Backstepping-based lag synchronization of a complex permanent magnet synchronous motor system*

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Through introducing the concept of complex current and resetting cross-coupling term, this paper proposes a novel complex permanent magnet synchronous motor system and analyzes its properties. Based on a complex permanent magnet synchronous motor system, we design controllers and achieve lag synchronizations both in real part and imaginary part with backstepping method. In our study, we take complex current, time delay, and structure of complex system into consideration. Numerical simulation results demonstrate the validity of controllers.

Keywords: complex permanent magnet synchronous motor system, time delay, lag synchronization, backstepping method

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1. Introduction

Chaos refers to a class of uncertain behaviors which happen in deterministic systems and features initial condition sensitiveness, ergodicity, noise-like and fractional-dimension. In 1963, meteorologist Lorenz found the first chaos attractor in his study of 3-dimensional autonomous systems.[1] Li and Yorke first used "chaos" and proposed "period-three implies chaos"[2] in 1973. In the last 40 years, chaos has been a hot field in nonlinear science. In 1990, Ott, Grebogi and Yorke controlled chaos with the OGY method[3] and Pecora and Carroll successfully got chaos synchronization with the PC method[4] in the same year. After that, synchronizations have been widely studied.[5–7]

Currently, the study of chaos has developed from low-dimension and integral order to high-dimension and fractional order. People have proposed many different kinds of synchronization schemes according to various systems. For example, Wang[8] studied the behavior of the Liu system and Lü[9] studied the lag synchronization in uncertain chaotic systems. Besides that, in high-dimensional chaotic systems, Yang[10] and Bu[11] achieved synchronization with different methods.

On the other hand, synchronization of fractional systems or synchronization of a fractional system and integer system, which shows fractional order has an influence on the synchronization, also attracts a lot of attention. For instance, the literature[12] reports the synchronization between a fractional system and integer system and the function projective synchronization was studied in Ref. [13]. In 2011, Zhou *et al*.[14] achieved function projective synchronization for fractional systems.

Even though previous methods are effective and robust, they just focused on real number systems and ignored the study of complex systems. Mahmoud[15,16] and Nian *et al*.[17] have committed to the study of synchronizations in complex systems.

Permanent magnet synchronous motors have higher performance, smaller size, lighter weight, and better maintainability compared with mechanically commutated motors, so they have important applications in industrial areas.[18,19] Currently, a lot of research has been done in the study of synchronization and control of permanent magnet synchronous motor systems.[20–22] Like the study of other kinds of chaotic systems, the research of complex motor systems are studied little, while complex number voltage and complex number current exist widely in motor systems and it is easier to analyze motor systems with complex systems. In this paper, we propose a complex permanent magnet synchronous motor system by modifying cross-coupled term. In addition, we take the time delay in the process of synchronization into account and design controllers with backstepping method[23] because a complex permanent magnet synchronous motor system is partly in strict feedback forms.

2. System descriptions

State equations of permanent magnet synchronous motor system in field-oriented rotor can be described as follows:

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$$\begin{cases} \dfrac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} = (-R_1 i_\mathrm{d} + \omega L_\mathrm{q} i_\mathrm{q} + u_\mathrm{d})/L_\mathrm{d}, \\ \dfrac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} = (-R_1 i_\mathrm{q} + \omega L_\mathrm{d} i_\mathrm{d} + u_\mathrm{q} - \omega\psi_r)/L_\mathrm{q}, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = [n_\mathrm{p}\psi_r i_\mathrm{q} + n_\mathrm{p}(L_\mathrm{d} - L_\mathrm{q}) i_\mathrm{d} i_\mathrm{q} - T_L - \beta\omega]/J, \end{cases} \tag{1}$$

where i_d, i_q, u_d, u_q are direct axis component and quadrature axis component of current and voltage, ω_r is angular velocity, ψ_r is rotor flux, R_S is stator resistance, n_p is pole-pairs, J is rotor inertia, β is viscous damping coefficient, T_L is load moment, L_d and L_q are direct axis and quadrature axis inductances.

