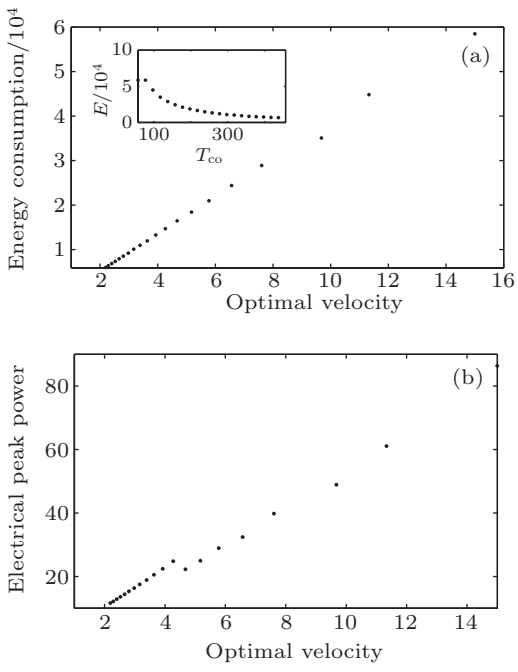


Fig. 5. (a) The total energy consumption and (b) the electrical peak power vary with the optimal velocity. The insert shows the total energy consumption versus the planned time.

6. Conclusion

Based on the optimal velocity car-following model, a new simulation model is proposed to capture the major characteristics of train movement. Using the proposed model, we analyze and discuss the optimal control of train movement. In our study, the train moves with the cruise velocity, so the energy consumption of train movement can be reduced under the constraint of a specified planned time. The numerical results demonstrate that our proposed model can not only well capture some major characteristics of train movement, but also is suitable for simulating its optimal control.

We believe our proposed model is a good choice, which provides a new way to obtain the optimal velocity curve. However, the proposed model also has some disadvantages, for example, some complex traffic environments and advanced control strategies have not been considered.

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