When the air gap is even, and the motor operates with no load and power outage, the dimensionless equations of a permanent magnet synchronous motor system can be depicted as

$$\begin{cases} \dot{z}_1 = a(z_2 - z_1), \\ \dot{z}_2 = bz_1 - z_2 - z_1 z_3, \\ \dot{z}_3 = z_1 z_2 - z_3, \end{cases} \tag{2}$$

where a, b are parameters. If current in the system (1) is plural and variables z_1, z_2 in the system (2) are complex numbers, by changing cross-coupled terms z_1 and z_2 to conjugate form, we can get a complex permanent magnet synchronous motor system as

$$\begin{cases} \dot{z}_1 = a(z_2 - z_1), \\ \dot{z}_2 = bz_1 - z_2 - z_1 z_3, \\ \dot{z}_3 = \dfrac{1}{2}(z_1\bar{z}_2 + \bar{z}_1 z_2) - z_3, \end{cases} \tag{3}$$

where $z_1 = u_1 + \mathrm{j}u_2$, $z_2 = u_3 + \mathrm{j}u_4$, $z_3 = u_5$. $\mathrm{j} = \sqrt{-1}$, $\bar{z}_1$ and $\bar{z}_2$ are conjugate complex numbers of z_1 and z_2. Replace complex variables in system (3) with real number variables and imaginary number variables, and we can get an equivalent system as follows:

$$\begin{cases} \dot{u}_1 = a(u_3 - u_1), \\ \dot{u}_2 = a(u_4 - u_2), \\ \dot{u}_3 = bu_1 - u_3 - u_1 u_5, \\ \dot{u}_4 = bu_2 - u_4 - u_2 u_5, \\ \dot{u}_5 = u_1 u_3 + u_2 u_4 - u_5. \end{cases} \tag{4}$$

When $1 \le a \le 11$, $10 \le b \le 20$, the Lyapunov exponent spectra of system (4) is shown in Fig. 1, where λ_1, λ_2, λ_3, λ_4, and λ_5 are corresponding Lyapunov exponents of system (4). From Fig. 1, we can see that when $6 \le a \le 11$, $14 \le b \le 20$, $\lambda_1 > 0$, $\lambda_2 = \lambda_3 = 0$, $\lambda_4 < 0$, and $\lambda_5 < 0$. In other words, there is one positive Lyapunov exponent, two Lyapunov exponents of zero, and two negative Lyapunov exponents which means the system is chaotic. In the following, we choose $a = 11$ and $b = 20$, the corresponding phase diagrams of real number part and imaginary part are shown in Fig. 2.

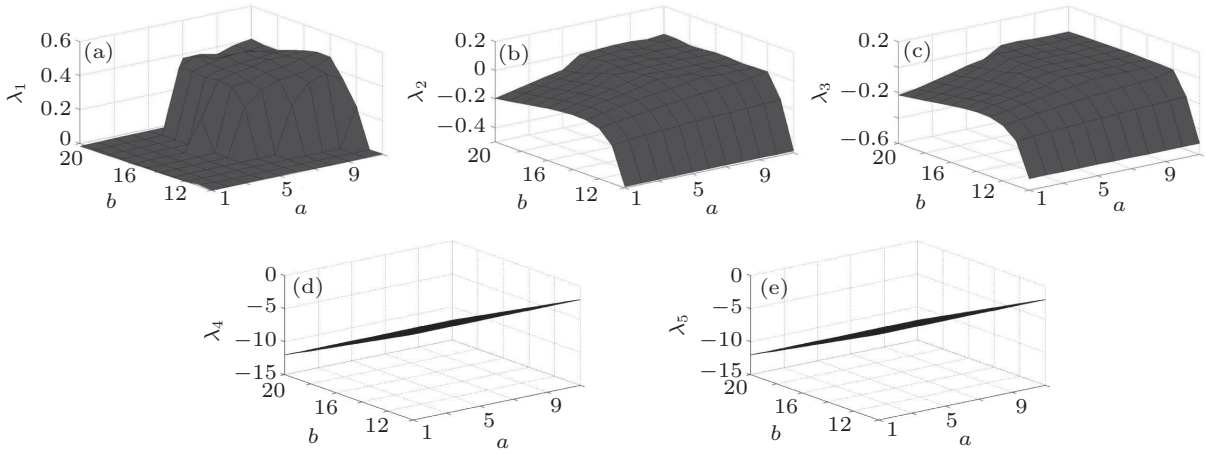


Fig. 1. Lyapunov exponent spectrum of complex system when $1 \le a \le 11$, $10 \le b \le 20$. (a) λ_1 corresponds to a, b, (b) λ_2 corresponds to a, b, (c) λ_3 corresponds to a, b, (d) λ_4 corresponds to a, b, (e) λ_5 corresponds to a, b.

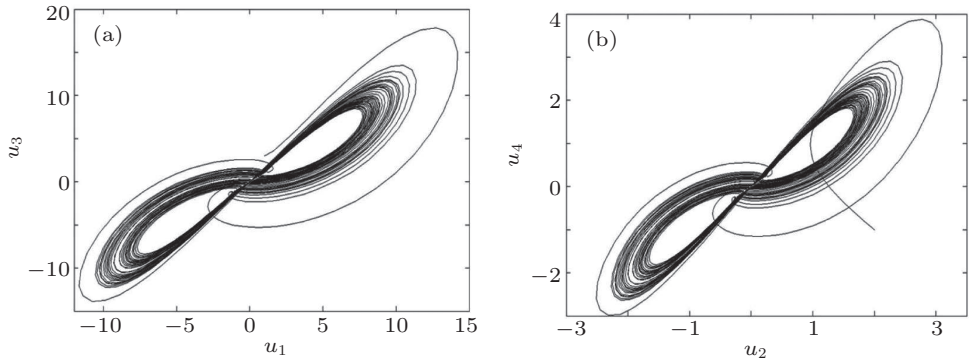


Fig. 2. Phase diagrams of complex system when $a = 11$, $b = 20$. (a) Phase diagram of real number part, (b) phase diagram of imaginary part.

3. Design of controllers

In practice, we need to get synchronization of two motor systems. However, because of operating time and transmission time, there exists a time delay in the process of synchronization. So it is necessary to design lag synchronization controllers for motor systems. In addition, because it is easier to analyze motor systems with the help of complex numbers, it is significant to design synchronization controllers for complex systems. In the following, we will design this kind of controllers based on the backstepping method. The driving complex system is

$$\begin{cases} \dot{x}_1 = a(x_3 - x_1), \\ \dot{x}_2 = a(x_4 - x_2), \\ \dot{x}_3 = bx_1 - x_3 - x_1x_5, \\ \dot{x}_4 = bx_2 - x_4 - x_2x_5, \\ \dot{x}_5 = x_1x_3 + x_2x_4 - x_5, \end{cases} \tag{5}$$

and the corresponding response system is

$$\begin{cases} \dot{y}_1 = a(y_3 - y_1), \\ \dot{y}_2 = a(y_4 - y_2), \\ \dot{y}_3 = by_1 - y_3 - y_1y_5 + L_1, \\ \dot{y}_4 = by_2 - y_4 - y_2y_5 + L_2, \\ \dot{y}_5 = y_1y_3 + y_2y_4 - y_5, \end{cases} \tag{6}$$

where L_1, L_2 are controllers of the real number part and imaginary part. Image time delay is τ, errors of system (5) and system (6) are

$$\begin{cases} w_1 = y_1(t) - x_1(t-\tau), \\ w_2 = y_2(t) - x_2(t-\tau), \\ w_3 = y_3(t) - x_3(t-\tau), \\ w_4 = y_4(t) - x_4(t-\tau), \\ w_5 = y_5(t) - x_5(t-\tau), \end{cases} \tag{7}$$

and error system can be described as:

$$\begin{cases} \dot{w}_1 = a(w_3 - w_1), \\ \dot{w}_2 = a(w_4 - w_2), \\ \dot{w}_3 = bw_1 - w_3 - y_5(t)w_1 - x_1(t-\tau)w_5 + L_1, \\ \dot{w}_4 = bw_2 - w_4 - y_5(t)w_2 - x_2(t-\tau)w_5 + L_2, \\ \dot{w}_5 = y_1(t)w_3 + x_3(t-\tau)w_1 + y_2(t)w_4 \\ \qquad + x_4(t-\tau)w_2 - w_5. \end{cases} \tag{8}$$

Set $e_1 = w_1$, $e_2 = w_2$, $e_3 = w_3 - \alpha_1$, $e_4 = w_4 - \alpha_2$, $e_5 = w_5$, where α_i $(i = 1,2)$ are virtual controllers. e_i $(i = 1,2,\ldots,5)$ are errors of error system (8) and virtual controllers. e_i $(i = 1,2,5)$ can be seen as errors of error system and zero.

We now design our lag synchronization controllers following the steps given below.

First step:

$$\dot{e}_1 = \dot{w}_1 = a(e_3 + \alpha_1) - aw_1 = ae_3 + a\alpha_1 - aw_1,$$

set $V_1 = \frac{1}{2}e_1^2$, there is

$$\dot{V}_1 = e_1\dot{e}_1 = ae_1e_3 + e_1(a\alpha_1 - aw_1).$$

Design

$$\alpha_1 = \frac{1}{a}(-c_1e_1 + aw_1),$$

where c_1 is a positive constant and

$$\dot{e}_1 = ae_3 - c_1e_1, \quad \dot{V}_1 = ae_1e_3 - c_1e_1^2.$$

The same procedure may be easily adapted to obtain

$$\alpha_2 = \frac{1}{a}(-c_2e_2 + aw_2),$$

where c_2 is a positive constant and

$$\dot{e}_2 = ae_4 - c_2e_2, \quad \dot{V}_2 = ae_2e_4 - c_2e_2^2.$$

Second step:

$$\begin{aligned} \dot{e}_3 &= \dot{w}_3 - \frac{\partial\alpha_1}{\partial w_1}\dot{w}_1 = bw_1 - w_3 - y_5(t)w_1 \\ &\quad - x_1(t-\tau)w_5 + L_1 - (a-c_1)(w_3 - w_1) \\ &= A + L_1, \end{aligned}$$

where

$$\begin{aligned} A &= bw_1 - w_3 - y_5(t)w_1 - x_1(t-\tau)w_5 \\ &\quad - (a-c_1)(w_3 - w_1). \end{aligned}$$

Set

$$V_3 = V_1 + \frac{1}{2}e_3^2,$$

and

$$\begin{aligned} \dot{V}_3 &= \dot{V}_1 + e_3\dot{e}_3 = -c_1e_1^2 + e_3(ae_1 + \dot{e}_3) \\ &= -c_1e_1^2 + e_3(ae_1 + A + u_1). \end{aligned}$$

Design

$$L_1 = -c_3e_3 - ae_1 - A,$$

where c_3 is a positive constant and

$$\dot{V}_3 = -c_1e_1^2 - c_3e_3^2 < 0.$$

The same procedure may be easily adapted to obtain

$$\begin{aligned} B &= bw_2 - w_4 - y_5(t)w_2 - x_2(t-\tau)w_5 \\ &\quad - (a-c_2)(w_4 - w_2), \\ V_4 &= V_2 + \frac{1}{2}e_4^2. \end{aligned}$$

Design

$$L_2 = -c_4e_4 - ae_2 - B,$$

where c_4 is a positive constant and

$$\dot{V}_4 = -c_2e_2^2 - c_4e_4^2 < 0.$$

Third step:

When the real number part and the imaginary part are synchronized, the conjugate term converges to zero, so

$$\dot{w}_5 = -w_5,$$

set

$$V_5 = V_3 + V_4 + \frac{1}{2}e_5^2.$$

It is easy to get

$$\begin{aligned} \dot{V}_5 &= \dot{V}_3 + \dot{V}_4 + e_5\dot{e}_5 \\ &= -c_1e_1^2 - c_2e_2^2 - c_3e_3^2 - c_4e_4^2 - e_5^2 < 0, \end{aligned}$$

and system (5) and system (6) get lag synchronization.

4. Numerical simulations

In numerical simulations, set time delay $\tau = 0.5$ s, time span is $t = 20$ s; parameters of the complex system are $a = 11$, $b = 20$. Discretize the system with the Euler method and choose step length as $h = 0.0001$. Initial values of driving and response system are $(1+2\mathrm{j}, 3+4\mathrm{j}, 5)$ and $(6+16\mathrm{j}, 15+14\mathrm{j}, 30)$. In other words, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$ and $y_1 = 6$, $y_2 = 16$, $y_3 = 15$, $y_4 = 14$, $y_5 = 30$. Controllers are designed as $L_1 = -c_3 e_3 - a e_1 - A$ and $L_2 = -c_4 e_4 - a e_2 - B$. Synchronization diagrams of lag synchronization are shown in Fig. 3.

Error diagrams of lag synchronization are shown in Fig. 4.

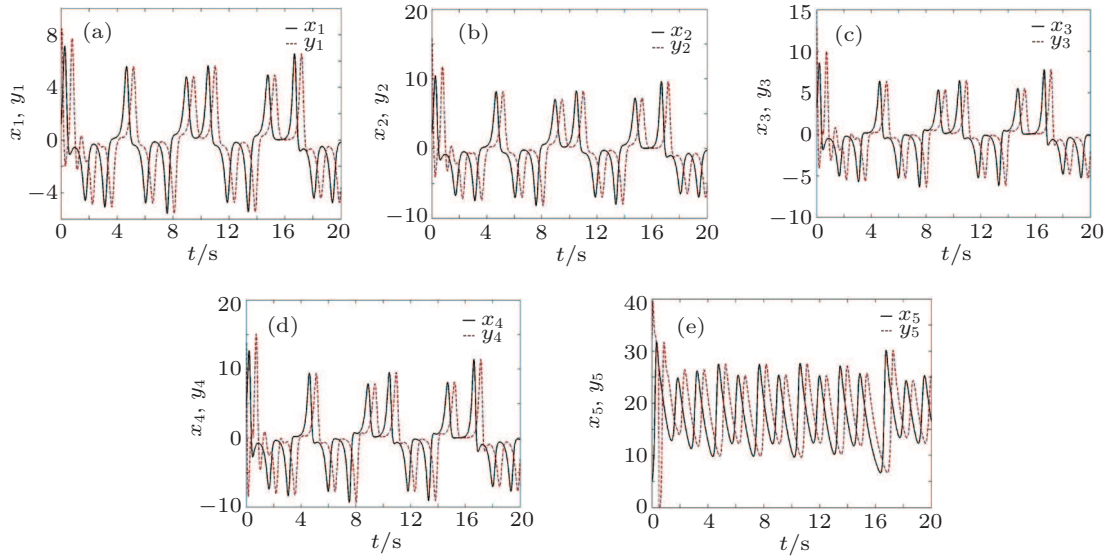


Fig. 3. (color online) Synchronization diagrams of complex system when $a = 11$, $b = 20$. (a) x_1, y_1 with time, (b) x_2, y_2 with time, (c) x_3, y_3 with time, (d) x_4, y_4 with time, (e) x_5, y_5 with time.

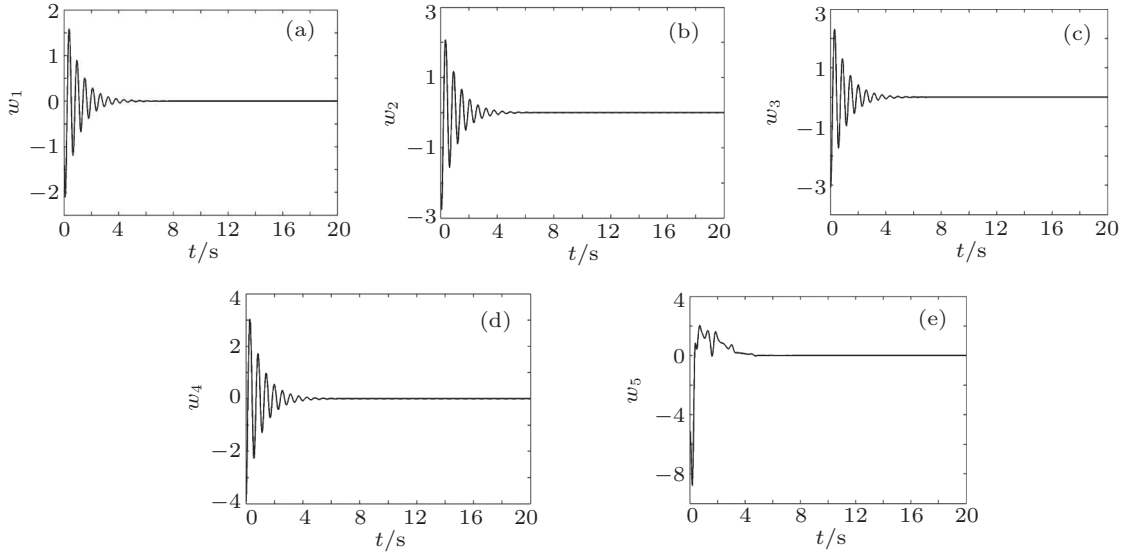


Fig. 4. Error diagrams of complex system when $a = 11$, $b = 20$. (a) w_1 with time, (b) w_2 with time, (c) w_3 with time, (d) w_4 with time, (e) w_5 with time.

From Fig. 3 we can see that systems get lag synchronization both in the real number part and the imaginary part with time delay $\tau = 0.5$ s. From Fig. 4, we find that errors converge to zero at 6 s and driving system and response system get lag synchronization.

5. Conclusion

In this paper, by replacing cross-coupled term with conjugate forms, we proposed a complex permanent magnet synchronous motor system and analyzed its properties. Based on the proposed system and taking time delay in practice into consideration, we designed the lag synchronization controllers with backstepping method because a complex permanent magnet synchronous motor system is partly strict feedback. Because the study of the complex system is just beginning and

it is easier to analyze a motor system with complex number current and complex number voltage, the problem of lag synchronization in complex motor systems is promising. In particular, controllers are designed respectively according to the real number part and imaginary part, so we just need to control a single variable in practice. Numerical simulation results demonstrate the validity of the controllers.

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